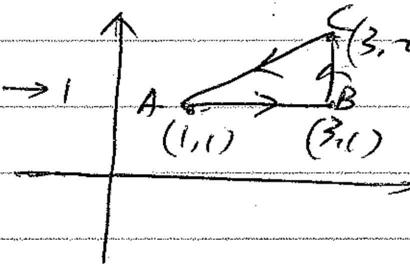
11/19/06

§ Worked Example : Ex 9.8 #33.

$$\vec{F}(x,y) = (x+2y)\vec{i} + (6y-2x)\vec{j}$$ acting

counterclockwise around the triangle with vertices $(1,1)$, $(3,1)$, & $(3,2)$

<u>Method 1</u>: by parametric representation



A → B: $\vec{r}_1 = (1+2t)\vec{i} + \vec{j}$; $t: 0 \to 1$

$\Rightarrow x = 1+2t$

$y = 1$

$$\vec{F}_1 = (x+2y)\vec{i} + (6y-2x)\vec{j}$$
$$= ((1+2t+2)\vec{i} + (6-2(1+2t))\vec{j}$$
$$= (3+2t)\vec{i} + (4-4t)\vec{j}$$

$$\therefore \int_A^B \vec{F}_1 \cdot d\vec{r}_1 = \int_0^1 (3+2t)\,2\,dt = \int_0^1 (6+4t)\,dt$$

$$[\, d\vec{r}_1 = 2\,dt\,\vec{i} + 0\,\vec{j} \,]$$

$$= 6t + 2t^2 \Big|_0^1 = \boxed{8}$$

B → C : $\vec{r}_2 = 3\vec{i} + (1+t)\vec{j}$

$x = 3$; $y = 1+t$ $t = 0 \to 1$

$d\vec{r}_2 = 0\vec{i} + dt\,\vec{j}$

$$\vec{F_2} = (x+2y)\vec{i} + (6y-2x)\vec{j}$$

$$= (3+2(1+t))\vec{i} + (6(1+t) - 2(3))\vec{j}$$

$$= (5+2t)\vec{i} + 6t\,\vec{j}$$

$$\int_B^C \vec{F_2}\cdot d\vec{r_2} = \int_0^1 (6t)\,dt = 3t^2\Big|_0^1 = \boxed{3}$$

$C \longrightarrow A$: $\vec{r_3} = (3-2t)\vec{i} + (2-t)\vec{j}$

$$\left.\begin{array}{l} x = 3-2t \\ y = 2-t \end{array}\right\} \quad t = 0 \longrightarrow 1$$

$$d\vec{r_3} = -2\,dt\,\vec{i} - dt\,\vec{j}$$

$$\vec{F_3} = (x+2y)\vec{i} + (6y-2x)\vec{j}$$

$$= ((3-2t) + 2(2-t))\vec{i} + (6(2-t) - 2(3-2t))\vec{j}$$

$$= (7-4t)\vec{i} + (6-2t)\vec{j}$$

$$\int_C^A \vec{F_3}\cdot d\vec{r_3} = \int_0^1 (7-4t)(-2dt) + (6-2t)(-dt)$$

$$= \int_0^1 (-20+10t)\,dt = -20t + 5t^2$$

$$= \boxed{-15}$$

$$\therefore \int_C \vec{F}\cdot d\vec{r} = \sum = \int_A^B \vec{F_1}\cdot d\vec{r_1} + \int_B^C \vec{F_2}\cdot d\vec{r_2}$$

$$+ \int_C^A \vec{F_3}\cdot d\vec{r_3}$$

$$= 8 + 3 - 15 = \boxed{-4}$$

<u>Method 2</u>:

$A \rightarrow B$: $\quad y = 1 \; ; dy = 0$

$$\int_A^B \vec{F_1}\cdot d\vec{r_1} = \int (x+2\underset{1}{y})\,dx + (6y-2x)\,\cancel{dy}^{\,0}$$

$$= \int_1^3 (x+2)\,dx = \left.\frac{(x+2)^2}{2}\right|_1^3$$

$$= \frac{5^2-3^2}{2} = \boxed{8.}$$

$B \rightarrow C$: $\quad x = 3 \; ; \; dx = 0$

$$\int_B^C \vec{F_2}\cdot d\vec{r_2} = \int (x+2y)\,\cancel{dx}^{\,0} + (6y-2\underset{3}{x})\,dy$$

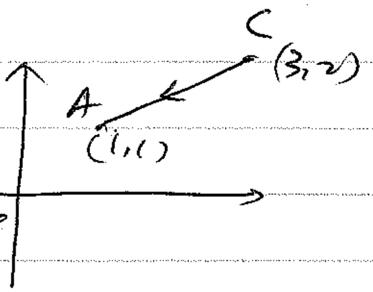
$$= \int_1^2 (6y-6)\,dy = \left. 3y^2 - 6y \right|_1^2$$

$$= (12-12) - (3-6)$$

$$= \boxed{3}$$

$C \longrightarrow A$: · Both x & y are varying.

· Also the path is not a simple horizontal or vertical line.



Eq^n : $\dfrac{y-1}{x-1} = \dfrac{2-1}{3-1} = \dfrac{1}{2}$

$\Rightarrow \boxed{x = 2y - 1}$ for $\overline{CA}$

Also $\boxed{dx = 2dy}$

$$\int_C^A \vec{F}_3 \cdot d\vec{r}_3 = \int (x+2y)dx + (6y-2x)dy$$

$$= \int_2^1 \left((2y-1)+2y\right)2dy + \left(6y - 2(2y-1)\right)dy$$

$$= \int_2^1 \left((8y-2) + (2y+2)\right)dy$$

$$= \int_2^1 10y\,dy = 5y^2 \Big|_2^1 = \boxed{-15}$$

Note:

The partial integrals are the same as in method as it should. $\therefore \int_C \vec{F} \cdot d\vec{r} = 8 + 3 - 15$
$= \boxed{-4}$