MAE 3360 ENGINEERING ANALYSIS

SPRING 2007

DEPARTMENT OF MECHANICAL
AND
AEROSPACE ENGINEERING

UNIVERSITY OF TEXAS AT ARLINGTON

Exam #2

CLOSED BOOKS and NO PROGRAMABLE CALCULATORS
(Laplace transform table is provided)

April 13, 2007
Time Limit : 50 min

(This exam has 6 pages)

Show all your intermediate work
Illegible writings or incomplete explanations
will result in loss of points

LAST NAME : Sol^n ____________

FIRST NAME : ____________

1.(30 pts) Find the general solution by the variation of parameters method to: (Lecture)

$$x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=\frac{21}{x^4}$$

homogeneous sol^{n}:

$$x^2y''-4xy'+6y=0$$

use $y=x^m$ - equidimensional

$$m(m-1)-4m+6=0 \Rightarrow m^2-5m+6=0$$
$$\Rightarrow (m-2)(m-3)=0$$
$$\Rightarrow m=2 \text{ or } 3$$

$\therefore$ $y_1=x^2$; $y_2=x^3$

Variation of parameter:

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}=\begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix}=3x^4-2x^4=x^4$$

$$r=\frac{21}{x^6} \quad (\text{note: } r \neq \frac{21}{x^4})$$

$$u'=\frac{-ry_2}{W}=-\frac{\frac{21}{x^6}x^3}{x^4}=-21x^{-7}$$

$$\Rightarrow u=\frac{21}{6}x^{-6}=3\tfrac{1}{2}x^{-6}$$

$$v'=\frac{ry_1}{W}=\frac{(21x^{-6})(x^2)}{x^4}=21x^{-8}$$

$$\Rightarrow v=\frac{-21}{7}x^{-7}=-3x^{-7}$$

$$\therefore y_p=uy_1+vy_2=3\tfrac{1}{2}x^{-6}x^2+(-3)x^{-7}x^3$$
$$=\boxed{\tfrac{1}{2}x^{-4}}$$

2a.(10 pts) Find the Laplace transform of : $x \cos 2x$ (H.W.#7)

$$= -\frac{d}{ds}\mathcal{L}(\cos 2x) = -\frac{d}{ds}\left(\frac{s}{s^2+4}\right)$$

$$= \frac{-[(s^2+4) - s(2s)]}{(s^2+4)^2} = \frac{s^2-4}{(s^2+4)^2}$$

2b.(10 pt) Find the inverse Laplace transform of $\frac{1}{s^3+5s}$ (H.W#7)

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3+5s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s^2+5)}\right\}$$

partial fraction: $\frac{1}{s(s^2+5)} = \frac{C_1}{s} + \frac{C_2 s + C_3}{s^2+5}$

$$C_1 = \left.\frac{1}{s^2+5}\right|_{s=0} = \frac{1}{5}$$

$$\frac{1}{s(s^2+5)} - \frac{1}{5s} = \frac{C_2 s + C_3}{s^2+5}$$

$$\frac{5-(s^2+5)}{5s(s^2+5)} = \frac{-s^2}{5s(s^2+5)} \Rightarrow C_2 = -\frac{1}{5}, \quad C_3 = 0$$

$$\therefore \frac{1}{s(s^2+5)} = \frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}s}{s^2+5}$$

$$\Downarrow$$

$$\frac{1}{5} \qquad \frac{1}{5}\cos\sqrt{5}t$$

$$\boxed{\therefore \mathcal{L}^{-1}\left\{\frac{1}{s^3+5s}\right\} = \frac{1}{5} - \frac{1}{5}\cos\sqrt{5}t}$$

2c.(10 pts) Find the inverse Laplace transform of $\dfrac{s}{(s-3)(s-6)(s-2)}$ (H.W. #7)

$$\frac{C_1}{(s-3)} + \frac{C_2}{(s-6)} + \frac{C_3}{(s-2)}$$

$$C_1 = \frac{s}{(s-6)(s-2)}\Bigg|_{s=3} = \frac{3}{(-3)(1)} = -1$$

$$C_2 = \frac{s}{(s-3)(s-2)}\Bigg|_{s=6} = \frac{6}{(3)(4)} = \frac{1}{2}$$

$$C_3 = \frac{s}{(s-3)(s-6)}\Bigg|_{s=2} = \frac{2}{(-1)(-4)} = \frac{1}{2}$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{s}{(s-3)(s-6)(s-2)}\right\} = \boxed{-e^{3t} + \frac{1}{2}e^{6t} + \frac{1}{2}e^{2t}}$$

H.W. #7
text book
#35

3.(20 pts) Find the solution y(t) using the Laplace transform method to:

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4y=0 \quad \text{with } y(0)=1 \ \textit{and} \ \frac{dy}{dt}(0)=0.$$

$$[s^2\tilde{y}-s\overset{1}{y(0)}-\overset{0}{y'(0)}]+5[s\tilde{y}-\overset{1}{y(0)}]+4\tilde{y}=0$$

$$(s^2+5s+4)\tilde{y}-s-\cancel{4}5=0$$

$$\Rightarrow \tilde{y}=\frac{s+5}{s^2+5s+4}=\frac{(s+5)}{(s+1)(s+4)}$$

$$=\frac{C_1}{(s+1)}+\frac{C_2}{(s+4)}$$

$$C_1=\left(\frac{s+5}{s+4}\right)_{s=-1}=\frac{4}{3} \ ; \quad C_2=\left(\frac{s+5}{s+1}\right)_{s=-4}=\frac{1}{-3}$$

$$\therefore \tilde{y}=\frac{4/3}{s+1}-\frac{1/3}{s+4}$$

$$\Rightarrow \boxed{y=\frac{4}{3}e^{-t}-\frac{1}{3}e^{-4t}}$$

4.(20pts)Using the Euler's method, with Δt=0.1, determine the solution of y, up to the third decimal point, at t=0.2 for the following initial value problem:

$$\frac{d^2y}{dt^2}+t\frac{dy}{dt}+2y=0 \quad ; \quad y(0)=1, \ \frac{dy}{dt}(0)=3$$

Show the details of the intermediate computations and enter the values of y in the table below.

t	0.0	0.1	0.2
y	1.000	1.300	1.580

$y' = u \qquad ; \ y(0) = 1$

$u' = -tu - 2y \qquad ; \ u(0) = 3$

$t_0 = 0; \ y_0 = 1, \ u_0 = 3, \ \Delta t = 0.1$

$y_1 = y_0 + \Delta t\, u_0 = 1 + (0.1)\,3 = 1.300$

$u_1 = u_0 + \Delta t\,(-t_0 u_0 - 2y_0)$

$= 3 + 0.1\,(-0 - 2(1)) = 2.800$

$y_2 = y_1 + \Delta t\, u_1 = 1.300 + (0.1)(2.800) = 1.580$

$u_2 = u_1 + \Delta t(-t_1 u_1 - 2y_1) = 2.800 + (0.1)\,(-(0.1)(2.800) - 2(1.3))$

$= 2.642$

(Note, u_2 is actually not needed)

4.(20pts)Using the Euler's method, with Δt=0.1, determine the solution of y, up to the third decimal point, at t=0.2 for the following initial value problem:

$$\frac{d^2y}{dt^2}+t\frac{dy}{dt}+y=0 \quad ; \quad y(0)=1, \ \frac{dy}{dt}(0)=2$$

Show the details of the intermediate computations and enter the values of y in the table below.

t	0.0	0.1	0.2
y	1.000	1.200	1.390

$y' = u \quad ; \quad y(0) = 1 = y_0$

$u' = -tu - y \quad ; \quad u(0) = 2 = u_0$

$y_1 = y_0 + \Delta t\, u_0 = 1 + (0.1)(2) = \boxed{1.200}$

$u_1 = u_0 + \Delta t(-t_0 u_0 - y_0) = 2 + (0.1)(0 - 1) = 1.900$

$y_2 = y_1 + \Delta t\, u_1 = 1.200 + (0.1)(1.900) = \boxed{1.390}$

$u_2 = \text{-----}$ (Not needed)