SI Session

Exam - Review - 1

MAE 3360.

02/25/2008.

* Introduction to Ordinary Differential Equation:-

First order Homogeneous ODE:-

⇒ Exact Differential Method:-

For given ODE as

$F(x,y)\,dx + G(x,y)\,dy = 0$ the condition for exact differential is

$$\frac{\partial F}{\partial y} = \frac{\partial G}{\partial x}$$

Ex:- $(x + \sin y)\,dx + (x\cos y - 2y)\,dy = 0$

→ Now, $F(x,y) = x + \sin y$

$G(x,y) = x\cos y - 2y$

$$\frac{\partial F}{\partial y} = \cos y \quad \equiv \quad \frac{\partial G}{\partial x} = \cos y$$

∴ The given ODE is exact differential equation.

Now, $U(x,y) = \int (x + \sin y)\,dx + f(y)$

$$= \frac{x^2}{2} + x\sin y + f(y) \qquad \text{—(i)}$$

$$U(x,y) = \int (x\cos y - 2y)\,dy + g(x)$$

$$= x\sin y - y^2 + g(x) \qquad \text{—(ii)}$$

$$\boxed{\therefore y = \pm\sqrt{e^{x}(x-1)+c}}$$ ← solution

* **Bernoulli's Equation:-**

If the ODE is of form

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^{n} \quad ; \quad n \neq 0, 1$$

then use the substitution as

$$v = y^{1-n}$$

and solve the ODE in terms of v and finally substitute $v = y^{1-n}$ to get answer in terms of y.

* **Homogeneous Equations / Substitution Method:-**

If the given 1st order ODE is Homogeneous to n^{th} degree (n = const) then use the substitution of ($x = uy$) or ($y = vx$) and solve the equation.

→ For example refer to the SI notes on 04th Feb 2008.

$\therefore \quad \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$

$\therefore \quad \frac{d}{dx}\left(\frac{y}{x}\right) = 0$

Now Integrating both sides,

$y/x = c$

$\Rightarrow \boxed{y = cx}$

* <u>Separation of variable Method:</u>

If the given ODE is of

$$F(x,y)\,dx + G(x,y)\,dy = 0$$

$$F(x,y) = F_1(x)\,F_2(y)$$
$$G(x,y) = G_1(x)\,G_2(y)$$

then,
$$\frac{F_1(x)}{G_1(x)}\,dx + \frac{G_2(y)}{F_2(y)}\,dy = 0$$

Now Integrating both sides we can get the solution to it

ex: $y' = \frac{xe^x}{2y}$

$\rightarrow \quad \frac{dy}{dx} = \frac{xe^x}{2y}$

using separation of variables method,

$$2y\,dy = xe^x\,dx$$

Now Integrating both sides,

$$\int 2y\,dy = \int xe^x\,dx + c$$

$$\therefore \quad y^2 = xe^x - e^x + c$$

Now, comparing (i) & (ii) we have,

$$f(y) = -y^2 \quad \& \quad g(x) = x^2/2$$

$$\therefore U(x,y) = \frac{x^2}{2} + x \sin y - y^2$$

Now, solution is $U(x,y) = c$

$$\boxed{\therefore \frac{x^2}{2} + x \sin y - y^2 = c}$$

* Integrating Factor method:

If the given ODE is not exact and of the form

$$\frac{dy}{dx} + a_1(x)\, y = f(x)$$

Find our Integrating Factor,

$$I.F. = P(x) = e^{\int a_1(x)\, dx}$$

Now multiply with integrating factor to the given equation.

$$P(x)\frac{dy}{dx} + P(x)\, a_1(x)\, y = P(x)\, f(x)$$

ex: $y\,dx - x\,dy = 0$

$\rightarrow$ Now, $\frac{\partial F}{\partial y} \neq \frac{\partial G}{\partial x}$

$\therefore$ It is non-exact differential.

So, Now, $-x\frac{dy}{dx} + y = 0$

$$\therefore \frac{dy}{dx} - \frac{y}{x} = 0$$

Now, $P(x) = e^{\int -1/x\, dx} = e^{-\ln x} = x^{-1} = 1/x$