MAE 3360 ENGINEERING ANALYSIS

Spring 2008

DEPARTMENT OF MECHANICAL
AND
AEROSPACE ENGINEERING

UNIVERSITY OF TEXAS AT ARLINGTON

Exam #1

CLOSED BOOK and NO CALCULATORS

February 29, 2008
Time Limit : 50 min

(This exam has 7 pages)

LAST NAME : Sol^{n}

FIRST NAME : ____________________

SIGNATURE : ____________________

1a.(15 pts) Determine the general solution to: $(xy + y^2)dx - x^2 dy = 0$

Homogeneous to order 2

Let $y = ux$

$\Rightarrow dy = u\,dx + x\,du$

D.E. becomes:

$$(x\,ux + (ux)^2)\,dx - x^2(u\,dx + x\,du) = 0$$

$$\Rightarrow \quad \frac{dx}{x} = \frac{du}{u^2}$$

$$\Rightarrow \quad \boxed{\ln x = -\frac{x}{y} + C}$$

1b.(15 pts) Determine the general solution to: $\underset{M}{(x+y)^2}\,dx + \underset{N}{(2xy + x^2 - 1)}\,dy = 0$

$$\left.\begin{array}{l}\dfrac{\partial M}{\partial y} = 2x+2y \\ \dfrac{\partial N}{\partial x} = 2x+2y\end{array}\right\} \Rightarrow \text{Exact Differential}$$

&

$$M = \frac{\partial \phi}{\partial x}\ ;\ N = \frac{\partial \phi}{\partial y}$$

with $\phi = c$

(i) $\dfrac{\partial \phi}{\partial x} = M = \cancel{2x+2y}\ (x+y)^2 \ \cancel{\Rightarrow \phi = x^2} =$

$= x^2 + 2xy + y^2$

$\Rightarrow \phi = \dfrac{x^3}{3} + x^2y + xy^2 + f_1(y)$ ①

(ii) $\dfrac{\partial \phi}{\partial y} = N = 2xy + x^2 - 1$

$\Rightarrow \phi = xy^2 + x^2y - y + f_2(x)$ ②

Comparing ① & ② gives: $f_1(y) = -y$

& $f_2(x) = \dfrac{x^3}{3}$

which gives:

$$\boxed{xy^2 + x^2y - y + \frac{x^3}{3} = c}$$

1c.(15 pts) Determine the solution to: $x\frac{dy}{dx}+y=e^x$; $y(1)=2$

$$\frac{dy}{dx}+\frac{1}{x}y=\frac{1}{x}e^x$$

Integrating factor $= e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$$\Rightarrow \quad x\frac{dy}{dx}+y = e^x$$

$$\Rightarrow \quad \frac{d}{dx}(xy) = e^x$$

$$\Rightarrow \quad xy = \int e^x\,dx + c$$

$$= e^x + c$$

$$\Rightarrow \quad \boxed{y = \frac{e^x}{x} + \frac{c}{x}}$$

Apply I.C.

$$y(1) = 2$$

$$2 = \frac{e}{1} + \frac{c}{1} \Rightarrow c = 2-e$$

$$\Rightarrow \quad \boxed{y = \frac{e^x}{x} + \frac{2-e}{x}}$$

2.(25 pts) Determine the general solution to:

(a) $4\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ $\quad y=e^{mx} \Rightarrow 4m^2+m=0 \Rightarrow m=0,-\frac{1}{4}$

$\therefore \boxed{y=C_1+C_2e^{-\frac{1}{4}x}}$

(b) $\frac{d^2y}{dx^2}-10\frac{dy}{dx}+25y=0$ $\quad y=e^{mx} \Rightarrow m^2-10m+25=0$

$\Rightarrow (m-5)^2=0 \quad m=5$ (double root)

$\therefore \boxed{y=e^{5x}(C_1+C_2x)}$

(c) $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=0$ $\quad y=x^m \Rightarrow m(m-1)+4m+2=0$

$\Rightarrow m^2+3m+2=0$

$\Rightarrow (m+1)(m+2)=0$

$\Rightarrow m=-1,-2$

$\therefore \boxed{y=\frac{C_1}{x}+\frac{C_2}{x^2}}$

(d) $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=0$ $\quad y=x^m \Rightarrow m(m-1)+3m+1=0$

$\Rightarrow m^2+2m+1=0$

$\Rightarrow (m+1)^2=0$

$\Rightarrow m=-1$ (double root)

$\therefore \boxed{y=\frac{1}{x}(C_1+C_2\ln x)}$

(e) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+5y=0$ $\quad y=e^{mx} \Rightarrow m^2-4m+5=0$

$\Rightarrow m=\frac{4\pm\sqrt{4^2-(4)(5)}}{2}$

$=2\pm i$

$\therefore \boxed{y=e^{2x}(C_1\cos x+C_2\sin x)}$

3.(30pts) Write down the appropriate trial form of the non-homogeneous (particular) solution for the following problems. You **do not** have to determine the coefficients in your proposed solution form.

(a) $$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=e^{2x}(\cos x-3\sin x)$$

y_h : $e^{mx} \Rightarrow m^2-2m+2=0 \Rightarrow m=\dfrac{2\pm\sqrt{4-(4)(2)}}{2}$

$= 1 \pm i$

$\therefore y_h = e^{x}(C_1\cos x + C_2 \sin x)$

$y_p = e^{2x}(A\cos x + B\sin x)$ No correction needed.

(b) $$\frac{d^2y}{dx^2}+4y=3\sin 2x$$

y_h : $e^{mx} \Rightarrow m^2+4=0 \Rightarrow m=\pm 2i$

$\Rightarrow y_h = C_1\cos 2x + C_2 \sin 2x$

$y_p = x(A\cos 2x + B\sin 2x)$ correction needed.

(c) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=2x-5$

$y_h: e^{mx} \Rightarrow m^2+m=0 \Rightarrow m(m+1)=0$

$\Rightarrow m=0, -1$

$\therefore y_h = c_1 + c_2 e^{-x}$

$\boxed{y_p = A_1 x^2 + A_2 x}$ correction needed

(d) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=2x^2-3xe^x$

$y_h: m^2-2m+5=0 \Rightarrow m=\frac{2\pm\sqrt{4-(4)(5)}}{2}$

$= 1 \pm 2i$

$\Rightarrow y_h = e^x(c_1\cos 2x + c_2 \sin 2x)$

$\boxed{y_p = A_1 x^2 + A_2 x + A_3 + (A_4 x + A_5)e^x}$

No correction needed