MAE 3360 ENGINEERING ANALYSIS

Fall 2006

DEPARTMENT OF MECHANICAL
AND
AEROSPACE ENGINEERING

UNIVERSITY OF TEXAS AT ARLINGTON

Exam #1

CLOSED BOOK and NO CALCULATORS

Oct 6, 2006
Time Limit : 50 min

(This exam has 6 pages)

LAST NAME : Sol^N^

FIRST NAME : ____________________

1.(10 pt) TRUE OR FALSE

a correct answer scores 2 points, there is no penalty for incorrect response. Note that if a statement would only be true under certain conditions (conditionally true), then it is false.

*Write "T" for true statement and "F" for false statement.

T $xy' + y = \sin x$ is a linear first order ODE.

T $f(x,y) = x^6 + 2x^2y^4 + xy^5$ is a homogeneous function.

T $x^2y'' + xy' + 3y = \sin x$ is a linear ODE.

F $y' = x\sin y + e^x$ is a first order linear ODE.

F $y' + \frac{x}{y} = 0$ is a linear first order ODE.

2.(15 pts) Determine the general solution to : $x\frac{dy}{dx}+y=e^{x}$

$$\frac{dy}{dx}+\frac{1}{x}y=\frac{1}{x}e^{x}$$

Integrating factor: $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$$\therefore \quad x\frac{dy}{dx}+y=e^{x}$$

$$\Rightarrow \quad \frac{d}{dx}(xy)=e^{x}$$

$$\Rightarrow \quad xy=\int e^{x}dx + C$$

$$\Rightarrow \quad \boxed{y=\frac{1}{x}e^{x}+\frac{C}{x}}$$

3.(15 pts) Determine the general solution to : $(x - y^3 + y^2 \sin x)dx = (3xy^2 + 2y\cos x)dy$

Check for Exactness : M ... $-N$

$$M = x - y^3 + y^2 \sin x \Rightarrow \frac{\partial M}{\partial y} = -3y^2 + 2y \sin x$$

$$-N = 3xy^2 + 2y \cos x \Rightarrow \frac{\partial N}{\partial x} = -3y^2 + 2y \sin x$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{Exact Differential}$$

$$M = \frac{\partial U}{\partial x} \Rightarrow U = \int (x - y^3 + y^2 \sin x)\,dx + f_1(y)$$

$$= \frac{x^2}{2} - xy^3 - y^2 \cos x + f_1(y) \quad ①$$

$$N = \frac{\partial U}{\partial y} \Rightarrow U = \int (-3xy^2 - 2y\cos x)\,dy + f_2(x)$$

$$= -xy^3 - y^2 \cos x + f_2(x) \quad ②$$

Comparing ① & ② gives : $f_2(x) = x^2/2$ & $f_1(y) = C_1$

or $U = \boxed{\frac{x^2}{2} - y^3 x - y^2 \cos x = C}$

Alternatively :

From ①: $U = \frac{x^2}{2} - xy^3 - y^2 \cos x + f_1(y)$

$$\Rightarrow \frac{\partial U}{\partial y} = -3y^2 x + 2y \cos x + f_1'(y)$$

Comparing with N gives: $f_1'(y) = 0$ or $f_1(y) = C_1$

$$\therefore \boxed{\frac{x^2}{2} - xy^3 - y^2 \cos x = C}$$

4a.(10 pts) Determine the general solution to : $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+25y=0$

$m^2+10m+25=0 \Rightarrow (m+5)^2=0$

$m=-5$ (double root)

$\therefore \boxed{y = A_1 e^{-5x} + A_2 x e^{-5x}}$

4b.(10 pts) Determine the general solution to : $2\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$

$2m^2+2m+1=0$

$m = \frac{-2 \pm \sqrt{2^2-(4)(2)(1)}}{4} = -\frac{1}{2} \pm \frac{1}{2}i$

$\therefore \boxed{y = e^{-\frac{1}{2}x}\left(C_1 \cos\frac{x}{2} + C_2 \sin\frac{x}{2}\right)}$

4c.(10 pts) Determine the general solution to : $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=0$

Equidimensional : $y=x^m$

$m(m-1)+3m+1=0 \Rightarrow m^2+2m+1=0$

$\Rightarrow (m+1)^2=0$

$\therefore m=-1$ (double root)

$\therefore \boxed{y = C_1 x^{-1} + C_2 x^{-1}\ln x}$

or $= \boxed{\frac{C_1 + C_2 \ln x}{x}}$

5.(30pts) Write down the appropriate trial form of the non-homogeneous solution for the following problems. **DO NOT** solve for the coefficients in your proposed solution.

(a) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=x+1+e^{-2x}$

y_h: $m^2+2m=0 \Rightarrow m(m+2)=0 \Rightarrow m=0, or -2$

$\therefore y = C_1 + C_2 e^{-2x}$

y_p: $\boxed{y_p = A_1 x^2 + A_2 x + A_3 x e^{-2x}}$

(b) $\frac{d^2y}{dx^2}+y=2e^{3x}+3\sin x$

y_h: $m^2+1=0 \Rightarrow m=\pm i$

$y_h = C_1 \cos x + C_2 \sin x$

y_p: $\boxed{y_p = A_1 e^{3x} + A_2 x\cos x + A_3 x \sin x}$

(c) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x$

y_h: $m^2-2m+1=0 \Rightarrow (m-1)^2=0 \Rightarrow m=1$ double root

$\therefore y_h = C_1 e^x + C_2 x e^x$

y_p: $\boxed{y_p = A x^2 e^x}$