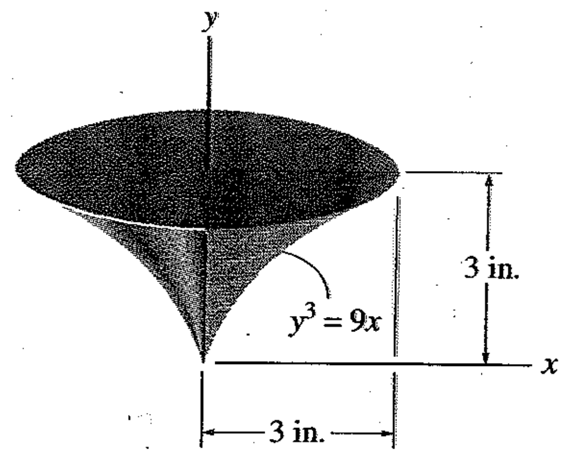
Problem 17.5 (Page 387)

A solid is formed by revolving the shaded area completely around the y-axis as shown. Determine the radius of gyration, k_y for this figure generated. The specific weight of the material $\gamma = 380$ lb/ft^3.

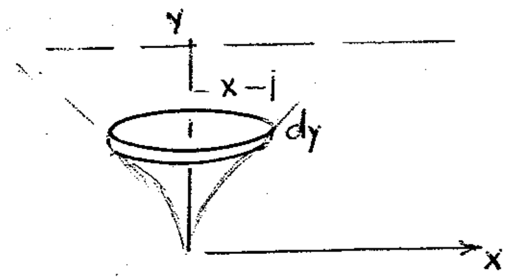


$$dI_y = \frac{1}{2}\, dm\, x^2$$

$$= \frac{1}{2}(\pi x^2 \rho\, dy)\, x^2$$

$$= \frac{1}{2}\pi \rho x^4\, dy$$

$$= \frac{1}{2(9)^4}\rho \pi y^{12}\, dy$$



$$I_y = \int dI_y = \frac{\rho \pi}{2(9)^4}\int_0^3 y^{12}\, dy$$

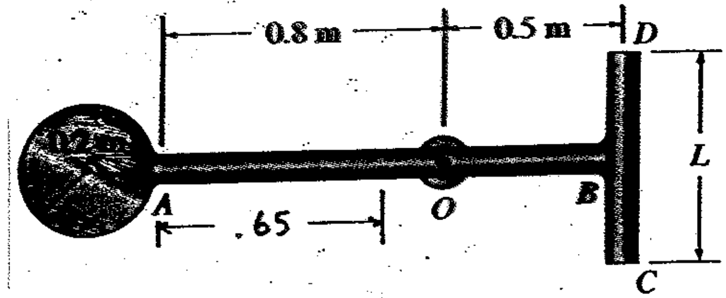
$$= 29.632\, \rho$$

$$m = \int dm = \frac{1}{81}\rho \pi \int_0^3 y^6\, dy = 12.117\, \rho$$

$$K_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{29.632\, \rho}{12.117\, \rho}} = 1.56 \text{ in.}$$

Problem 17.14 (Page 389)

The assembly shown consists of a disk having a mass of 6 kg, and two slender rods, AB and DC, each having a mass of 2 kg/m. If L = 0.75 m, determine the moment of inertia of the assembly about an axis perpendicular to the screen and passing through O.



$$I_o = I_{o_{disk}} + I_{o_{AB}} + I_{o_{CD}}$$

$$I_{o_{disk}} = \frac{1}{2}(6)(.2)^2 + 6(1)^2 = 6.12 \text{ Kg}\cdot\text{m}^2$$

$$I_{o_{AB}} = \frac{1}{12}\underbrace{(2)(1.3)}_{m_{AB}}(1.3)^2 + 2(1.3)(.15)^2 = .425 \text{ Kg m}^2$$

$$I_{o_{CD}} = \frac{1}{12}\underbrace{(2)(.75)}_{m_{CD}}(.75)^2 + 2(.75)(.5)^2 = .445 \text{ Kg m}^2$$

$$I_o = 6.12 + .425 + .445 = \underline{6.99 \text{ Kg m}^2}$$

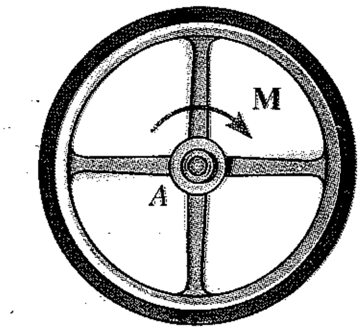
$$\text{Total mass} = 6 + 2(1.3) + 2(.75) = 10.1 \text{ Kg}$$

$$I_o = K^2 m = 6.99 = 10.1 K^2$$

$\underline{K = .832 \text{ m}}$ (Radius of Gyration)

Problem 17.54 (Page 414)

The 10-kg wheel shown has a radius of gyration k_A = 200 mm. If the wheel is subjected to a moment M = 5 t (N·m), where t is in seconds, determine its angular velocity when t = 3 sec starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.



$$\overset{+}{\rightarrow} \sum F_x = A_x = 0$$

$$+\uparrow \sum F_y = A_y - 10(9.81) = 0$$

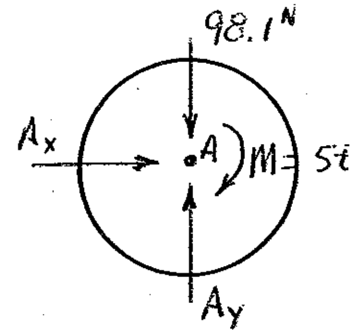
$$A_y = 98.1^{N}$$

$$\circlearrowleft + \sum M_A = 5t = I_A \alpha$$

$$\alpha = \frac{d\omega}{dt} = 12.5t$$

$$\omega = \int_0^3 12.5t\,dt = \frac{12.5}{2}(3)^2$$

$$= 56.2 \text{ rad/sec}$$

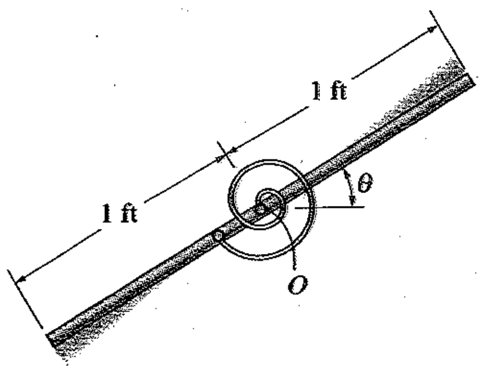


$$I_A = 10(.2)^2$$

$$= 0.4 \text{ Kg m}^2$$

Problem 17.59 (Page 415)

A 10-lb bar is pinned at its center O as shown and is connected to a torsional spring. The spring has a stiffness $k = 5$ ft-lb/rad, providing a torque $M = 5\,\theta$ (ft-lb), where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.



$$\circlearrowleft + \sum M_o = -5\theta = \frac{1}{12}\left(\frac{10}{32.174}\right)(2)^2 \alpha$$

$$-48.3\,\theta = \alpha = \omega \frac{d\omega}{d\theta}$$

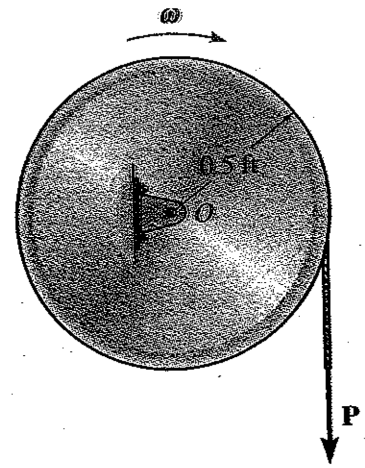
$$-\int_{\pi/2}^{0} 48.3\,\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

$$\frac{48.3}{2}\left(\frac{\pi}{2}\right)^2 = \frac{1}{2}\omega^2$$

$$\omega = 10.9 \text{ rad/sec} \circlearrowright$$

Problem 17.56 (Page 414)

The drum shown has a weight of 80 lbs with a radius of gyration $k_o = 0.4$ ft. A cable is wrapped around the drum and is subjected to a vertical force P = 15 lbs. Determine the time needed to increase the drum's angular velocity from $\omega_1 = 5$ rad/sec to $\omega_2 = 25$ rad/sec. Neglect the mass of the cable.



$$\circlearrowright + \sum M_o = I_o \alpha$$

$$15(.5) = \frac{80}{32.174}(.4)^2 \alpha$$

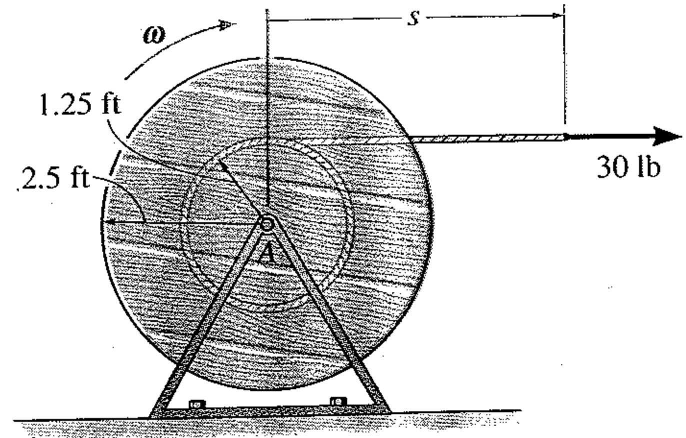
$$\alpha = 18.87 \text{ rad/sec}^2 = \frac{d\omega}{dt}$$

$$\omega_2 = \omega_1 + \alpha t$$

$$25 = 5 + 18.87t \qquad t = 1.06 \text{ sec}$$

Problem 17.58 (Page 415)

A cord is wrapped around the inner core of a spool. If the cord is pulled with a constant tension of 30 lbs and the spool is originally at rest, determine the spool's angular velocity when s = 8 ft of cord has unwound. Neglect the weight of the 8-ft portion of cord. The spool and the entire cord have a total weight of 400 lbs, and the radius of gyration about the axle is $k_A = 1.3$ ft.



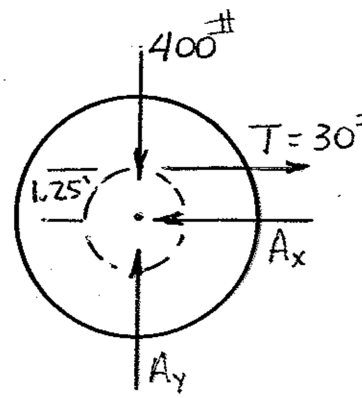
$$I_A = m k_A^2 = \frac{400}{32.2}(1.3)^2$$

$$= 20.99 \text{ slug ft}^2$$

$$\circlearrowleft + \sum M_A = I_A \alpha$$

$$= 30(1.25) = 20.99\,\alpha$$

$$\alpha = 1.786 \text{ r/s}^2$$



$$\theta = \frac{s}{r} = \frac{8}{1.25} = 6.4 \text{ rad}$$

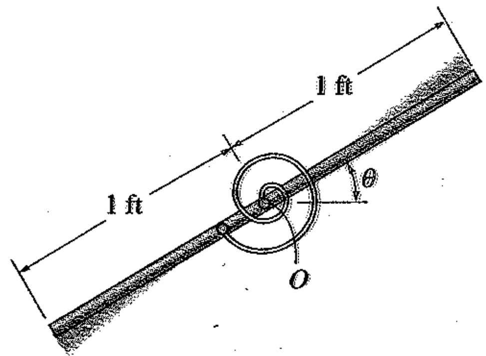
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$= 0 + 2(1.786)(6.4 - 0)$$

$$\omega = 4.78 \text{ r/s}$$

Problem 17.60 (Page 415)

A 10-lb bar is pinned at its center O as shown and is connected to a torsional spring. The spring has a stiffness $k = 5$ ft-lb/rad, providing a torque $M = 5\,\theta$ (ft-lb), where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 45°$.



$$\circlearrowleft + \sum M_O = -5\theta = \frac{1}{12}\left(\frac{10}{32.174}\right) 2^2 \alpha$$

$$\alpha = -48.3\,\theta = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$$

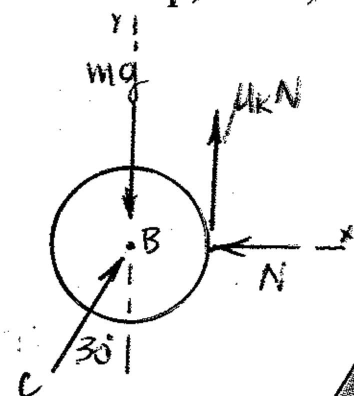
$$-\int_{\pi/2}^{\pi/4} 48.3\,\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

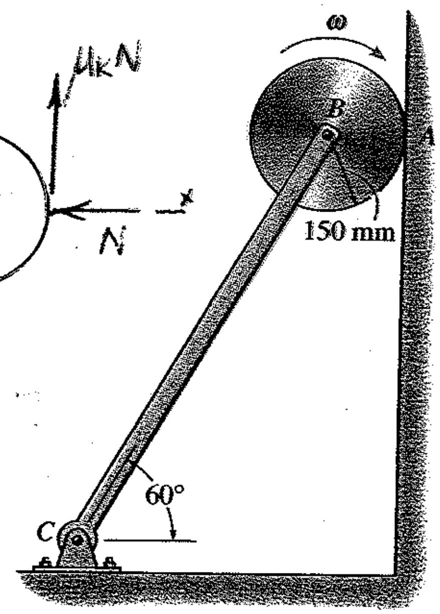
$$-24.15\left[\left(\frac{\pi}{4}\right)^2 - \left(\frac{\pi}{2}\right)^2\right] = \frac{1}{2}\omega^2$$

$$\omega = 9.45 \text{ rad/sec}$$

Problem 17.73 (Page 418)

The disk shown has a mass of 20 kg and is originally spinning at the end of a strut with an angular velocity of $\omega = 60$ rad/sec. If it is then placed against the wall that has a coefficient of kinetic friction $\mu_k = 0.3$, determine a) the time required for the motion to stop, and b) the force in the strut BC during this time.





$$+\sum M_B = .3N(.15) = I_B \alpha$$

$$I_B = \frac{1}{2}(20)(.15)^2 = .225 \text{ kg m}^2$$

$$\alpha = .2N$$

$$\sum F_x = -N + B_c \sin 30^\circ = 0$$

$$\sum F_y = B_c \cos 30^\circ - mg + .3N = 0$$

$$N = .5 B_c = -2.887 B_c + 654$$

$$B_c = 193.09 N$$

$$N = 96.54 N \leftarrow$$

$$\alpha = -.2(96.54) = 19.31 \text{ r/s}^2 = \frac{d\omega}{dt}$$

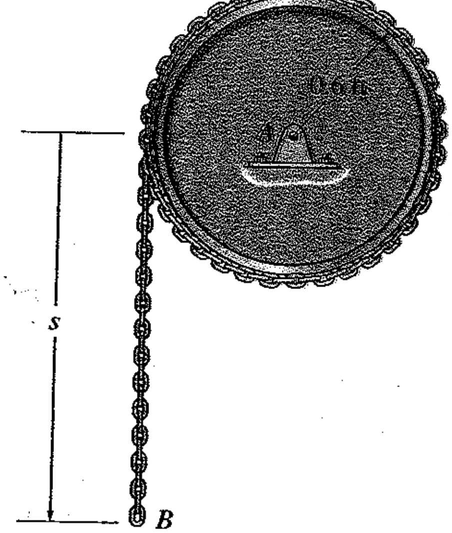
$$\omega = \omega_0 + \alpha t$$

$$0 = 60 - 19.31 t$$

$$t = 3.11 \text{ sec}$$

Problem 17.86 (Page 421)

The drum shown below has a weight of 50 lbs and has a radius of gyration of $k_A = 0.4$ ft. A 35 foot chain having a weight of 2 lb/ft is wrapped around the outer surface of the drum so that a chain length of s = 3 feet is suspended as shown. If the drum is originally at rest, determine its angular velocity after the end B of the chain has descended to s = 13 feet. Neglect the thickness of the chain.

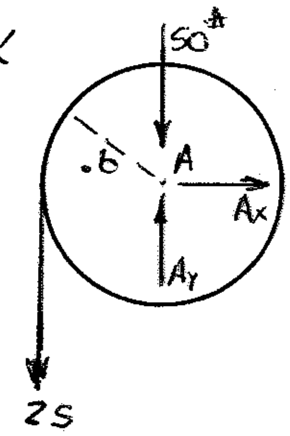


$$I_{A_{drum}} = \left(\frac{50}{32.2}\right)(.4)^2 = .249 \text{ slug ft}^2$$

$$I_{A_{chain}} = \frac{2(35-s)}{32.2}(.6)^2 = .0224(35-s)$$

$$\sum F_y = 2s = \frac{2s}{32.2}\underbrace{(.6)\alpha}_{a_{chain}}$$

$$\circlearrowleft + \sum M_A = .6(2s) - \frac{2s}{32.2}(.6)\alpha(.6) = I_{A_{drum}}\alpha + I_{A_{chain}}\alpha$$



$$\alpha = 1.164s = \omega\frac{d\omega}{d\theta}$$

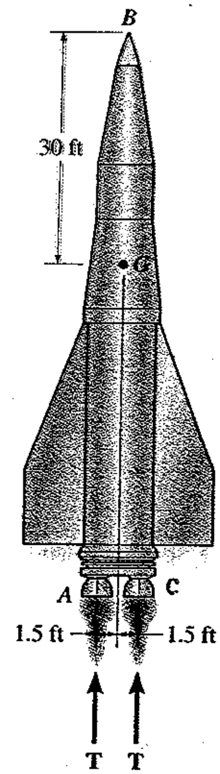
$$\int_3^{13} 1.164s\left(\frac{ds}{.6}\right) = \int_0^{\omega} \omega\, d\omega$$

$$1.9398\left[\frac{13^2}{2} - \frac{3^2}{2}\right] = \frac{1}{2}\omega^2$$

$$\omega = 17.6 \text{ r/s}$$

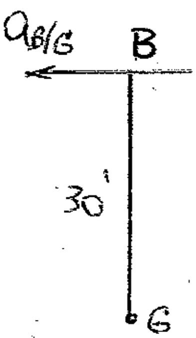
Problem 17.90 (Page 429)

A rocket weighs 20,000 lbs and has a radius of gyration about the mass center G of $k_G = 21$ ft. when it is fired. Each of its engines has a thrust of $T = 50,000$ lbs. At a given instant in time, engine A suddenly fails to operate. Determine the angular acceleration α of the rocket and the linear acceleration a_B of its nose B at this instant.



$$\circlearrowleft + \sum M_G = 1.5(50{,}000) = (21)^2\left(\frac{20{,}000}{32.174}\right)\alpha$$

$$\alpha = 0.2736 \text{ rad/sec}^2$$



$$\bar{a}_B = \bar{a}_G + \bar{a}_{B/G}$$

$$+\uparrow \sum F_y = 50{,}000 - 20{,}000$$

$$= \frac{20{,}000}{32.174} a_G$$

$$a_G = 48.26 \text{ ft/sec}^2$$

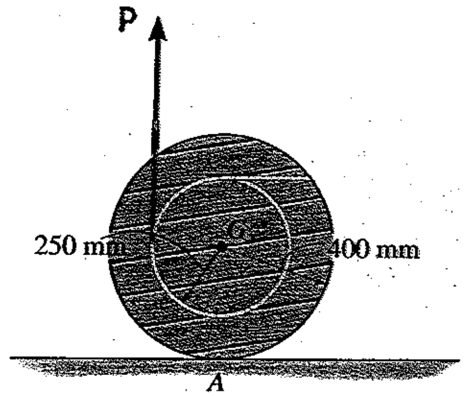
$$\bar{a}_B = -(.2736)(30)\,i + 48.26\,j$$

$$= -8.208\,i + 48.26\,j$$

$$a_B = 48.95 \text{ f/s}^2 \quad 80.3$$

Problem 17.96 (Page 430)

The spool shown has a mass of 100 kg and a radius of gyration of $k_G = 0.3$. If the coefficient of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if P is vertically up at 50 N



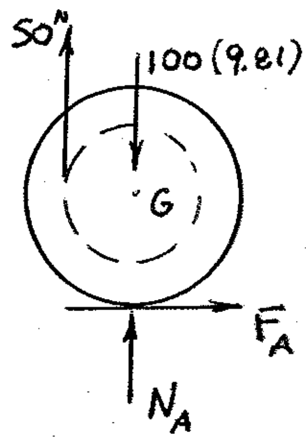
$\xrightarrow{+} \sum F_x = F_A = 100\, a_G$

$+\uparrow \sum F_y = N_A + 50 - 100(9.81)$

$= 0$

$\circlearrowleft + \sum M_G = 50(.25) - .4 F_A$

$= 100(.3)^2 \alpha$



For no slipping $\quad a_G = .4\alpha$

$\alpha = 0.5\ r/s^2$

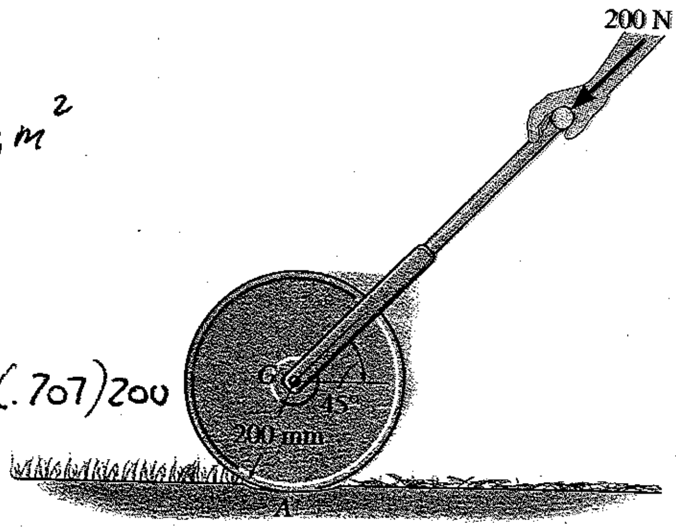
$a_G = .4(.5) = 0.2\ m/s^2$

$N_A = 931\ N \qquad F_A = 20\ N$

$F_{A_{max}} = .2(931) = 186.2\ N > 20\ N$ ($\therefore$ no slipping)

Problem 17.102 (Page 431)

The lawn roller shown has a mass of 80 kg and a radius of gyration $k_G = 0.175$ m. If the roller is pushed forward with a force 200 N when the handle is at 45°, determine its angular acceleration. The coefficient of static and kinetic friction between the ground and the roller are $\mu_s = 0.12$ and $\mu_k = 0.10$.



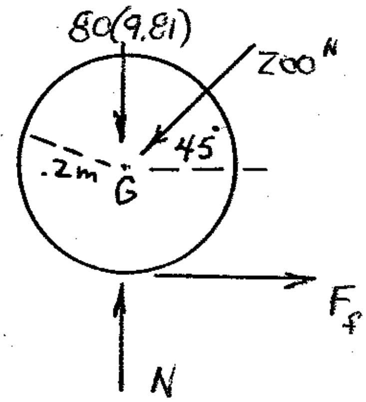
$$I_G = (.175)^2(80) = 2.45 \text{ Kg m}^2$$

$$\circlearrowleft + \sum M_G = F_f(.2) = 2.45\alpha$$

$$+\uparrow \sum F_y = N - 80(9.81) - (.707)200$$

$$N = 926.2^N$$

$$\xrightarrow{+} \sum F_x = F_f - 200(.707) = -80(.2)\alpha$$



$$F_f = 12.25\alpha = 141.4 - 16\alpha$$

$$\alpha = 5.005 \text{ r/s}^2$$

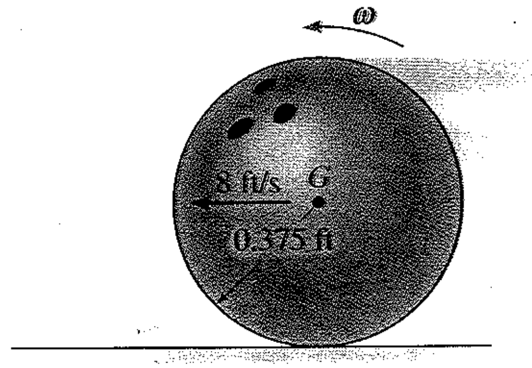
$$F_f = 61.31^N$$

$$F_{max} = .12(926.2) = 111.14^N$$

$\therefore$ roller rolls w/o slipping

Problem 17.107 (Page 432)

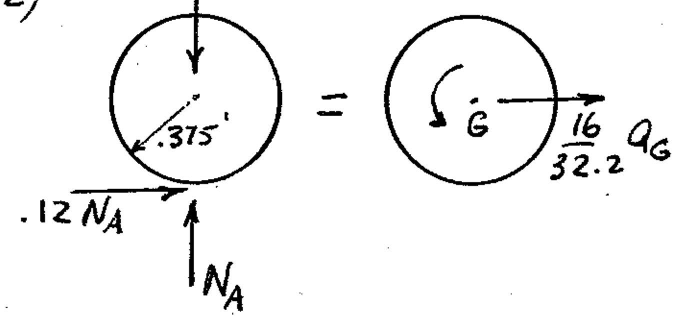
The 16-lb bowling ball is cast horizontally onto a lane such that initially $\omega = 0$ and its mass center has a velocity $v = 8$ ft/sec. If the coefficient of kinetic friction between the lane and the ball is $\mu_k = 0.12$, determine the distance the ball travels before it rolls without slipping. For the calculations, neglect the finger holes in the ball and assume the ball has a uniform density.



$$\xrightarrow{+} \sum F_x = .12 N_A = \frac{16}{32.2} a_G$$

$$+\uparrow \sum F_y = N_A - 16 = 0$$

$$\circlearrowleft + \sum M_G = .12 N_A (.375) = \frac{2}{5}\left(\frac{16}{32.2}\right)(.375)^2 \alpha$$



$N_A = 16$ lb

$a_G = 3.864$ f/s^2

$\alpha = 25.76$ r/s^2

When ball rolls w/o slipping $\quad V = .375\,\omega$

$$\omega = \omega_0 + \alpha t = \frac{V}{.375} = 0 + 25.76\,t$$

$$V = 9.66\,t = V_0 + at = 8 - 3.864\,t$$

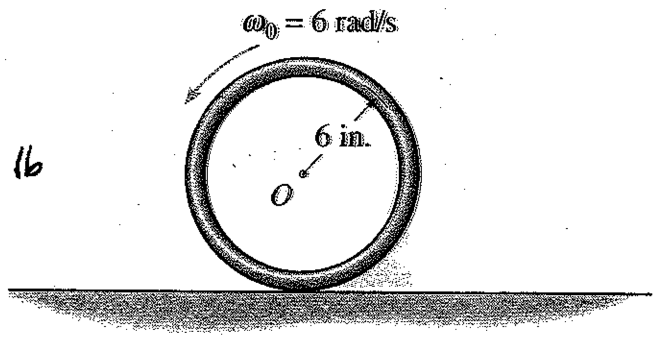
$$t = 0.592 \text{ sec}$$

$$S = S_0 + V_0 t + \frac{1}{2} a t^2 = 0 + 8(.592) - \frac{1}{2}(3.864)(.592)^2$$

$$= 4.06 \text{ ft}$$

Problem 17.108 (Page 433)

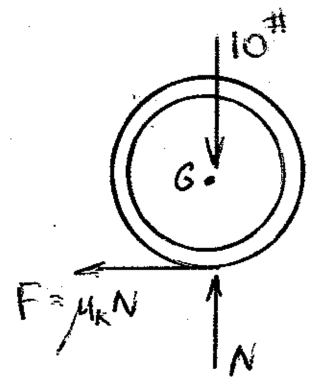
A 10-lb thin ring is given an initial angular velocity of 6 rad/sec when it is placed on a surface. If the coefficient of kinetic friction between the ring and the surface is $\mu_k = 0.3$, determine the distance the ring moves before it stops slipping.



$+\uparrow \sum F_y = N - 10 = 0 \qquad N = 10 \text{ lb}$

$\overset{+}{\rightarrow} \sum F_x = -.3(10) = \frac{10}{32.174} a_G$

$a_G = 9.66 \text{ f/s}^2 \leftarrow$



$\circlearrowleft + \sum M_O = I_O \alpha = .3(10)\left(\frac{6}{12}\right)$

$= \frac{10}{32.174}\left(\frac{6}{12}\right)^2 \alpha$

$\alpha = -19.32 \text{ r/s}^2 = \frac{d\omega}{dt}$

$V_G = r\omega = \left(\frac{6}{12}\right)\omega$

$\omega = \omega_0 + \alpha t$

$= 6 - 19.32\, t$

$V_G = V_0 + a_G t$

$= 0 + 9.66 t = .5\,\omega$

$38.64 t = 6$

$t = 0.155 \text{ sec}$

$V_G = 1.5 \text{ f/s}$

$= \frac{ds}{dt}$

$\omega = 3 \text{ rad/sec}$

$S = S_0 + V_{G_0} t + \frac{1}{2} a_G t^2$

$= 0 + 0 + \frac{1}{2}(9.66)(.155)^2$

$= 0.116 \text{ ft} = 1.4 \text{ in}$