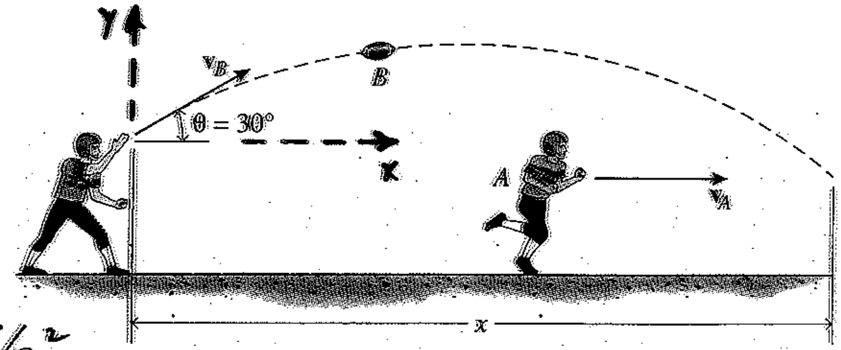
A football player throws a pass to a receiver as shown. The initial velocity of the football is 35 f/s at $\theta = 30°$.. If the receiver runs at a constant speed of 15 f/s, determine

a) the distance x at which the receiver will catch the football.

b) the relative velocity of the ball with respect to the receiver when he catches the ball.



For the Ball

$a_x = 0. \quad V_x = \text{constant}$

$= 35 \cos 30°$

$= 30.31 \text{ f/s}$

$a_y = -g = -32.174 \text{ f/s}^2$

$= \frac{dV_y}{dt}$

$$\int_{35 \sin 30°}^{V_y} dV_y = -32.174 \int_0^t dt \qquad \therefore V_y = 17.5 - 32.174t = \frac{dy}{dt}$$

$$y = 17.5t - \frac{32.174t^2}{2}$$

In vector form $\quad \bar{a}_B = -32.174 \,\bar{j}$

$\bar{V}_B = 30.31 \,\bar{i} + (17.5 - 32.174t)\,\bar{j}$

$\bar{r}_B = 30.31t \,\bar{i} + (17.5t - 16.1t^2)\,\bar{j}$

Receiver A catches the ball when $y = 17.5t - 16.1t^2 = 0$

$t = 1.087 \text{ sec} \qquad X = 30.31(1.087) = 32.9 \text{ ft}$

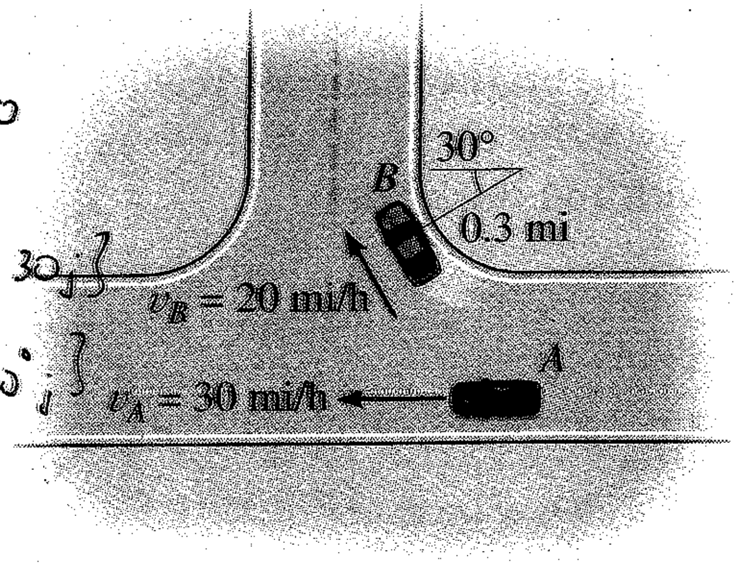
$\bar{V}_{B/A} = \bar{V}_B - \bar{V}_A = 30.31\,\bar{i} + (17.5 - 32.174t)\,\bar{j} - 15\,\bar{i}$

$= 15.31\,\bar{i} - 17.5\,\bar{j}$ or

$23.3 \text{ f/s} \searrow 48.8°$

Problem 12.197 (Page 92)

At the instant shown, cars A and B are traveling at speeds of 30 mi/hr and 20 mi/hr, respectively. If B is increasing its speed by 1200 mi/hr^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.



$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A} \qquad \bar{a}_A = 0$

$a_{B/A} = 1200\left[-\sin 30^\circ \,\bar{i} + \cos 30^\circ \,\bar{j}\right] + 1333\left[\cos 30^\circ \,\bar{i} + \sin 30^\circ \,\bar{j}\right]$

$= 554\,\bar{i} + 1705.5\,\bar{j}$

$V_A = -30\,\bar{i} \qquad V_B = 20\left[-\sin 30^\circ \,\bar{i} + \cos 30^\circ \,\bar{j}\right]$

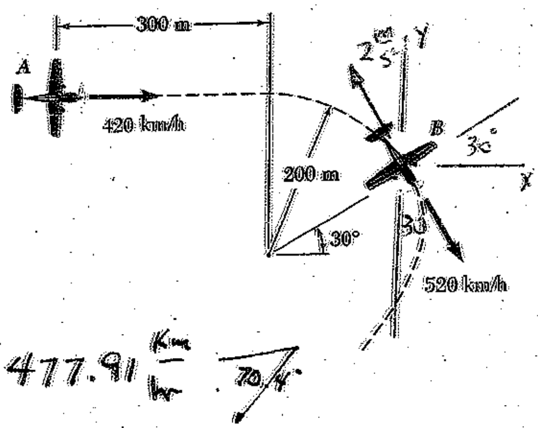
$\bar{V}_{B/A} = \bar{V}_B - \bar{V}_A = 20\,\bar{i} + 17.32\,\bar{j}$

Direction of velocity: $\theta = \tan^{-1} \dfrac{17.32}{20} = 40.9^\circ$

Direction of Acceleration:

$\theta = \tan^{-1} \dfrac{1705.5}{554} = 72^\circ$

1. At a given instant in time airplane A is flying horizontal at 420 km/hr in a straight line while its speed is increasing at a rate of 6 m/s^2. Airplane B is flying at the same altitude along the curved path at 520 km/hr in the position shown, but is decreasing its speed at 2 m/s^2. Determine a) the velocity of B with respect to A, and b) the acceleration of B with respect to A.



$V_A = 420\,\bar{i}$

$V_B = 520 \sin 30^\circ\,\bar{i} - 520 \cos 30^\circ\,\bar{j}$

$\overline{V_{B/A}} = \bar{V}_B - \bar{V}_A$

$= 160\,\bar{i} - 450.33\,\bar{j}$ $\quad V = 477.91 \frac{km}{hr}$ $70.4°$

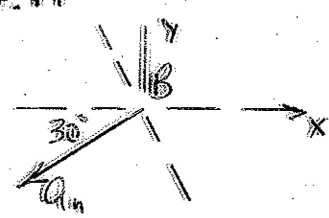
$\bar{a}_A = 6\,\bar{i}$ $\quad \bar{a}_{B_t} = -2 \sin 30^\circ\,\bar{i} + 2 \cos 30^\circ\,\bar{j}$

$a_{B_n} = \frac{V_t^2}{\rho} = \frac{(144.44)^2}{200} = 104.3\ m/s^2$

$V = V_t = \frac{477.91\,(1000)}{360}$

$= 144.44\ m/s$

$\bar{a}_{B_n} = -104.3 \cos 30^\circ\,\bar{i} - 104.3 \sin 30^\circ\,\bar{j}$



$\bar{a}_B = \bar{a}_t + \bar{a}_n$

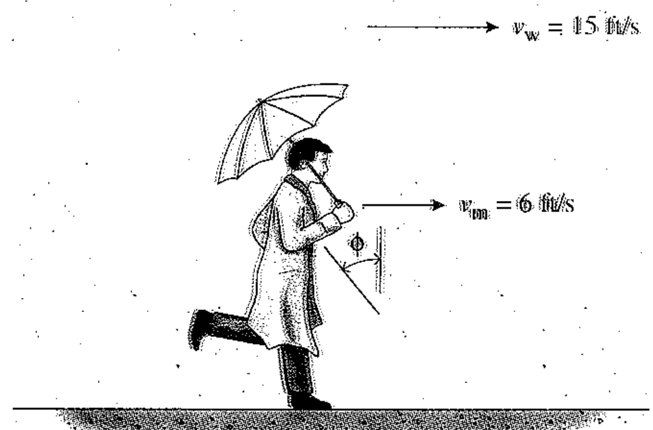
$= -91.34\,\bar{i} - 50.42\,\bar{j}$

$a_{B/A} = \bar{a}_B - a_A = -91.34\,\bar{i} - 50.42\,\bar{j} - 6\,\bar{i}$

$= -97.34\,\bar{i} - 50.42\,\bar{j}$

$a = 109.62 \frac{m}{s^2}$ $27.38°$

Rain falls vertically at 90 ft/sec, but is blown sideways by a horizontal wind at 15 ft/sec. For a man walking briskly at 6 ft/sec, determine the angle ϕ at which the man should hold an umbrella he is walking a) with the wind, and b) into the wind.

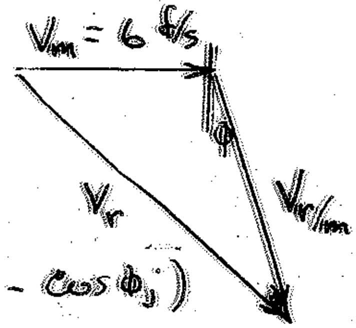


$$\bar{V}_r = \bar{V}_m + \bar{V}_{r/m}$$

$$\bar{V}_r = 15\bar{i} - 90\bar{j} \quad f/s$$

$$\bar{V}_m = V_m \bar{i}$$

$$\bar{V}_{r/m} = V_{r/m}(\sin\phi\,\bar{i} - \cos\phi\,j)$$



For $V_m = 6$ f/s

$$15\bar{i} - 90\bar{j} = 6\bar{i} + V_{r/m}(\sin\phi\,i - \cos\phi\,j)$$

$$x: \quad 15 = 6 + V_{r/m}\sin\phi$$

$$y: \quad -90 = -V_{r/m}\cos\phi \qquad \phi = 5.71^\circ$$

For $V_m = -6$ f/s

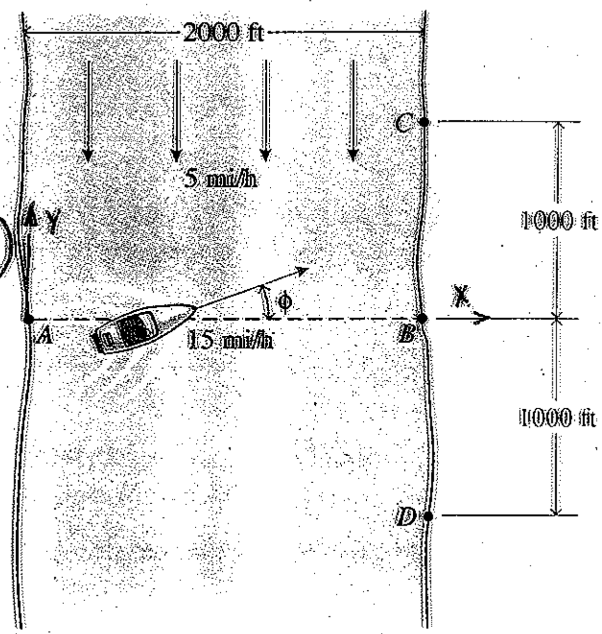
$$x: \quad 15 = -6 + V_{r/m}\sin\phi$$

$$y: \quad -90 = -V_{r/m}\cos\phi \qquad \phi = 13.13^\circ$$

A boat wants to travel straight across a river as shown. The river is 2000 ft. wide and has a current of 5 mi/hr. If the boat travels at 15 mi/hr, determine

a) The time T required to travel straight across the river from A to B,

b) The angle ϕ at which the boat must head to travel straight from A to B.



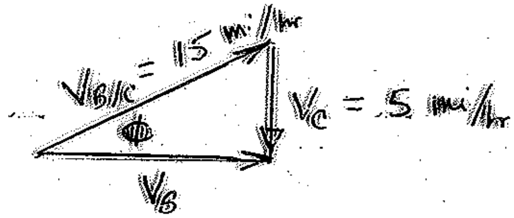
$$\bar{V}_B = \bar{V}_c + \bar{V}_{b/c}$$

$$V_B \bar{i} = -5\bar{j} + 15(\cos\phi\,\bar{i} + \sin\phi\,\bar{j})$$

$$x:\quad V_B = 15\cos\phi$$

$$y:\quad 0 = -5 + 15\sin\phi$$

$$\phi = \sin^{-1}\frac{5}{15} = 19.47^\circ$$



$$V_B = 15\cos(19.47)$$

$$= 14.14\ \text{mi/hr}$$

$$= 20.742\ \text{ft/sec} = \frac{dx}{dt}$$

$$x = 20.742\,t$$

$$2000\ \text{ft} = 20.742\ \frac{\text{f}}{\text{s}}\ t$$

$t = 96.4$ sec $= 1.6$ min

time to cross the river

An airplane desires to fly straight north while the wind is blowing due east at 20 m/sec. The speed of the airplane (indicated air speed) is 250 km/hr.

Determine: a) Direction the airplane must fly in order to go north.
b) Time required to fly 250 km due north.

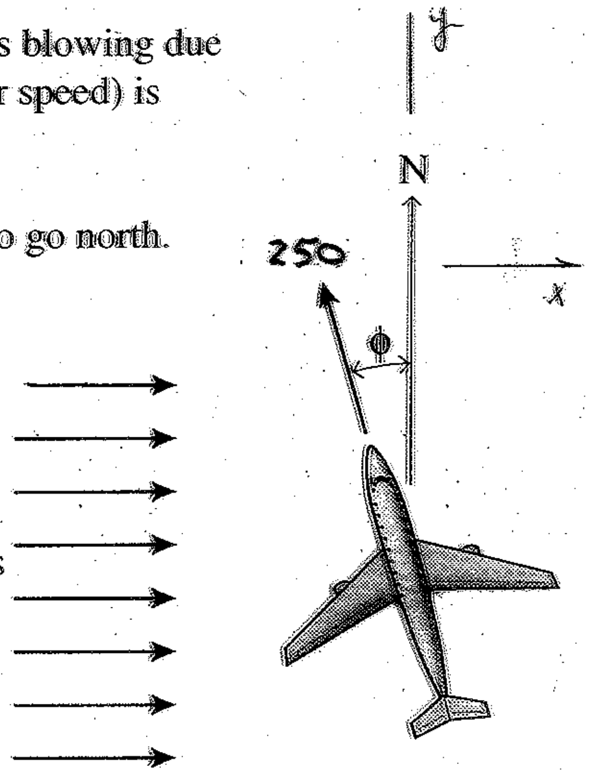
Wind velocity $\bar{v}_w = 20\bar{i}$

Desired velocity of the airplane

$\bar{v}_a = V\bar{j}$ m/s

Velocity of the airplane relative to the wind is

20 m/s



$$\bar{V}_{a/w} = -250 \sin\phi\, \bar{i} + 250 \cos\phi\, \bar{j} \quad \text{Km/hr}$$

$$= -69.44 \sin\phi\, \bar{i} + 69.44 \cos\phi\, \bar{j}$$

Note: $\frac{250 \text{ Km}}{\text{hr}} \frac{\text{hr}}{3600 \text{ sec}} \frac{(1000) \text{ m}}{\text{Km}} = 69.44 \text{ m/s}$

$$\bar{V}_a = \bar{V}_w + \bar{V}_{a/w}$$

$$V\bar{j} = 20\bar{i} + \left[-69.44 \sin\phi\, \bar{i} + 69.44 \cos\phi\, \bar{j}\right]$$

X: $0 = 20 - 69.44 \sin\phi$ $\qquad \phi = \sin^{-1}\frac{20}{69.44} = 16.74^\circ$

y: $V = 69.44 \cos 16.74^\circ$

$$V = 66.5 \text{ m/s} = \frac{Y}{t}$$

$$t = \frac{250}{66.5}\left(\frac{1000}{3600}\right) = \underline{\underline{1.04 \text{ hr}}}$$