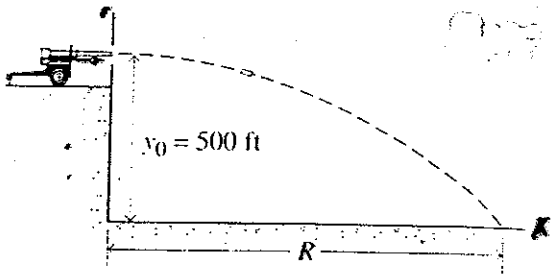
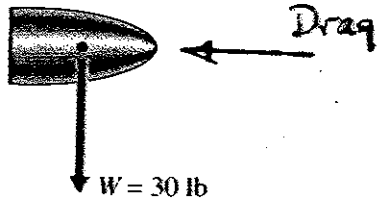
Projectile Including Aerodynamic Resistance



$$\Sigma F_x = m a_x = -D$$

$$m \frac{dv_x}{dt} = -\frac{1}{2} \rho V_x^2 C_D A_{ref}$$



$$\frac{dv_x}{v_x^2} = -\frac{\rho C_D A_{ref}}{2m} dt$$

$$\int_{v_0}^{v_x} v_x^{-2} dv_x = -\frac{\rho C_D \pi d^2}{8m} \int_0^t dt$$

$$-\frac{1}{v_x}\Big|_{v_0}^{v_x} = -\frac{\rho C_D \pi d^2 t}{8m}$$

let $K = \frac{\rho C_D \pi d^2}{8m}$

$$\frac{1}{v_0} - \frac{1}{v_x} = -Kt$$

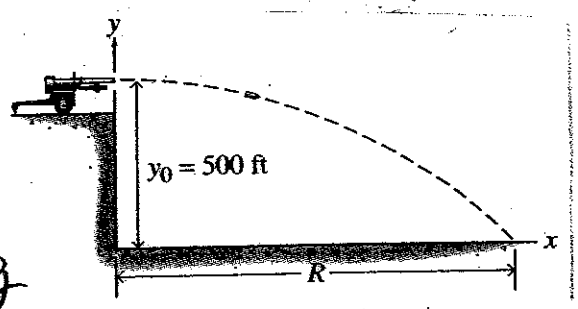
$$\frac{1}{v_x} = \frac{1}{v_0} + Kt = \frac{1 + K v_0 t}{v_0}$$

$$v_x = \frac{dx}{dt} = \frac{v_0}{1 + K v_0 t}$$

$$\int_0^x dx = \int_0^t \frac{v_0 \, dt}{1 + K v_0 t}$$

$$x = \frac{8m}{\rho C_D \pi d^2} \ln\left[1 + \frac{\rho C_D \pi d^2 v_0 t}{8m}\right]$$

Projectile Including Aerodynamic Resistance (Continued)



y direction:

$$\Sigma F_y = m a_y = -W = -mg$$

$$\frac{dv_y}{dt} = -g$$

$$v_y = \overset{0}{\widetilde{v_{y_0}}} - gt = -gt$$

$$\frac{dy}{dt} = -gt \qquad \int_0^{-500} dy = -g\int_0^t t\,dt$$

$$-500 = -g\frac{t^2}{2} \qquad t \text{ for } X_R = \frac{1000}{g}$$

$$= 5.57 \text{ sec}$$

let: $m = \frac{30 \text{ lb}}{32.2 \text{ lb}}$ slugs $\qquad v_0 = 750$ f/s

$C_D = 0.3$, $\rho = .0024$ slugs/ft^3 $\qquad d = 4/12$ ft

Range X_R

$$X_R = \frac{g(30)}{32.2(.0024)(.3)\pi\left(\frac{4}{12}\right)^2} \ln\left[1 + \frac{(.0024)(.3)\pi\left(\frac{4}{12}\right)^2(750)(5.57)}{g\left(\frac{30}{32.2}\right)}\right]$$

$$= 3905 \text{ ft. including air resistance}$$

$$4180 \text{ ft excluding " "}$$