HW#10 SP07

#1.

After discarding any constants of integration

(a) determine the appropriate value of the unknown velocities u or v that satisfy the equation of two-dimensional incompressible continuity for

(i) $u = x^2y$ *(ii)* $u = x^2 – xy$ *(iii)* $v = y^2 - xy$

(b) determine the appropriate value of the unknown velocities w or v that satisfy the equation of three-dimensional incompressible continuity for

(i) $u = x^2yz$ $v = - y^2x$ *(ii)* $u = x^2 + 3z^2x$ $w = - z^3 + y^2$

#2.

(a) Consider the following steady, three-dimensional velocity field in Cartesian coordinates: $\vec{V} = (u, v, w) = (axy^2 - b)\vec{i} + cy^3\vec{j} + dxy\vec{k}$, where a,b,c and d are constants. Under what conditions is this flow field incompressible?

(b) Consider the following steady, two-dimensional, incompressible velocity field: $\vec{V} = (u, v) = (ax + b)\vec{i} + (-ay + cx)\vec{j}$, Find out whether this flow field rotational or irrotational?

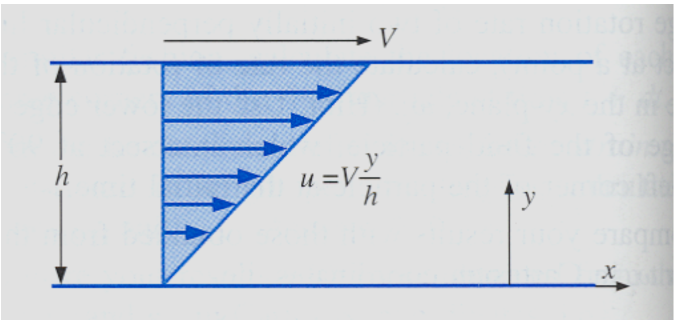
#3.

a.) Determine an expression for the *vorticity* of the flow field described by $V = -xy^3\, i + y^4\, j$. Is the flow irrational?

b.) For a certain incompressible, two-dimensional flow field the velocity component in the *y* direction is given by the equation, $v = 3xy + x^2y$. Determine the velocity component in *x* direction so that the continuity equation is satisfied.

#4.

Consider fully developed *Couette flow* – flow between two infinite parallel plates separated by distance h, with the top plate moving and the bottom plate stationary. The flow is steady, incompressible and two dimensional in xy-plane. The velocity field is given by $\vec{V} = (u, v) = V\frac{y}{h}\vec{i} + 0\vec{j}$. Is this flow rotational or *irrotational*? If it is rotational, calculate the *vorticity*. Do fluid particles in this flow rotate clockwise or counterclockwise?



#5.

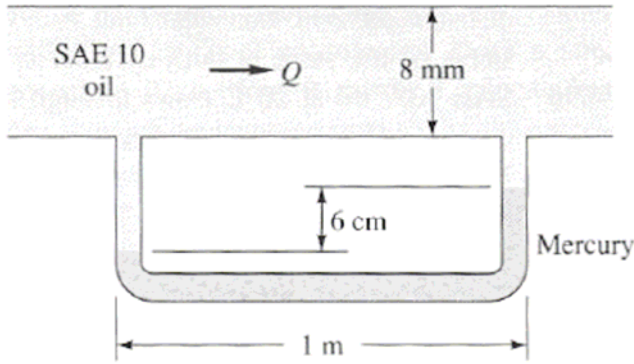
A frictionless, incompressible steady flow field is given by

$V = 2xy\mathrm{i} - y^2\mathrm{j}$

in arbitrary units. Let the density be ρ_0 = constant and neglect gravity. Find an expression for the pressure gradient in the x direction.

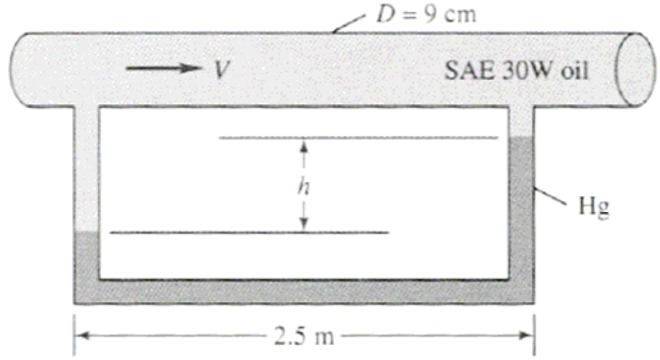
#6.

Oil flows between two parallel plates 8 cm apart as shown. A mercury manometer, with wall pressure taps 1m apart, registers a 6-cm height, as shown. Estimate the flow rate of the oil for this condition.



#7.

Oil flows through the 9-cm-diameter pipe as shown. The average flow velocity is 4.3 m/s. *(a)* Determine the volume flow rate in m^3/h. *(b)* calculate the expected reading h of the mercury manometer, in cm.

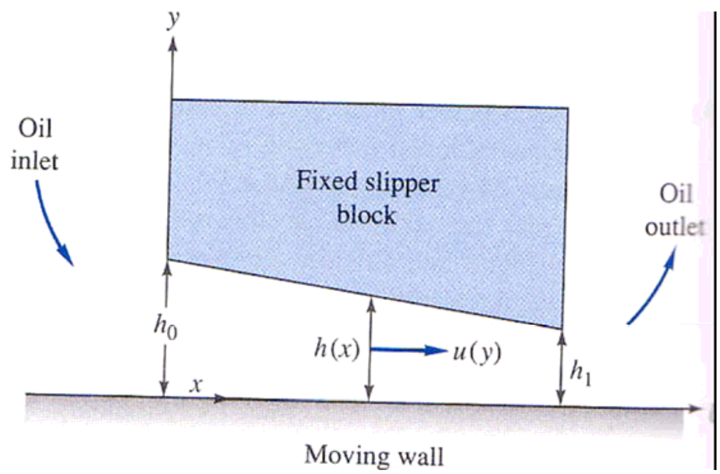


#8.

The flow pattern in bearing lubrication can be illustrated by the figure below, where a viscous oil *(ρ,μ)* is forced into the gap *h(x)* between a fixed slipper block and a wall moving at a velocity *U*. If the gap is thin, $h<<L$, it can be shown that the pressure and velocity distribution are of the form $p=p(x)$, $u=u(y)$, $v=w=0$. Neglecting gravity, reduce the Navier-Stokes equations to a single differential equation for *u(y)*. What are the proper boundary conditions? Integrate and show that

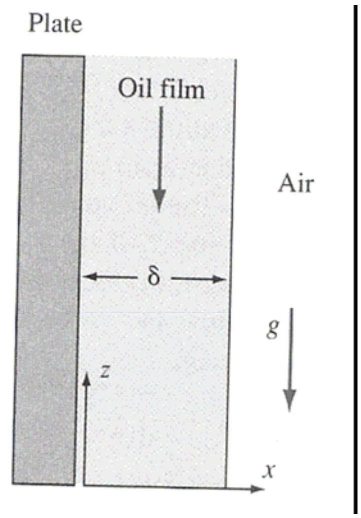
$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - yh\right) + U\left[1 - \frac{y}{h}\right]$$

Where $h=h(x)$ may be an arbitrary, slowly varying gap width.



#9.

Oil, of density ρ and viscosity μ, drains steadily down the side of a vertical plate as shown. After a development region near the top of the plate, the oil film will become independent of z and of constant thickness δ. Assume $w = w(x)$ only and that the atmosphere offers no shear resistance to the surface of the film. *(a)* Solve the *Navier-Stokes* equation for $w(x)$, and sketch its approximate shape. *(b)* Suppose that film thickness δ and the slope of the velocity profile at the wall $[dw/dx]_{wall}$ are measured with a laser-Doppler anemometer. Find an expression for oil viscosity μ as a function of ρ, δ, g, $[dw/dx]_{wall}$.



#10.

Consider steady, incompressible, parallel, laminar flow of a viscous fluid falling between two infinite vertical walls. The distance between the walls is *h*, and gravity acts in negative *z*-direction. There is no applied (forced) pressure driving the flow – the fluid falls by gravity alone. The pressure is constant everywhere in the flow field. Calculate the velocity field.