

Solutions to HW#11 SPO7

#1.

Water flows through an inclined 8-cm-diameter pipe. At sections A and B the following data are taken: $p_A = 186 \text{ kPa}$, $V_A = 3.2 \text{ m/s}$, $z_A = 24.5 \text{ m}$, and $p_B = 260 \text{ kPa}$, $V_B = 3.2 \text{ m/s}$, $z_B = 9.1 \text{ m}$. Which way is the flow going? What is the head loss in meters?

Solution: Guess that the flow is from A to B and write the steady flow energy equation:

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_f, \quad \text{or:} \quad \frac{186000}{9790} + 24.5 = \frac{260000}{9790} + 9.1 + h_f,$$

or: $43.50 = 35.66 + h_f$, solve: $h_f = +7.84 \text{ m}$ Yes, flow is from A to B. Ans. (a, b)

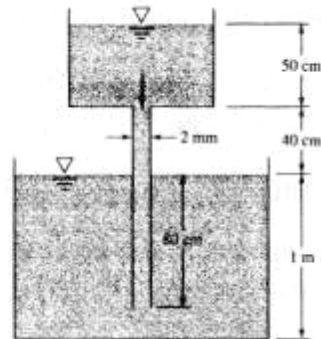
#2.

For the configuration shown below, the fluid is ethyl alcohol and the tanks are very wide. Find the flow rate which occurs in m^3/h . Is the flow laminar?

Solution: For ethanol, take $\rho = 789 \text{ kg/m}^3$ and $\mu = 0.0012 \text{ kg/m}\cdot\text{s}$. Write the energy equation from upper free surface (1) to lower free surface (2):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f,$$

with $p_1 = p_2$ and $V_1 \approx V_2 \approx 0$



$$\text{Then } h_f = z_1 - z_2 = 0.9 \text{ m} = \frac{128\mu L Q}{\pi \rho g d^4} = \frac{128(0.0012)(1.2 \text{ m})Q}{\pi(789)(9.81)(0.002)^4}$$

Solve for $Q \approx 1.90\text{E-}6 \text{ m}^3/\text{s} = \mathbf{0.00684 \text{ m}^3/\text{h}}$. *Ans.*

Check the Reynolds number $Re = 4\rho Q/(\pi\mu d) \approx 795$ – **OK, laminar flow.**

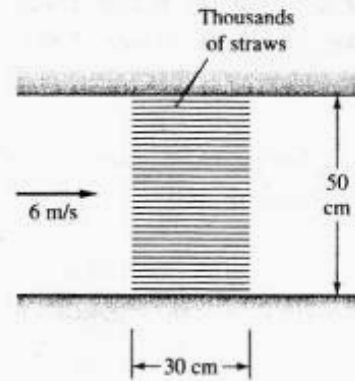
#3.

For straightening and smoothing an airflow in a 50-cm-diameter duct, the duct is packed with a “honeycomb” of thin straws of length 30 cm and diameter 4 mm, as in Fig. The inlet flow is air at 110 kPa and 20°C, moving at an average velocity of 6 m/s. Estimate the pressure drop across the honeycomb.

Solution: For air at 20°C, take $\mu \approx 1.8\text{E-}5$ kg/m·s and $\rho = 1.31$ kg/m³. There would be approximately 12000 straws, but each one would see the average velocity of 6 m/s. Thus

$$\Delta p_{\text{laminar}} = \frac{32\mu LV}{d^2} = \frac{32(1.8\text{E-}5)(0.3)(6.0)}{(0.004)^2} \approx \mathbf{65 \text{ Pa}} \quad \text{Ans.}$$

$$\text{Check } Re = \rho Vd/\mu = (1.31)(6.0)(0.004)/(1.8\text{E-}5) \\ \approx 1750 \quad \text{OK, laminar flow.}$$



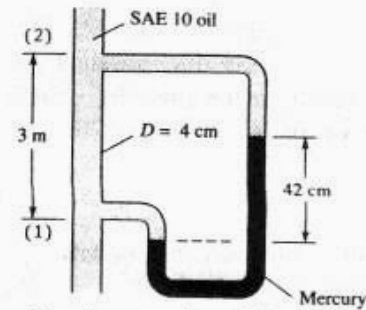
#4.

SAE 10 oil flows through the 4-cm-diameter vertical pipe as shown. For the mercury manometer reading $h = 42$ cm shown, (a) calculate the volume flow rate in m^3/h and (b) state the direction of flow.

Solution: For SAE 10 oil, take $\rho = 870 \text{ kg/m}^3$ and $\mu = 0.104 \text{ kg/m}\cdot\text{s}$. The pressure at the lower point (1) is considerably higher than p_2 according to the manometer reading:

$$p_1 - p_2 = (\rho_{\text{Hg}} - \rho_{\text{oil}})g\Delta h = (13550 - 870)(9.81)(0.42) \approx 52200 \text{ Pa}$$

$$\Delta p/(\rho_{\text{oil}}g) = 52200/[870(9.81)] \approx 6.12 \text{ m}$$



This is more than 3 m of oil, therefore it must include a friction loss: **flow is up.** Ans. (b)

The energy equation between (1) and (2), with $V_1 = V_2$, gives

$$\frac{p_1 - p_2}{\rho g} = z_2 - z_1 + h_f, \text{ or } 6.12 \text{ m} = 3 \text{ m} + h_f, \text{ or: } h_f \approx 3.12 \text{ m} = \frac{128\mu LQ}{\pi\rho g d^4}$$

$$\text{Compute } Q = \frac{(6.12 - 3)\pi(870)(9.81)(0.04)^4}{128(0.104)(3.0)} = 0.00536 \frac{\text{m}^3}{\text{s}} \approx \mathbf{19.3 \frac{\text{m}^3}{\text{h}}} \text{ Ans. (a)}$$

Check $Re = 4\rho Q/(\pi\mu d) = 4(870)(0.00536)/[\pi(0.104)(0.04)] \approx 1430$ (OK, laminar flow).

#5.

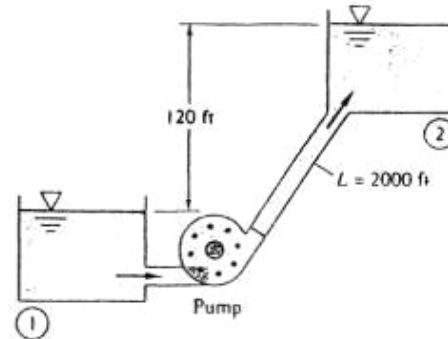
Water is to be pumped through 2000 ft of pipe from reservoir 1 to 2 at a rate of $3 \text{ ft}^3/\text{s}$, as shown. If the pipe is cast iron of diameter 6 in and the pump is 75% efficient, what horsepower pump is needed? (roughness value for cast iron, $\epsilon = 0.00085 \text{ ft}$)

Solution: For water at 20°C , take $\rho = 1.94 \text{ slug/ft}^3$ and $\mu = 2.09\text{E-}5 \text{ slug/ft}\cdot\text{s}$. For cast iron, take $\epsilon \approx 0.00085 \text{ ft}$, or $\epsilon/d = 0.00085/(6/12) \approx 0.0017$. Compute V , Re , and f :

$$V = \frac{Q}{A} = \frac{3}{(\pi/4)(6/12)^2} = 15.3 \frac{\text{ft}}{\text{s}}$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.94(15.3)(6/12)}{2.09\text{E-}5} \approx 709000$$

$$\epsilon/d = 0.0017, \quad f_{\text{Moody}} \approx 0.0227$$



The energy equation, with $p_1 = p_2$ and $V_1 \approx V_2 \approx 0$, yields an expression for pump head:

$$h_{\text{pump}} = \Delta z + f \frac{L}{d} \frac{V^2}{2g} = 120 \text{ ft} + 0.0227 \left(\frac{2000}{6/12} \right) \frac{(15.3)^2}{2(32.2)} = 120 + 330 \approx 450 \text{ ft}$$

$$\text{Power: } P = \frac{\rho g Q h_p}{\eta} = \frac{1.94(32.2)(3.0)(450)}{0.75} = 112200 \div 550 \approx \mathbf{204 \text{ hp}} \quad \text{Ans.}$$