HW # 1 SP07 Solutions

25 #1.

a). $p = \frac{F}{A} = \frac{\text{mass} \times \text{acc}}{\text{area}} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

Also $p = \frac{\text{force}}{\text{area}} = \frac{N}{m^2} = \underline{Pa \quad (\text{SI unit})}$ / $\frac{kg}{ms^2}$ ok

b). $C_D = \frac{D}{\frac{1}{2}\rho V^2 A} = \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2 L^2} = \frac{MLT^{-2}}{ML^{-3+2+2}T^{-2}}$ [$\frac{1}{2}$ has no dim.]

$= \frac{MLT^{-2}}{MLT^{-2}} = M^0L^0T^0 \Rightarrow$ Dimensionless

No units as it is dimensionless.

c). $Fr = \frac{V^2}{gL} = \frac{(LT^{-1})^2}{(LT^{-2})L} = \frac{L^2T^{-2}}{L^2T^{-2}} \Rightarrow M^0L^0T^0 \Rightarrow$ Dimensionless.

hence No unit.

d) $Re = \frac{\rho L V}{\mu} = \frac{(ML^{-3})(L)(LT^{-1})}{\mu}$ — (I)

For dimensional formula of μ consider

$$\tau = \mu \frac{dV}{dy} \Rightarrow \mu = \frac{\tau}{dV/dy}$$

so $\mu = \frac{\text{stress}}{\text{Vel. grad.}} \Rightarrow \frac{ML^{-1}T^{-2}}{LT^{-1}/L} = ML^{-1}T^{-1}$

so $Re = \frac{ML^{-3}\cdot L \cdot LT^{-1}}{ML^{-1}T^{-1}} = \frac{ML^{-1}T^{-1}}{ML^{-1}T^{-1}} \Rightarrow$ Dimensionless (No unit)

e.) $\tau = \mu \frac{\partial V}{\partial y}$ as done above for μ dimensional formula

is $ML^{-1}T^{-1}$

i.e. $\mu = \frac{\tau}{dV/dy} = \frac{ML^{-1}T^{-2}}{T^{-1}} = ML^{-1}T^{-1}$

$\mu = \frac{\text{Stress}}{\text{vel./disp}} = \frac{\text{stress}}{1/\text{Time}} = \text{stress} \times \text{time} = \frac{N\cdot s}{m^2}$ or $Pa\cdot s$ $\underline{\text{SI unit}}$

kg/ms ok

2

Given : $T = 2\pi\sqrt{\frac{m}{k}}$

$$k = \frac{\text{force}}{\text{displacement}} = \frac{MLT^{-2}}{L} = MT^{-2}$$

$$\text{Dimension}(2\pi) = \frac{T}{\sqrt{m/k}} = \frac{T}{\sqrt{\frac{M}{MT^{-2}}}} = \frac{T}{\sqrt{T^2}} = \cancel{\frac{T}{T}}$$

$$= M^0L^0T^0 \Rightarrow \text{Dimensionless}$$

So 2π is a constt

Ans

#3. Given :

$$\frac{1}{V}\frac{DV}{Dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \qquad (V = \text{Volume})$$

$$\text{Dim}(LHS) = \frac{1}{\cancel{L^3}}\cdot\frac{\cancel{L^3}}{T} = M^0L^0T^{-1}$$

$$\text{Dim}\left(\frac{\partial u}{\partial x}\right) = \frac{\text{velocity}}{\text{Disp.}} = \frac{\cancel{L}T^{-1}}{\cancel{L}} = M^0L^0T^{-1}$$

Similarly : $\text{Dim}\left(\frac{\partial v}{\partial y}\right) = \text{Dim}\left(\frac{\partial w}{\partial z}\right) = M^0L^0T^{-1}$

So each term in the equation has same dimensional formula ($M^0L^0T^{-1}$) Hence it is Dimensionally homogenous.

Ans

#4

a). $T_1 = 70^\circ c = 70 + 273 = 343\ K$

Isentsopic relation.

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 343\,(1.5)^{\frac{0.4}{1.4}} \qquad \left[\because \gamma = 1.4\right.$$

$$= 343 \times (1.5)^{0.285}$$

$$= 385.1\ ^{\circ}K$$

$$= 385.1 - 273 = 112^{\circ}C. \quad \text{Ans}$$

b). A frictionless, Adabatic process is isentropic process. So

-30% if Ans wrong

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{\gamma-1} = (35^{\circ}C + 273)\left(\frac{10}{1}\right)^{1.4-1}$$

$$= 308 \times 10^{0.4} = 773.7\ K.$$

$$= 500.7\ ^{\circ}C \quad \text{Ans}$$

#5

$$V_1 = 0.4\ m^3$$

$$p_1 = 100 \times 10^3\ N/m^2$$

$$T_1 = 80^{\circ}C = 80^{\circ}C + 273 = 353\ K$$

$$V_2 = 0.1\ m^3$$

Temperature remains constt $\Rightarrow$ Isothermal process.

Work done during Isothermal process

$$W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 100000 \times 0.4 \times \ln\left(\frac{0.1}{0.4}\right)$$

$$= -55451.8\ J = -55.452\ kJ \quad \text{Ans}$$

$W = -55.45\ kJ \Rightarrow$ Work done on the system.

#6

since $V = \nabla \phi$

a). $\Rightarrow \phi = \int V ds$ = Velocity x displacement

so $\text{Dim}(\phi) = LT^{-1} \cdot L = L^2T^{-1}$

$= \underline{M^0L^2T^{-1}}$ Ans

b) $\nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j \cong$ velocity

so $\text{Dim}(\nabla \phi) = \text{Dim}(\text{Vel})$

$= \underline{M^0L^1T^{-1}}$ Ans

c) $\frac{\partial \phi}{\partial x} = u\, i$ = Velocity in x-direction

so $\text{Dim}\left(\frac{\partial \phi}{\partial x}\right) = LT^{-1} = \underline{M^0L^1T^{-1}}$ Ans

d). $\frac{\partial^2 \phi}{\partial y \partial x} = \frac{\partial \left(\frac{\partial \phi}{\partial x}\right)}{\partial y} = \frac{\partial u}{\partial y} \cong \frac{\text{Velocity}}{\text{Displacement}}$.

So $\text{Dim}\left(\frac{\partial^2 \phi}{\partial y \partial x}\right) = \frac{\cancel{L}T^{-1}}{\cancel{L}} = \underline{M^0L^0T^{-1}}$ Ans

e). $\int \left(\frac{\partial \phi}{\partial x}\right) dy = \int u\, dy = u \int dy = uy$

So $\text{Dim}\left(\int \left(\frac{\partial \phi}{\partial x}\right) dy\right) = LT^{-1} \cdot L = L^2T^{-1}$

$= \underline{M^0L^2T^{-1}}$ Ans