HW #1

#1.

a.) Determine the mass and the weight of the air contained in a room whose dimensions are 6 m × 6 m × 8 m. Assume the density of air is 1.16 kg/m^3.

b.) A liquid has a viscosity of 0.005 kg/m.s and a density of 850 kg/m^3. Calculate the kinematic viscosity in (i) SI units (ii) British units (iii) viscosity in British units

(a) $\forall = 6 \times 6 \times 8 = 288 \text{ m}^3$

$\rho_a = 1.16 \text{ kg m}^{-3}$

mass, $m = \rho \forall = 1.16 \times 288$

$= 334.08 \text{ kg}$

wt, $mg = 334.08 \times 9.8$

$\simeq 3274 \text{ N}$ Ans

(b) $\mu = 0.005 \text{ kg m}^{-1}\text{s}^{-1}$

$\rho = 850 \text{ kg m}^{-3}$

(i) Kinematic Visc, $\nu = \frac{\mu}{\rho} = \frac{0.005}{850} = 5.882 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$ Ans

(ii) In B.U. : $\nu = \left(5.882 \times 10^{-6} \frac{\text{m}^2}{\text{s}}\right) \times \left(\frac{1 \text{ ft}}{0.3040 \text{ m}}\right)^2 = 6.33 \times 10^{-5} \frac{\text{ft}^2}{\text{s}}$ Ans

(iii) $\mu = \left(0.005 \frac{\text{kg}}{\text{m s}}\right)\left(\frac{1 \text{ slug/ft.s}}{47.9 \text{ kg/m.s}}\right) = 0.0001044 \text{ slug/ft.s}$ Ans

#2.

a.) Determine the density, specific gravity and the mass of the air in a room whose dimensions are 6 m × 6 m × 8 m at 100 kPa and 25°C.

b.) A 20-m^3 tank contains Nitrogen at 25°C and 800 kPa. Some Nitrogen is allowed to escape until the pressure in the tank drops to 600 kPa. If the temperature at this point is 20°C, determine the amount of Nitrogen that has escaped?

(a) For air: $R = 0.287\ kPa \cdot m^3/kg \cdot K$

From ideal-gas relation $p = \rho R T$

$$\rho = \frac{p}{RT} = \frac{100}{0.287 \times (25+273)} = 1.17\ kg/m^3 \quad \text{Ans}$$

$$SG = \frac{\rho}{\rho_w} = \frac{1.17}{1000} = 0.00117$$

Volume of room, $V = 6 \times 6 \times 8 = 288\ m^3$ Ans

mass, $m = \rho V = 1.17 \times 288 = 337\ kg$. Ans

(b) $p_1 = 800\ kN/m^2 \qquad T_1 = 25+273 = 298\ K$

$$R_N = \frac{R_u}{M} = \frac{8.314 \times 10^3\ J/kg \cdot K}{28} = 0.297\ kJ/kg \cdot K$$

$$\therefore \rho_1 = \frac{p_1}{R_N T} = \frac{800 \times 10^3}{0.297 \times 10^3 \times 298} = 9.039\ kg/m^3$$

$P_2 = 600\ kN/m^2 \qquad T_2 = 20+273 = 293\ K$

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \Rightarrow \rho_2 = \frac{p_2 \rho_1 T_1}{T_2 p_1}$$

$$\rho_2 = \frac{600 \times \cancel{10^3} \times 9.039 \times 298}{293 \times 800 \times \cancel{10^3}} = 6.895\ kg/m^3$$

Amount of N-escaped $= d\rho \times V$

$= (9.039 - 6.895) \times 20 = 42.9\ kg$ Ans

#3.

Drag force F on a sphere of diameter D in a fluid stream of low velocity V, density ρ and viscosity μ is proposed to be given by relation:

$$F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$$

Does this represent a valid physical relation? Prove it using dimensional analysis.

Given $F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$ ——— (i)

$\text{Dim}\{F\} = MLT^{-2}$ ——— (ii)

$$\mu = \frac{\tau}{\frac{dV}{dy}} = \frac{MLT^{-2}/L^2}{LT^{-1}/L} = \frac{MLT^{-2} \times \cancel{L}}{L^2 \times \cancel{L}T^{-1}} = ML^{-1}T^{-1}$$

$$\text{so Dim}\{3\pi\mu DV\} = (3\pi)(ML^{-1}T^{-1})(L)(LT^{-1})$$

$$= MLT^{-2} \text{ ——— (iii)}$$

$$\text{Dim}\left\{\frac{9\pi}{16}\rho V^2 D^2\right\} = \frac{9\pi}{16} \times \frac{M}{L^3} \times \frac{L^2}{T^2} \times L^2$$

$$= MLT^{-2} \text{ ——— (iv)}$$

so for eqn (i) Dimensions of each term are

$$MLT^{-2} = MLT^{-2} + MLT^{-2}$$

since each term has same dimensional formula.
equation (i) represents a valid physical relation.

#4.

In potential flow, the velocity field can be expressed as gradient of a scalar function, ψ, such that

$$\frac{\partial \psi}{\partial y}\hat{\mathbf{i}} - \frac{\partial \psi}{\partial x}\hat{\mathbf{j}} = u\hat{\mathbf{i}} + v\hat{j} \qquad \text{.......... (in 2D)}$$

Where u and v are the velocity components in x and y directions respectively. Find the dimensions of the quantities (a) ψ, (b) $\nabla\psi$, (c) $\frac{\partial \psi}{\partial x}$, (d) $\frac{\partial^2 \psi}{\partial x \partial y}$ and (e) $\int \frac{\partial \psi}{\partial y} dx$

since $\frac{\partial \psi}{\partial y} = u \Rightarrow \psi = \int u\,dy$

$\Rightarrow \psi$ = velocity x displacement.

(a) Therefore : $\text{Dim}[\psi] = LT^{-1} \times L = L^2T^{-1}$

$= M^0L^2T^{-1}$ Ans

(b) $\nabla\psi = \frac{\partial \psi}{\partial y}\hat{i} - \frac{\partial \psi}{\partial x}\hat{j} = u\hat{i} + v\hat{j}$ = Velocity.

$\therefore \text{Dim}[\nabla\psi] = LT^{-1} = M^0L^1T^{-1}$ Ans

(c) $\frac{\partial \psi}{\partial x} = -v$ = Velocity in y-dir.

$\therefore \text{Dim}\left[\frac{\partial \psi}{\partial x}\right] = M^0L^1T^{-1}$ Ans

(d) $\frac{\partial^2 \psi}{\partial x \partial y} = \frac{\partial\left(\frac{\partial \psi}{\partial y}\right)}{\partial x} = \frac{\partial u}{\partial x} = \frac{\text{Velocity}}{\text{Disp.}}$

$\therefore \text{Dim}\left[\frac{\partial^2 \psi}{\partial x \partial y}\right] = \frac{LT^{-1}}{L} = T^{-1} = M^0L^0T^{-1}$ Ans

(e) $\int \frac{\partial \psi}{\partial y} dx = \int u\,dx \;\hat{=}$ velocity x disp.

$\therefore \text{Dim}\left[\int \frac{\partial \psi}{\partial y} dx\right] = LT^{-1} \times L = M^0L^2T^{-1}$ Ans

#5.

Perfect gas equation and Lift acting on a wing are given as $p=\rho RT$ *and*

$L=C_L\frac{1}{2}\rho V^2S$ respectively. Where

p : pressure, ρ : density, T : temperature, R : gas constant, L: lift force, V : velocity, C_L : lift coefficient and S : Wing area

What are the dimensions of R and C_L?

(a)

$$\text{Dim}[p] = \frac{MLT^{-2}}{L^2} = M^1L^{-1}T^{-2}.$$

$$\text{Dim}[\rho] = \frac{M}{L^3} = M^1L^{-3}T^0$$

$$\text{Dim}[T] = M^0L^0T^0\theta^1$$

$$p = \rho RT$$

$$\Rightarrow R = \frac{p}{\rho T}$$

$$\text{Dim}[R] = \text{Dim}\left[\frac{p}{\rho T}\right] = \frac{M^1L^{-1}T^{-2}}{M^1L^{-3}T^0 \times \theta} = M^0L^2T^{-2}\theta^{-1} \quad \text{Ans}$$

(b)

$$C_L = \frac{2L}{\rho V^2 S}$$

$$\text{Dim}[C_L] = \frac{\text{Dim}[2L]}{\text{Dim}[\rho V^2 S]} = \frac{MLT^{-2}}{M^1L^{-3}T^0 \times L^2T^{-2} \times L^2}$$

$$= \frac{M\cancel{L}T^{-2}}{M\cancel{L}T^{-2}} = M^0L^0T^0 \quad \text{Ans}$$

$$= \text{Dimensionless}$$

#6.

a.) A liquid compressed in a cylinder has a volume of 1 liter ($L = 1000\ cm^3$) at 1 MN/m^2 and a volume of 995 cm^3 at 2 MN/m^2. What is its bulk modulus?

b.) A rigid tank contains air at a pressure of 90 psia and a temperature of 60°F. By how much will the pressure increase when the temperature is increased to 110°F?

(a)

$$\forall_1 = 1000\ cm^3 = 1000 \times (10^{-2} m)^3 = 1000 \times 10^{-6} m^3$$

$$p_1 = 1 \times 10^6\ N/m^2$$

$$\forall_2 = 995 \times 10^{-6}\ m^3$$

$$p_2 = 2 \times 10^6\ N/m^2$$

$$\text{Bulk Modulus, } E_V = -\frac{\Delta p}{\Delta\forall/\forall} = -\frac{[2-1]\times 10^6}{\dfrac{[995-1000]\times \cancel{10^{-6}}}{1000 \times \cancel{10^{-6}}}}$$

$$= \frac{1000 \times 10^6}{5} = 200 \times 10^6\ N/m^2 \quad \text{Ans}$$

(b)

$$p = \rho R T$$

$$\text{for rigid body } \rho = \frac{m}{\forall} = \text{const.}$$

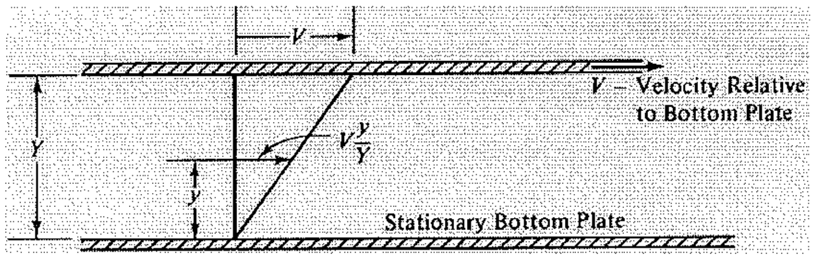
$$\text{So } \frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow p_2 = \frac{p_1 T_2}{T_1}$$

$$\Rightarrow p_2 = 90 \times \frac{(110+460)}{(60+460)} = 90 \times \frac{570}{520}$$

$$p_2 = 98.7\ \text{psia} \quad \text{Ans}$$

#7.

The viscosity of mercury at 68°F is 1.58×10^{-3} N.s/m^2. Determine the force necessary to maintain a relative velocity of 2 m/s between two plates as shown. The plates are separated by 0.1 m and have an area of 0.1 m^2. Consider only viscous effects.



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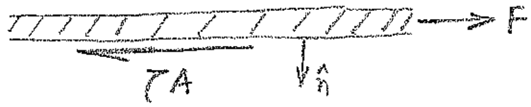


Shear stress on top plate:

$$\tau = \mu(\hat{n} \cdot \nabla u) = \mu\left[\langle \hat{n}_x, \hat{n}_y \rangle \cdot \left\langle \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right\rangle\right]$$

$$= \mu\left[\langle 0, -1 \rangle \cdot \left\langle 0, \frac{\partial}{\partial y}\left(V\frac{y}{Y}\right) \right\rangle\right]$$

$$= -\mu \frac{V}{Y}$$

$$\tau A = -\mu \frac{V}{Y} A = 1.58 \times 10^{-3} \times \frac{2}{0.1} \times 0.1 = -3.16 \times 10^{-3}\ \text{N}$$

For eqvm:

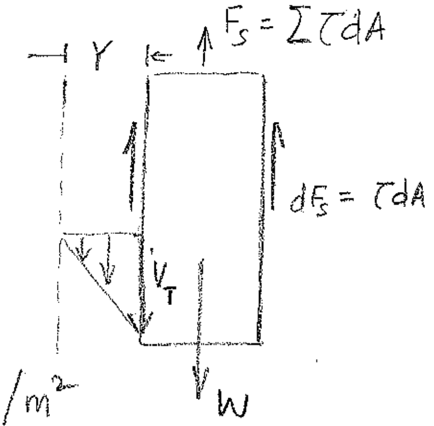
$$F + \tau A = 0$$

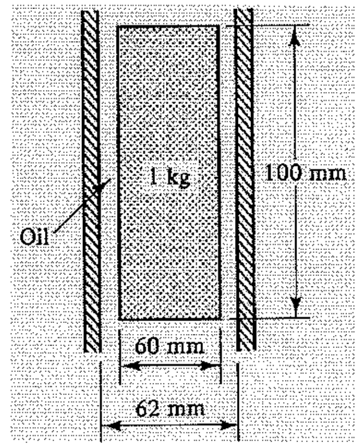
$$F - 3.16 \times 10^{-3} = 0$$

$$\therefore \quad F = 3.16 \times 10^{-3}\ \text{N} \qquad \text{Ans}$$

#8.

A solid cylinder having a mass of 1 kg slides down a pipe as shown. An oil having the viscosity of 8×10^{-2} N.s/m^2 keeps the cylinder concentric in the pipe. Determine the terminal speed of the cylinder, considering only the effects of viscosity.





$L = 0.1$ m

$d_o = 62$ mm
$= 0.062$ m

$d_i = 60$ mm
$= 0.06$ m

$\mu = 8 \times 10^{-2}$ Ns/m^2

$W = mg = 1 \times 9.8 = 9.8$ N.

A = surface area of the cylinder $= \int dA$

$= \pi d_i L = \pi \times 0.06 \times 0.1 = 0.0188$ m^2

spacing $Y = \dfrac{d_o - d_i}{2} = \dfrac{0.062 - 0.06}{2} = 0.001$ m.

$F_s = \mu [\hat{n} \cdot \nabla u] = \mu \dfrac{V_T}{Y} A$

$= 8 \times 10^{-2} \times \dfrac{V_T}{0.001} \times 0.0188$

For eqvm: $F_s = W$

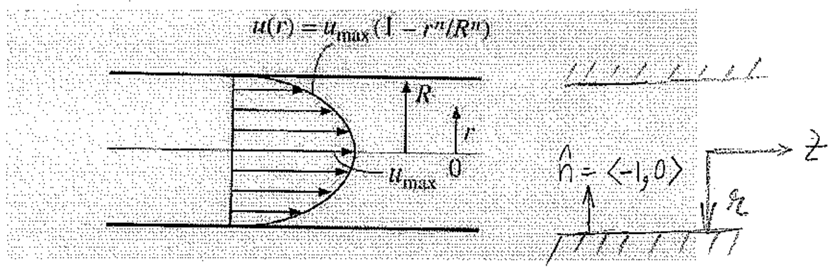
$\Rightarrow 8 \times 10^{-2} \times \dfrac{V_T}{0.001} \times 0.0188 = 9.8$

$V_T = \dfrac{9.8 \times 0.001}{8 \times 10^{-2} \times 0.0188} = 6.52$ m/s

Ans

#9.

Consider the flow of a fluid with viscosity μ through a circular pipe. The velocity profile in the pipe is given as $u(r) = u_{max}(1 - r^n/R^n)$, where u_{max} is the maximum flow velocity, which occurs at the centerline; r is the radial distance from the centerline; and $u(r)$ is the flow velocity at the position r. Develop a relation for the drag force exerted on the pipe wall by the fluid in the flow direction per unit length of the pipe.



$$u(r) = u_{max}\left[1 - \left(\frac{r}{R}\right)^n\right]$$

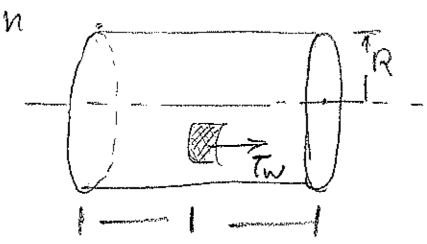
$$\tau_w = \mu[\hat{n} \cdot \nabla u]_{r=R} = \mu\left[\langle -1, 0\rangle \cdot \left\langle \frac{\partial u}{\partial r}, \frac{\partial u}{\partial z}\right\rangle\right]_{r=R}$$

$$= -\mu \frac{\partial u}{\partial r}\bigg|_{r=R}$$

$$= -\mu \cdot u_{max}\left[0 - \frac{n r^{n-1}}{R^n}\right]_{r=R} = \mu u_{max} \frac{n r^{n-1}}{R^n}\bigg|_{r=R}$$

$$\text{So } \tau_w = \mu u_{max} \frac{n R^{n-1}}{R^n} = \frac{n \mu u_{max}}{R}.$$

τ_w is the shear force/unit area on the inner wall. That means Total drag force acting:

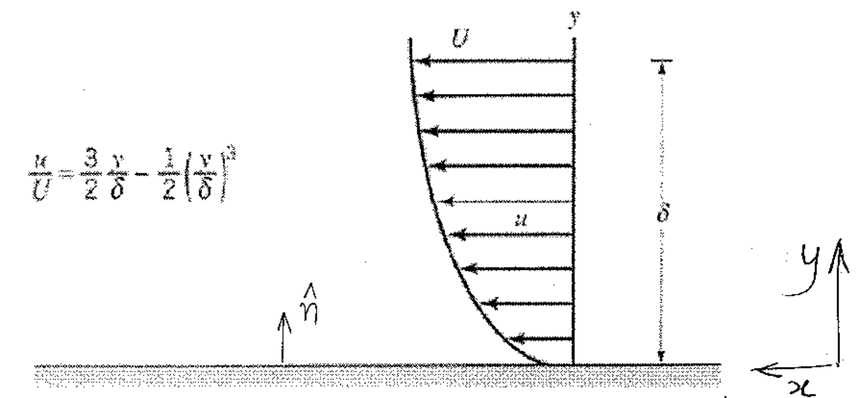


$$D = \tau_w \times \text{Area}$$

$$= \frac{n\mu u_{max}}{\cancel{R}} \cdot 2\pi \cancel{R} \times 1 = 2n\pi\mu u_{max} \quad \text{Ans}$$

#10.

1.58 A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4} m^2/s$ flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero (as shown in Video V1.2), and the velocity profile near the surface is shown in Fig. P1.58. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



$$\mu_f = SG \times \rho_w \times \nu_f = 0.92 \times 1000 \times 4 \times 10^{-4}$$

$$= 0.368 \text{ kg/ms}$$

$$u = U\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]$$

$$\tau_s = \mu_f\left[\hat{n} \cdot \nabla u\right]_{y=0} = \mu_f\left[\langle 0, 1\rangle \cdot \langle \cancel{\frac{\partial u}{\partial x}}_{0}, \frac{\partial u}{\partial y}\rangle\right]_{y=0}$$

Here $\frac{\partial u}{\partial y} = U\left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right]$.

$$\tau_s = \mu_f\left[\frac{\partial u}{\partial y}\right]_{y=0} = \mu_f\left[U\left(\frac{3}{2\delta} - \cancel{\frac{3y^2}{2\delta^3}}_{0}\right)\right]$$

$$\therefore \tau_s = \mu_f \frac{3U}{2\delta} = 0.368 \times \frac{3 \times U}{2\delta}$$

$$= 0.552 \frac{U}{\delta}$$ Ans

τ_s is in $-ve$ x direction.
i.e. in the direction of velocity.