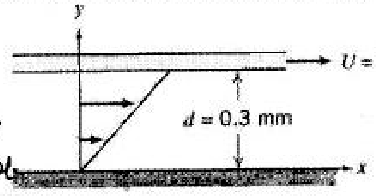
HW #2 (Spring 2007) MAE 2314

#1.

An infinite plate is moved over a second plate on a layer of liquid. For small gap width, *d*, we assume a linear velocity distribution in the liquid. The liquid viscosity μ is 1.36×10^{-5} lbf·s/ft^2 and its specific gravity is 0.88. Determine *(a)* Kinematic viscosity ν in SI units. *(b)* Shear stress on the upper plate in lbf/ft^2. *(c)* Shear stress on the lower plate in Pa. *(d)* The direction of each shear stress calculated in parts *(b)* and *(c)*.

Normal vector for lower surface $\vec{n} = \langle 0, 1 \rangle$



$\vec{n} = \langle 0, -1 \rangle$

Normal Vector for upper surface

Given : $\mu = 1.36 \times 10^{-5}$ lbf·s/ft^2.

a). Kinematic Viscosity, $\nu = \frac{\mu}{\rho} = \frac{\mu}{SG \times \rho_{H_2O}}$

$$= 1.36 \times 10^{-5} \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2} \times \frac{\text{ft}^3}{0.88 \times 1.94\ \text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} \times (0.305)^2 \frac{\text{m}^2}{\text{ft}^2}$$

$$= 7.4 \times 10^{-7}\ \text{m}^2/\text{s}$$ Ans

b). $\tau_{upper} = \mu \frac{du}{dy}\Big|_{y=d} \Rightarrow \mu(\vec{n} \cdot \nabla V)_{y=d}$ → in vector form.

$$= \mu\left[\langle 0, -1 \rangle \cdot \langle \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y} \rangle\right]$$

$$= -\mu \frac{\partial V}{\partial y} = -1.36 \times 10^{-5} \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2} \times \frac{0.3}{0.3} \times 1000\ \text{s}^{-1}$$

$$= -0.0136\ \text{lbf}/\text{ft}^2$$ Ans

c) $\tau_{lower} = \mu\left[\langle 0, 1 \rangle \cdot \langle \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y} \rangle\right]$

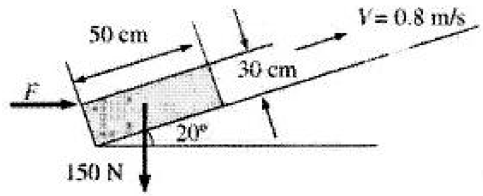
$$= \mu \frac{\partial V}{\partial y} = 0.0136 \frac{\text{lbf}}{\text{ft}^2} \times \frac{4.45\ \text{N}}{\text{lbf}} \times \frac{\text{ft}^2}{(0.305)^2\ \text{m}^2}$$

$$\Rightarrow 0.651 \frac{\text{N}}{\text{m}^2} = 0.651\ \text{Pa}$$ Ans.

d). Direction : b) ← In negative X-direction. c) → In positive X-direction } Ans

#2.

A 50-cm × 30-cm × 20-cm block weighing 150N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27. *(a)* Determine the force F that needs to be applied in horizontal direction. *(b)* If a 0.4-mm-thick oil film with a dynamic viscosity of 0.012 Pa·s is applied between the block and the inclined surface, determine the percent reduction in the required force.

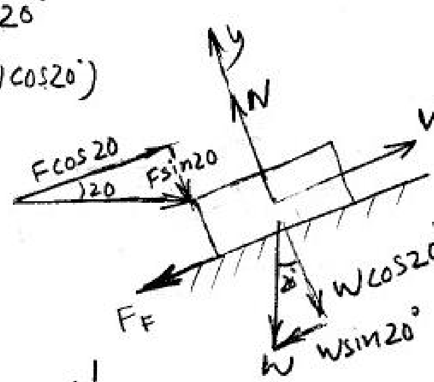


Area, $A = 50 \times 20\ cm^2 = 0.1\ m^2$

a) Normal force, $N = F\sin 20^\circ + W\cos 20^\circ$

Force of friction $= \mu N = \mu(F\sin 20^\circ + W\cos 20^\circ)$

For equm. in x-direction:



$F\cos 20^\circ = W\sin 20^\circ + \mu(F\sin 20^\circ + W\cos 20)$

$F\cos 20 - \mu F\sin 20 = W\sin 20 + \mu W\cos 20$

$F(0.9396 - 0.09234) = 51.303 + 38.06$

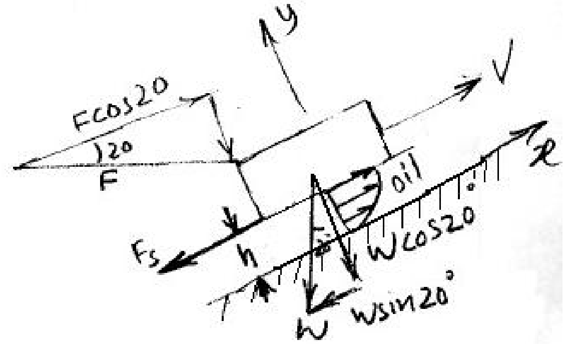
$F = 89.36/0.8472 = 105.5\ N$ Ans.

b) $\mu_{oil} = 0.012\ Pa.s$

$h = 0.4\ mm = 0.0004\ m$

$\tau_w = \mu_{oil} \left.\dfrac{du}{dy}\right|_{y=h} = 0.012 \times \dfrac{0.8}{0.0004}$

$= 24\ N/m^2.$



$F_s = \tau_w A = 24 \times 0.1 = 2.4\ N$

For equm in x-direction:

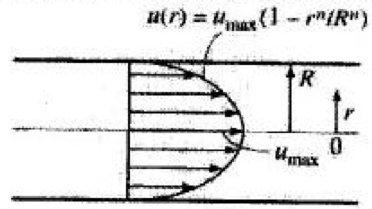
$F\cos 20^\circ = F_s + 150\sin 20$

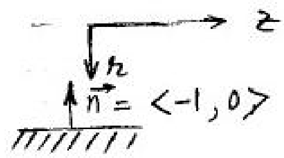
$F = \dfrac{2.4 + 150\sin 20^\circ}{\cos 20^\circ} = 57.15\ N$ Ans.

% Reduction $= \dfrac{105.5 - 57.15}{105.5} \times 100 = 45.8\ \%$ Ans

#3.

Consider the flow of a fluid with viscosity μ through a circular pipe. The velocity profile in the pipe is given as $u(r) = u_{max}(1 - r^n/R^n)$, where u_{max} is the maximum flow velocity, which occurs at the centerline; r is the radial distance from the centerline; and $u(r)$ is the flow velocity at the position r. Develop a relation for the drag force exerted on the pipe wall by the fluid in the flow direction per unit length of the pipe.





$$u(r) = u_{max}\left[1 - \left(\frac{r}{R}\right)^n\right]$$

$$\tau_w = \mu[\hat{n} \cdot \nabla u] = \left[\langle -1, 0\rangle \cdot \left\langle \frac{\partial u(r)}{\partial r}, \cancel{\frac{\partial u(r)}{\partial z}} \right\rangle\right]$$

0

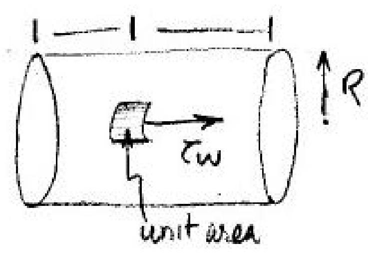
$$= -\mu \left.\frac{\partial u(r)}{\partial r}\right|_{r=R}$$

$$= -\mu \cdot u_{max}\left[0 - \frac{n r^{n-1}}{R^n}\right]_{r=R} = \mu u_{max} \left.\frac{n r^{n-1}}{R^n}\right|_{r=R}$$

$$\tau_w = \mu u_{max} \frac{n R^{n-1}}{R^n} = \frac{n \mu u_{max}}{R}$$

τ_w is the shear force acting on unit area on the inner wall.

So Total Drag force acting



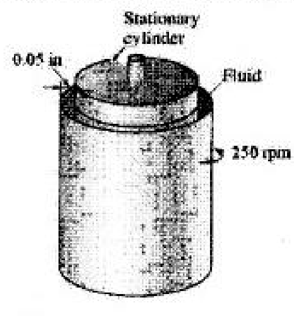
$$D = \tau_w \times \text{Area}$$

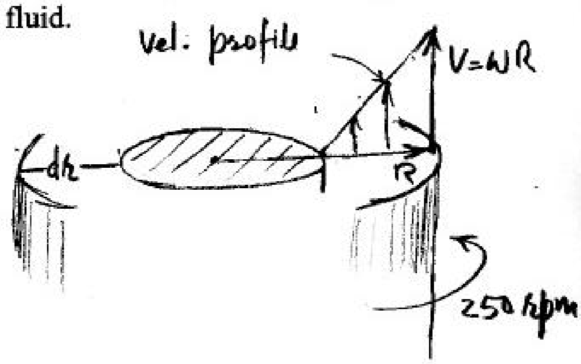
$$= \frac{n \mu u_{max}}{R} \cdot 2\pi R(1)$$

$$D = \frac{n \mu u_{max} \cdot 2\pi \cancel{R}}{\cancel{R}} = 2\pi n \mu u_{max} \quad \text{Ans}$$

#4.

The viscosity of a fluid is to be measured by a viscometer constructed of two 3-ft-long concentric cylinders. The inner diameter of the outer cylinder is 6 in, and the gap between the two cylinders is 0.05 in. The outer cylinder is rotated at 250 rpm, and the torque is measured to be 1.2 lbf·ft. Determine the viscosity of the fluid.





Length, $L = 3\,ft$

$R = 6/2 = 3\,in = 0.25\,ft.$

Gap, $dr = 0.05\,in = 0.05/12\,ft$

Angular velocity of outer cylinder, $\omega = \frac{2\pi \times 250}{60} = \frac{25\pi}{3}\,rad/s$

Tangential velocity on inner surface of the outer cylinder, $V_\theta = \omega R = \frac{25\pi}{3} \times 0.25 = \frac{6.25\pi}{3}\,ft\,s^{-1}$

$$\frac{dV}{dr} = \frac{\frac{6.25\pi}{3}}{\frac{0.05}{12}} = \frac{6.25\pi \times 12}{3 \times 0.05} = 500\pi\,s^{-1}$$

Total force $= \tau_w \cdot \text{Area} = \mu \frac{dV}{dr} \cdot \text{Area}$

$$= \mu \times 500\pi \times 2\pi R L = \mu \times 500\pi \times 2\pi \times 0.25 \times 3$$

$$= 750\pi^2 \mu\ lbf.$$

Torque, $T = \text{force} \times R = 750\pi^2 \mu R.$

But we have been given $T = 1.2\,lbf \cdot ft$

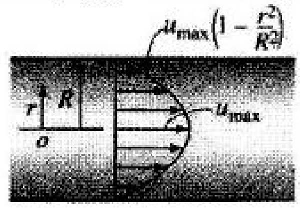
so $750\pi^2 \mu R = 1.2.$

Ans

$$\mu = \frac{1.2}{750\pi^2 R} = \frac{1.2 \times 4}{750\pi^2} = 0.000648\ \frac{lb.s}{ft^2}$$

#5.

In regions far from the entrance, fluid flow through a circular pipe is one-dimensional, and the velocity profile for the laminar flow is given by $u(r) = u_{max}(1 - r^2/R^2)$, where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{max} is the maximum flow velocity, which occurs at the center. Obtain *(a)* a relation for the drag force applied by the fluid on a section of the pipe of length L and *(b)* The value of the drag force for water flow with $R = 0.08$ m, $L = 15$ m, $u_{max} = 5$ m/s and $\mu = 0.0010$ kg/m·s.



$n = \langle -1, 0 \ldots \rangle$

$$u(r) = u_{max}\left[1 - \frac{r^2}{R^2}\right]$$

$$\tau_w = [\vec{n} \cdot \nabla u] = \mu\left[\langle -1, 0\rangle \cdot \left\langle \frac{\partial u}{\partial r}, \cancel{\frac{\partial u}{\partial z}} \right\rangle\right]_{r=R}$$

$$= -\mu \left.\frac{\partial u(r)}{\partial r}\right|_{r=R}$$

$$= -\mu\, u_{max}\left[0 - \frac{2r}{R^2}\right] = 2\mu\, u_{max} \left.\frac{r}{R^2}\right|_{r=R}$$

$$\tau_w = 2\mu\, u_{max} \frac{R}{R^2} = \frac{2\mu\, u_{max}}{R}.$$

$$\text{Drag force} = \tau_w \times \text{Area} = \frac{2\mu\, u_{max}}{\cancel{R}} \times 2\pi \cancel{R} L$$

(a) $$D = 4\mu\pi\, u_{max} L$$ Ans

(b) $$D = 4 \times 0.001 \times \pi \times 5 \times 15$$

$$= 0.3 \times \pi \text{ N}$$

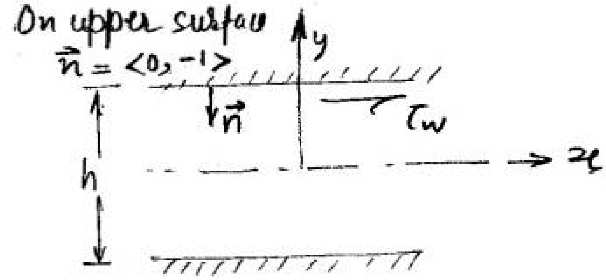
$$= 0.942 \text{ N}$$ Ans

#6.

The velocity distribution for the laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

Where h is the distance separating the plates and the origin is placed midway between the plates. Consider the flow of water with u_{max} = 0.10 m/s and h = 0.25 mm. Calculate the shear stress on the upper plate and give its direction. (μ of water is 1.14×10^{-3} N·s/m^2)



$$u = u_{max}\left[1 - \left(\frac{2y}{h}\right)^2\right]$$

$$h = 0.25 \text{ mm} = 0.00025 \text{ m}.$$

$$\mu_{water} = 1.14 \times 10^{-3} \text{ N.s/m}^2.$$

$$\tau_w = \mu\left[\vec{n} \cdot \nabla u\right]_{wall}.$$

$$= \mu\left[\langle 0, -1\rangle \cdot \langle \underbrace{\cancel{\frac{\partial u}{\partial x}}}_{0}, \frac{\partial u}{\partial y}\rangle\right] = -\mu \frac{\partial u}{\partial y}\bigg|_{y = h/2}$$

$$= -\mu \times u_{max}\left[0 - \frac{8y}{h^2}\right] = \frac{8\mu u_{max} \cdot y}{h^2}\bigg|_{y = \frac{h}{2}}$$

$$\tau_w = \frac{8\mu u_{max} \cdot h/2}{h^2} = \frac{4\mu \cdot u_{max}}{h}.$$

$$\tau_w = \frac{4 \times 1.14 \times 10^{-3} \times 0.1}{0.00025} = \frac{4.56 \times 10^{-3} \times 0.1 \times 10^{5}}{25}$$

$$= \frac{45.6}{25} = 1.824 \text{ N/m}^2 \quad \text{Ans}$$

Direction $\longrightarrow$

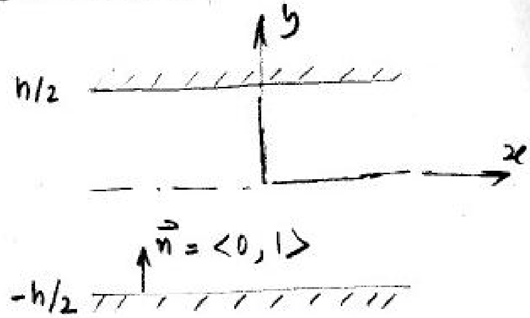
#7.

The velocity distribution for the laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

Where h is the distance separating the plates and the origin is placed midway between the plates. Consider the flow of water with the maximum speed of 0.05 m/s and $h = 1$ mm. Calculate the force on a 1 m^2 section of the lower plate and give its direction.

Given $u = u_{max}\left[1 - \left(\frac{2y}{h}\right)^2\right]$



$$\tau = \mu[\vec{n} \cdot \nabla u]$$

$$= \mu\left[\langle 0, 1 \rangle \cdot \left\langle \underbrace{\cancel{\frac{\partial u}{\partial x}}}_{0}, \frac{\partial u}{\partial y} \right\rangle\right]$$

$$= \mu \frac{\partial u}{\partial y}$$

$$\tau_{lower} = \mu \frac{\partial u}{\partial y}\bigg|_{y = -\frac{h}{2}} = \mu u_{max}\left[0 - \frac{8y}{h^2}\right]_{y = -\frac{h}{2}}$$

$$= -\frac{8\mu u_{max}}{h^2} \cdot -\frac{h}{2} = \frac{4\mu u_{max}}{h}$$

$$F_{Lower} = \tau_{Lower} \times Area = \tau_{Lower} \times 1$$

$$= \frac{4\mu u_{max}}{h} = \frac{4 \times 1.14 \times 10^{-3} \times 0.05}{1 \times 10^{-3}} = 0.228 \text{ N}$$

Ans

Direction : $\rightarrow$, positive x-dir.

#8.

Crude oil, with specific gravity SG = 0.85 and the viscosity $\mu = 2.15 \times 10^{-3}$ lbf·s/ft^2, flows steadily down a surface inclined $\theta = 30°$ below the horizontal in a film of thickness $h = 0.125$ in. The velocity profile is given by

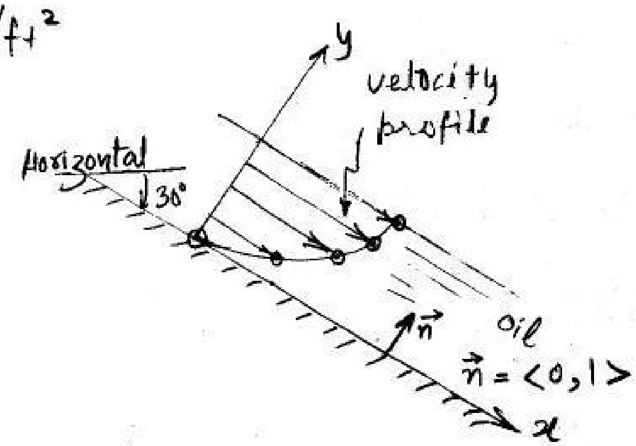
$$u = \frac{\rho g}{\mu}\left(hy - \frac{y^2}{2}\right)\sin\theta$$

(Coordinate x is along the surface and y is normal to the surface.) Plot the velocity profile. Determine the magnitude and direction of the shear stress τ that acts on the surface.

Given : $\mu = 2.15 \times 10^{-3}$ lbf s/ft^2

$h = 0.125$ in

$= \frac{0.125}{12}$ ft.



$\rho_{water} = 1.94$ slug/ft^3

$$u = \frac{\rho g}{\mu}\left[hy - \frac{y^2}{2}\right]\sin\theta$$

$$= \frac{(0.85 \times 1.94) \times 32.4}{2.15 \times 10^{-3}}\left[\frac{0.125}{12}y - \frac{y^2}{2}\right]\sin 30°$$

$$u = u(y) = 24850\left[0.010417\,y - 0.5y^2\right] \times 0.5$$

$$= 129.43\,y - 6212.5\,y^2$$

$$\tau_w = \mu\left[\langle 0,1 \rangle \cdot \langle \underbrace{\cancel{\tfrac{\partial u}{\partial x}}}_{0}, \frac{\partial u}{\partial y} \rangle\right] = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$= \mu\left[129.43 - 12425\,y\right]_{y=0}$$

$$= \mu \times 129.43 = 2.15 \times 10^{-3} \times 129.43$$

Ans

$$\tau_w = 0.27827 \text{ lbf/ft}^2 \text{ in positive } x\text{-dir.}$$

Take few values like $y = (0, 0.25h, 0.5h, 0.75h, h)$ and get u values. The profile will look something like shown in diagram.