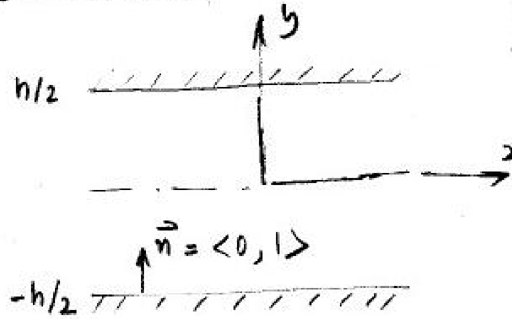
The velocity distribution for the laminar flow between parallel plates is given by

$$\frac{u}{u_{max}} = 1 - \left(\frac{2y}{h}\right)^2$$

Where h is the distance separating the plates and the origin is placed midway between the plates. Consider the flow of water with the maximum speed of 0.05 m/s and $h = 1$ mm. Calculate the force on a 1 m^2 section of the lower plate and give its direction.

Given $u = u_{max}\left[1 - \left(\frac{2y}{h}\right)^2\right]$



$$\tau = \mu[\vec{n} \cdot \nabla u]$$

$$= \mu\left[\langle 0, 1 \rangle \cdot \left\langle \underbrace{\cancel{\frac{\partial u}{\partial x}}}_{0}, \frac{\partial u}{\partial y} \right\rangle\right]$$

$$= \mu \frac{\partial u}{\partial y}$$

$$\tau_{lower} = \mu \frac{\partial u}{\partial y}\Big|_{y=-\frac{h}{2}} = \mu u_{max}\left[0 - \frac{8y}{h^2}\right]_{y=-\frac{h}{2}}$$

$$= -\frac{8\mu u_{max}}{h^2} \cdot -\frac{h}{2} = \frac{4\mu u_{max}}{h}$$

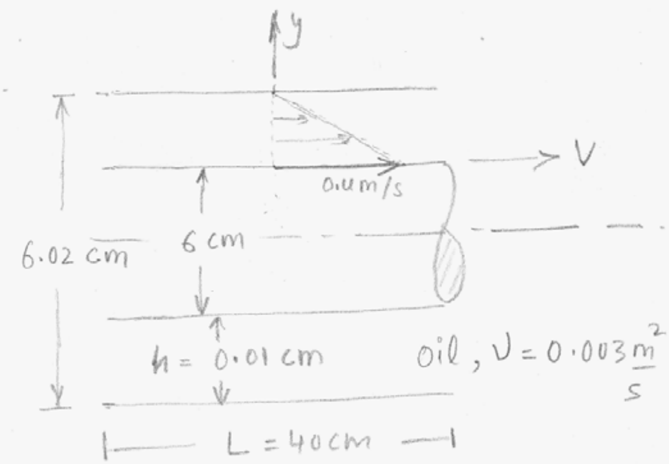
$$F_{Lower} = \tau_{Lower} \times \text{Area} = \tau_{Lower} \times 1$$

$$= \frac{4\mu u_{max}}{h} = \frac{4 \times 1.14 \times 10^{-3} \times 0.05}{1 \times 10^{-3}} = 0.228 \text{ N}$$

Ans

Direction : $\longrightarrow$, positive x-dir.

A shaft 6.00 cm in diameter is being pushed axially through a bearing sleeve 6.02 cm in diameter and 40 cm long. The clearance, assumed uniform, is filled with oil whose properties are $\nu = 0.003\ m^2/s$ and SG = 0.88. Estimate the force required to pull the shaft at a steady velocity of 0.4 m/s.



Gap, $h = 0.01$ cm

$= 0.0001$ m

$\nu_{oil} = 0.003\ m^2/s$

$SG = 0.88$

$\rho_{oil} = 1000 \times 0.88$

$= 880\ kg/m^3$

So $\mu = \rho_{oil}\nu_{oil} = 880 \times 0.003 = 2.64\ kg\ m^{-1}s^{-1}$

$\tau = \mu \dfrac{dv}{dy} = 2.64 \times \dfrac{0.0 - 0.4}{0.0001} = -10560\ N/m^2$

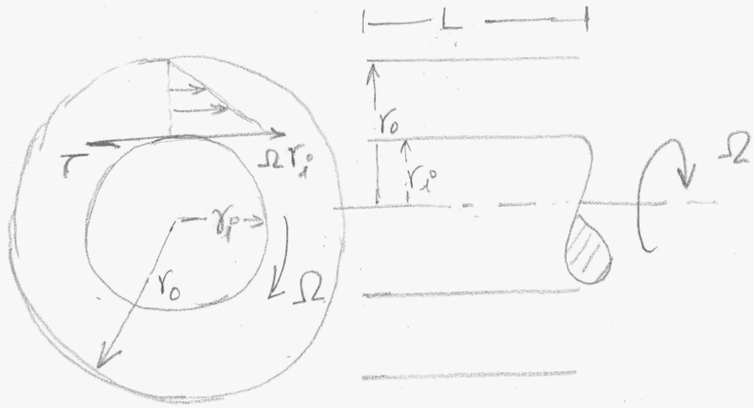
Surface area of shaft, $A = 2 \times \pi \times 0.03 \times 0.4$

$= 0.0754\ m^2$

$F = \tau A = -10560 \times 0.0754$

$= 796.2$ N

An amazing number of commercial and laboratory devices have been developed to measure fluid viscosity. Consider a concentric shaft, as in the previous problem but now fixed axially and rotating inside the sleeve. Let the inner and the outer cylinders have radii r_i and r_o, respectively, with total sleeve length L. Let the rotational rate be Ω (rad/s) and the applied torque be M. Using these parameters, derive a theoretical relation for the viscosity μ of the fluid between the cylinders.



Applied torque,
$= M$

Velocity at $r_i = \Omega r_i$

at $r_o = 0$

$$\frac{dv}{dr} = \frac{\Omega r_i}{r_o - r_i}$$

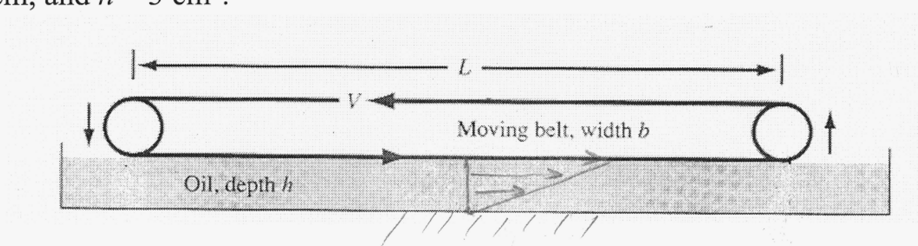
$$\tau = \mu \frac{\Omega r_i}{r_o - r_i}$$

$$M = \tau A r_i = \frac{\mu \Omega r_i}{r_o - r_i} \cdot 2\pi r_i L \cdot r_i$$

$$= \frac{\mu \Omega 2\pi r_i^3 L}{r_o - r_i}$$

so $$\mu = \frac{M(r_o - r_i)}{2\pi r_i^3 \Omega L}$$

The belt in the figure moves at a steady velocity V and skims the top of a tank of oil of viscosity μ, as shown. Assuming a linear velocity profile in the oil, develop a simple formula for the required belt-drive power P as a function of (h, L, V, b, μ). What belt-drive power P, in watts, is required if the belt moves at 2.5 m/s over SAE 30 oil, with $L = 2$ m, $b = 60$ cm, and $h = 3$ cm ?



$$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{h}$$

$$F = \tau A = \mu \frac{V}{h} . Lb$$

(a) $$P = FV = \mu \frac{V}{h} L b V = \frac{\mu V^2}{h} L b$$

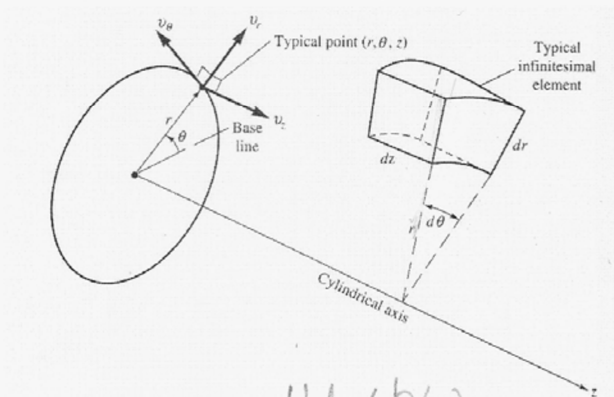
(b) $$P = 0.29 . \frac{2.5^2}{0.03} \times 2 \times 0.6 \cong 72.5 \text{ W}$$

μ_{oil} ; From the table in Prob # 2.

A highly viscous (nonturbulent) fluid fills the gap between two long concentric cylinders of radii a and $b > a$, respectively. If the outer cylinder is fixed and the inner cylinder moves steadily at an axial velocity U, the fluid will move at the axial velocity

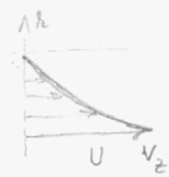
$$v_z = \frac{U \ln(b/r)}{\ln(b/a)}$$

See figure below for a definition of the velocity component v_z. Sketch this velocity distribution between the cylinders and comment. Find expressions for the shear stresses at both the inner and outer cylinder surfaces and explain why they are different.



Given $v_z = \dfrac{U \ln(b/r)}{\ln(b/a)}$

I. Velocity profile : Plot using fake values →



II. $\dfrac{dv_z}{dr} = -\dfrac{U}{r \ln(b/a)}$

$$\tau_a = \mu \frac{dv_z}{dr}\bigg|_{r=a} = -\frac{\mu U}{a \ln(b/a)}$$

$$\tau_b = \mu \frac{dv_z}{dr}\bigg|_{r=b} = -\frac{\mu U}{b \ln(b/a)}$$

Two are different because

I $\quad \dfrac{dv_z}{dr}\bigg|_{r=a} \neq \dfrac{dv_z}{dr}\bigg|_{r=b}$

II $\quad a \neq b$

a.) A liquid compressed in a cylinder has a volume of 1 liter ($L = 1000\ \text{cm}^3$) at 1 MN/m^2 and a volume of 995 cm^3 at 2 MN/m^2. What is its bulk modulus?

b.) A rigid tank contains air at a pressure of 90 psia and a temperature of 60°F. By how much will the pressure increase when the temperature is increased to 110°F?

ⓐ

$$\forall_1 = 1000\ \text{cm}^3 = 1000 \times (10^{-2}\ \text{m})^3 = 1000 \times 10^{-6}\ \text{m}^3$$

$$p_1 = 1 \times 10^6\ \text{N/m}^2$$

$$\forall_2 = 995 \times 10^{-6}\ \text{m}^3$$

$$p_2 = 2 \times 10^6\ \text{N/m}^2$$

$$\text{Bulk Modulus, } E_V = -\frac{\Delta p}{\Delta\forall/\forall} = -\frac{[2-1]\times 10^6}{\frac{[995-1000]\times \cancel{10^{-6}}}{1000 \times \cancel{10^{-6}}}}$$

$$= \frac{1000 \times 10^6}{5} = 200 \times 10^6\ \text{N/m}^2 \quad \text{Ans}$$

ⓑ

$$p = \rho R T$$

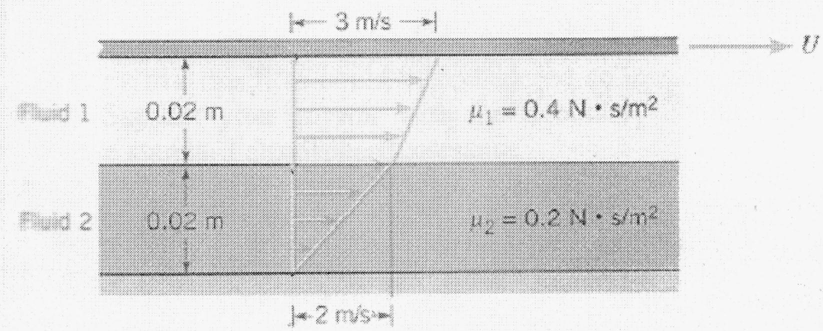
for rigid body $\rho = \frac{m}{\forall} = \text{constt.}$

So $\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow p_2 = \frac{p_1 T_2}{T_1}$

$$\Rightarrow p_2 = 90 \times \frac{(110 + 460)}{(60 + 460)} = 90 \times \frac{570}{520}$$

$$p_2 = 98.7\ \text{psia} \quad \text{Ans}$$

1.54 As shown in Video V1.2, the "no slip" condition means that a fluid "sticks" to a solid surface. This is true for both fixed and moving surfaces. Let two layers of fluid be dragged along by the motion of an upper plate as shown in Fig. P1.54. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the botton plate. Determine the ratio of these two shear stresses.

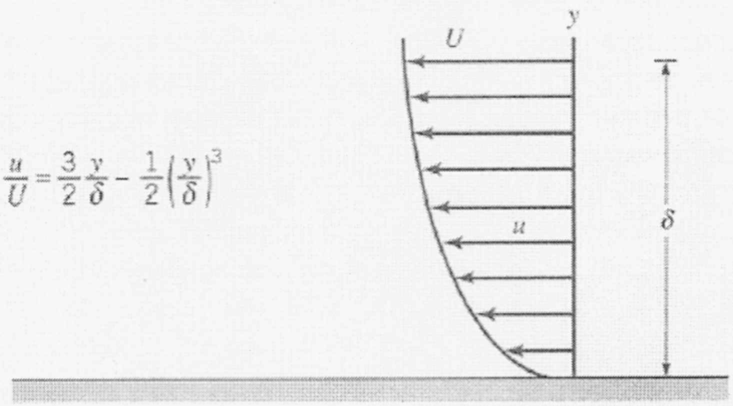


$$\tau_{top} = \mu_1 \frac{dv}{dz} = 0.4 \times \frac{3-2}{0.02} = 20 \text{N} m^{-2}$$

$$\tau_{bot} = \mu_2 \frac{dv}{dz} = 0.2 \times \frac{2-0}{0.02} = 20 \text{N} m^{-2}$$

$$\text{Ratio} = \frac{\tau_I}{\tau} = 1$$

1.58 A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4} m^2/s$ flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero (as shown in Video V1.2), and the velocity profile near the surface is shown in Fig. P1.58. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



$$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Given $u = U\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)$

$$\frac{du}{dy} = U\left(\frac{3}{2\delta} - \frac{3}{2\delta^3} \cdot y^2\right)$$

$$= \frac{3U}{2\delta}\left(1 - \frac{y^2}{\delta^2}\right)$$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{3U}{2\delta}$$

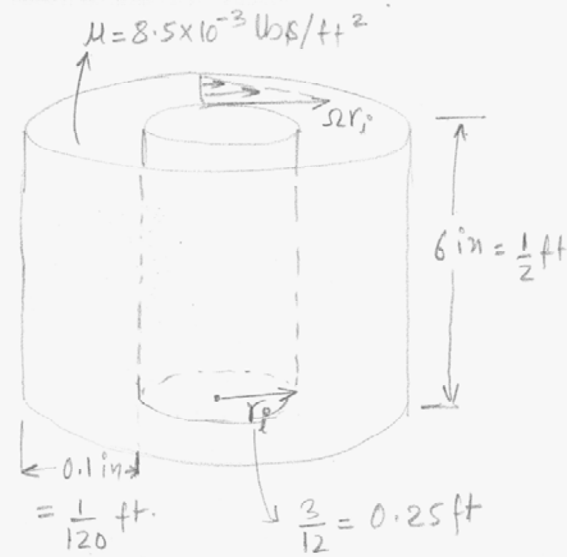
$$\tau_{surface} = \mu \left.\frac{du}{dy}\right|_{y=0} = \nu\rho \cdot \frac{3U}{2\delta}$$

$$= 4 \times 10^{-4} \times 0.92 \times 1000 \times \frac{3}{2}\frac{U}{\delta}$$

$$= 0.552 \frac{U}{\delta} \frac{N}{m^2}$$

Direction same as Velocity

1.62 The space between two 6-in.-long concentric cylinders is filled with glycerin (viscosity = 8.5×10^{-3} lb · s/ft²). The inner cylinder has a radius of 3 in. and the gap width between cylinders is 0.1 in. Determine the torque and the power required to rotate the inner cylinder at 180 rev/min. The outer cylinder is fixed. Assume the velocity distribution in the gap to be linear.



Rotation speed, Ω

$$= 180 \text{ RPM}$$

$$= \frac{180 \times 2\pi}{60} = 6\pi \ \frac{\text{rad}}{\text{s}}$$

$$V_i = \Omega r_i = 6\pi \times 0.25$$

$$= 1.5\pi \ \text{ft s}^{-1}$$

$$\tau_{surf} = \mu \left.\frac{dV}{dr}\right|_{r=r_i}$$

$$= \mu \cdot \frac{\Omega r_i - 0}{r_o - r_i} = 8.5\times10^{-3} \ \frac{1.5\pi}{(1/120)}$$

$$= 8.5\times10^{-3} \times 1.5\pi \times 120 = 4.807 \ \frac{\text{lbf}}{\text{ft}^2}$$

$$A = 2\pi \cdot r_i L = 2\pi \times 0.25 \times 0.5 = 0.7854 \ \text{ft}^2$$

Torque, $T = \tau A r_i = 4.807 \times 0.7854 \times 0.25 = 0.944$ lbf.ft

Power $P = T\Omega = 0.944 \times 6\pi \simeq 17.8 \ \frac{\text{lbft}}{\text{s}}$