

Solutions to HW#3 SUMM 07

#1.

Answer in 4 to 5 lines in the space provided for each question:

(a) A tank partially filled with water has a balloon well below the free surface and anchored to the bottom by a string. The tank is accelerated with an acceleration a_x to the right. Which direction the balloon would tilt to and why?

Buoyancy force acts in a direction opposite to pressure gradient. When the tank is accelerated to the right, the free water surface tilts towards right such that the direction of pressure gradient is towards left. Therefore the buoyancy force on the balloon acts in the right direction making it tilt in the right direction as against common sense.

(b) Write important assumptions used in derivation of Bernoulli's equation.

Main Assumptions:

- Inviscid flow – viscosity (internal friction) = 0
- Steady flow
- Incompressible flow – $\rho = \text{constant}$ along a streamline. ρ may vary from streamline to streamline, however.
- Generally, the equation applies along a streamline. For constant-density potential flow, it applies throughout the entire flow field.

(c) Apart from airplane wing, give an example based on Bernoulli's principle

Low speed wind tunnel is a direct application of Bernoulli's principle. It has a converging section (contraction), constant area section where models are placed (test section) and a divergent section (diffuser). Low speed wind in the test section is created by accelerating air through contraction using a fan. Kinetic energy is reconverted into pressure energy in the diffuser with minimum losses.

(d) Can Bernoulli's relation be applied inside boundary layer? Explain briefly.

Bernoulli's relation can't be applied inside boundary layer which is essentially a consequence of viscosity, therefore a region of dissipation of energy. It violates the first assumption i.e. Inviscid flow – viscosity (internal friction) = 0, hence can't be used inside a boundary layer.

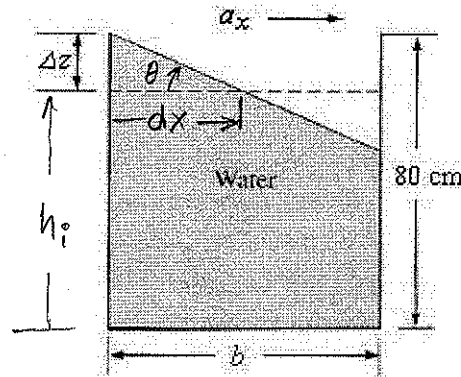
(e) Vortex flows are of two types: free vortex and forced vortex. Can we apply Bernoulli equation in a vortex flows?

For free vortex, $V = C/r$. Example of free vortex is hurricane. Flow inside free vortex can generally be assumed to be steady, incompressible and irrotational (excluding eye). Therefore Bernoulli equation can be applied.

For forced vortex, $V = Cr$. It represents rigid body rotation. Since it isn't irrotational flow field, we can't apply Bernoulli relation arbitrarily between any two points. We have to make sure that the two points lie on the same streamline.

#2.

An 80 cm high fish tank of cross section 2 m × 0.6 m that is initially filled with water is to be transported on the back of a truck. The truck accelerates from 0 to 90 km/h in 10 s. It is desired that no water spill during acceleration. Determine the maximum initial water height in the tank for no spillage. Would you recommend the tank to be aligned with the long or short side parallel to the direction of motion?



$$a_x = \frac{(90-0) \text{ km/h}}{10 \text{ s}} \times \frac{1 \text{ m/s}}{3.6 \text{ km/h}} = 2.5 \text{ ms}^{-2}$$

For no-spillage, vertical rise of the free surface Δz should be such that $(80 \text{ cm} - h_i) = \Delta z$ where:

h_i = maximum initial height of water.

$$\frac{\Delta z}{\Delta x} = \frac{a_x}{a_x + g} = \frac{2.5}{9.8} = 0.255$$

$$\Rightarrow \Delta z = 0.255 \Delta x = 0.255 \times \frac{b}{2}$$

b can be either 2 m or 0.6 m.

Case I $b = 2 \text{ m}$.

$$\Delta z = 0.255 \times \frac{2}{2} = 0.255 \text{ m} = 25.5 \text{ cm}$$

Case II $b = 0.6 \text{ m}$.

$$\Delta z = 0.255 \times 0.3 = 0.0765 \text{ m} = 7.65 \text{ cm}$$

We want to keep Δz minimum in order to maximise h_i

$$\therefore 80 \text{ cm} - h_i = 7.65 \text{ cm}$$

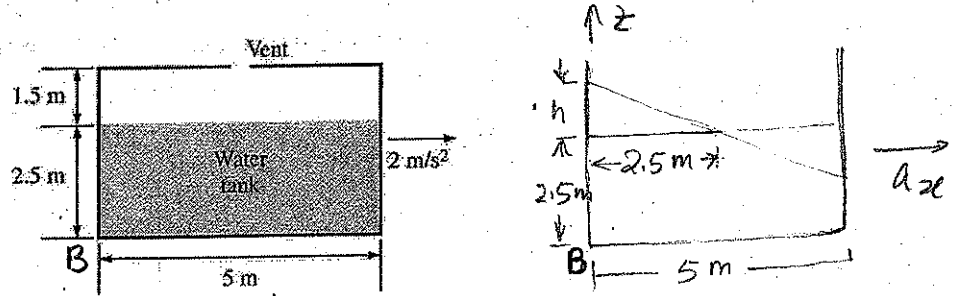
$$\Rightarrow h_i = 72.35 \text{ cm}$$

\therefore Case (II) is recommended

Ans

3.

A 5 m long, 4 m high tank contains 2.5 m deep water when not in motion and is open to the atmosphere through a vent in the middle. The tank is now accelerated to the right on a level surface at 2 m/s^2 . Determine the maximum pressure in the tank relative to the atmospheric pressure.



$$a_x = 2 \text{ m/s}^2$$

$$\frac{dz}{dx} = -\frac{a_x}{a_z + g} \quad (a_z = 0, \because \text{motion on level surface})$$

$$-\frac{h}{2.5} = -\frac{2}{9.8} \Rightarrow h = \frac{2 \times 2.5}{9.8} = 0.51 \text{ m}$$

$$\text{So total height} = h + 2.5 = 3.01 \text{ m}$$

Maximum pressure will be at point B:

$$p_{\max} = \gamma_w (h + 2.5)$$

$$= 9790 \times 3.01 = 29468 \text{ Pa.} \quad \underline{\text{Ans}}$$

4

A 16-cm-diameter open cylinder 27 cm high is full of water. Compute the rigid body rotation rate about its central axis, in rot/min, (a) for which one-third of the water will spill out and (b) for which the bottom will be barely exposed.

$$\begin{aligned}\text{Initial volume} &= \pi R^2 H \\ &= 0.00543 \text{ m}^3\end{aligned}$$

a)

After $\frac{1}{3}$ of water gone:

$$V = \frac{2}{3} \times 0.00543 = 0.00362 \text{ m}^3$$

$$\text{Height } H = \frac{0.00362}{(\pi)(0.08)^2} = 0.18 \text{ m}$$

H is in the middle of $(27 - h_0)$

$$\text{i.e. } \frac{0.27 - h_0}{2} = 0.18$$

$$\Rightarrow h_0 = 0.36 - 0.27 = 0.09 \text{ m}$$

$$\text{Now } H - h_0 = \frac{\omega^2 R^2}{4g} \Rightarrow \omega = \sqrt{\frac{(H - h_0) 4 \times 9.8}{R^2}}$$

$$\omega = 23.48 \text{ rad/s} \quad \text{or} \quad 224.12 \text{ RPM.} \quad \underline{\text{Ans}}$$

b). when surface goes all the way to bottom

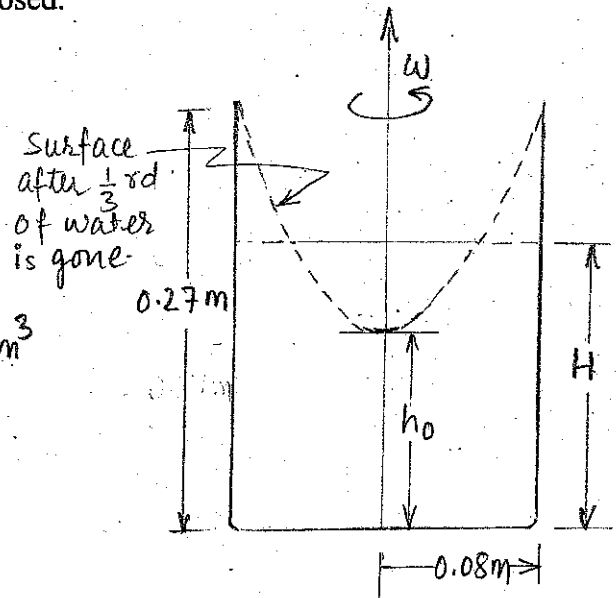
$$h_0 = 0$$

$$H = \frac{0.27}{2} = 0.135 \text{ m}$$

$$H - h_0 = \frac{\omega^2 R^2}{4g}$$

$$\omega = \sqrt{\frac{(H - h_0) 4g}{R^2}} = \sqrt{\frac{0.135 \times 4 \times 9.8}{(0.08)^2}}$$

$$\omega = 28.76 \text{ rad/s} \quad \text{or} \quad 274.5 \text{ RPM} \quad \underline{\text{Ans}}$$



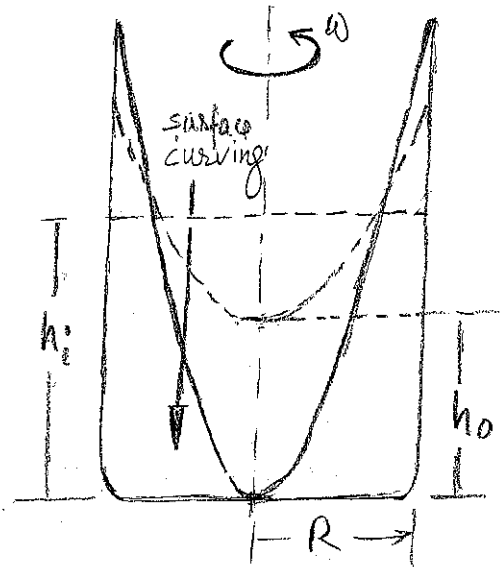
#

An open 1 m-diameter tank contains water at a depth of 0.7 m at rest. As the tank is rotated about its vertical axis, the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.

Given: Radius, $R = 0.5$ m.

Initial (non-rotating) level of water, $h_i = 0.7$ m.

Initial volume, $V_1 = \pi R^2 h_i$ — (I)



When fluid rotates as a rigid body, free surface takes the shape of paraboloid with h_0 as lowest point at center. As long as there is no spill, the volume under the paraboloid is given as:

$$V = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_0 \right)$$

In the problem, the free surface has to reach bottom of cylinder i.e. $h_0 = 0$.

So the Volume, $V_2 = \frac{\pi \omega^2 R^4}{4g}$ — (II)

Since there is no spill, therefore

$$\begin{aligned} V_2 &= V_1 \\ \frac{\pi \omega^2 R^4}{4g} &= \pi R^2 h_i \\ \Rightarrow \omega &= \sqrt{\frac{4g h_i}{R^2}} = \sqrt{\frac{4 \times 9.8 \times 0.7}{(0.5)^2}} = 10.5 \text{ rad/s} \end{aligned}$$

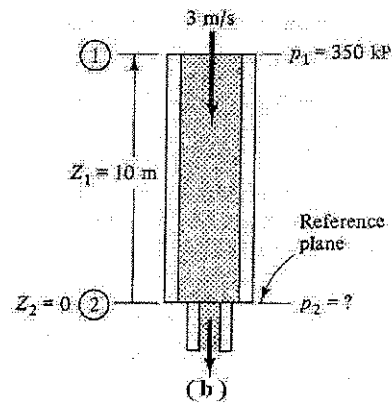
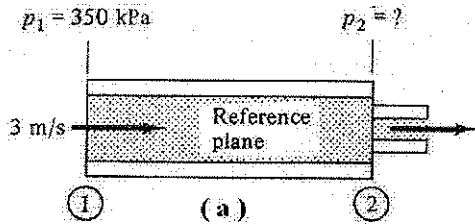
Ans

#6.

Water ($\gamma = 9810 \text{ N/m}^3$) flows in a pipe. At a section where the inside diameter is 150 mm, the velocity is 3 m/s and the pressure is 350 kPa. At a section located 10 m from the first section, the inside diameter reduces to 75 mm. Calculate the pressure at the second section.

(a) If the pipe is horizontal

(b) If the pipe is vertical and the flow is downward



$$D_1 = 150 \text{ mm} = 0.15 \text{ m} \quad V_1 = 3 \text{ m/s}$$

$$D_2 = 75 \text{ mm} = 0.075 \text{ m}$$

$$\text{Continuity Eqn: } A_2 V_2 = A_1 V_1$$

$$\Rightarrow V_2 = \frac{A_1 V_1}{A_2} = \frac{D_1^2}{D_2^2} V_1 = \frac{0.15^2}{0.075^2} \times 3 = 12 \text{ m/s}$$

Apply Bernoulli eqn:

$$\textcircled{a} \quad \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{350}{9810} + \frac{3^2}{2 \times 9.81} = \frac{12^2}{2 \times 9.81} + \frac{p_2}{9810}$$

$$\Rightarrow p_2 = 282.5 \text{ kPa} \quad \underline{\text{Ans}}$$

\textcircled{b} for vertical flow:

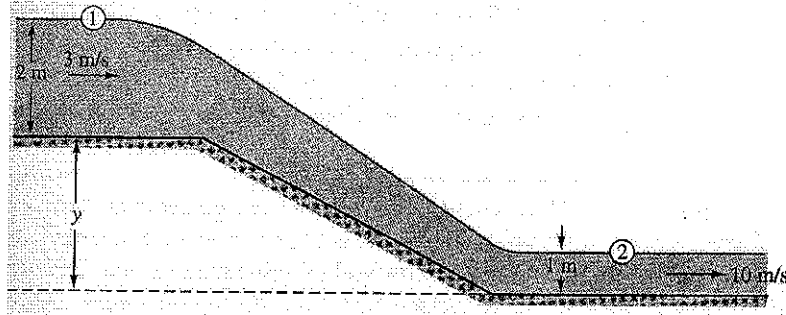
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{350}{9810} + \frac{3^2}{2 \times 9.8} + 10 = \frac{p_2}{9810} + \frac{12^2}{2 \times 9.8} + 0$$

$$\Rightarrow p_2 = 380.6 \text{ kPa} \quad \underline{\text{Ans}}$$

#7.

Water is flowing in an open channel at a depth of 2 m and a velocity of 3 m/s. It then flows down a contracting chute into another channel where the depth is 1 m and the velocity is 10 m/s. Assuming a frictionless flow, determine the difference in elevation of the channel floors, y .



Apply Bernoulli eqn between ① and ②

$$\frac{p_1}{\rho} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2g} + z_2$$

(0, gage)

$$z_1 - z_2 = \frac{1}{2g} [v_2^2 - v_1^2]$$

$$y + 2 - 1 = \frac{1}{2 \times 9.8} [10^2 - 3^2] = \frac{100 - 9}{2 \times 9.8}$$

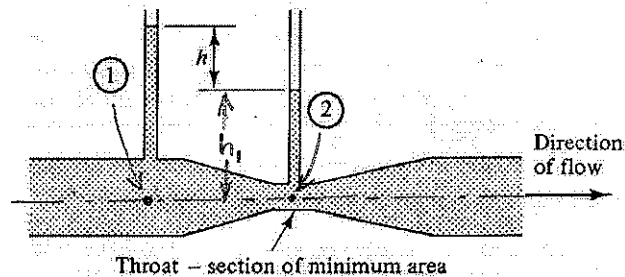
$$y + 1 = \frac{91}{2 \times 9.8}$$

$$y = \frac{91}{2 \times 9.8} - 1$$

$$= 4.643 - 1 = 3.643 \text{ m} \quad \underline{\text{Ans}}$$

#8.

The device in the figure is called *venturi meter*, which consists of a converging and diverging conical section of a pipe arranged to give an increase in velocity as the pipe converges, causing a measurable drop in pressure. The diverging section is used to reconvert the increased kinetic energy of the fluid stream into pressure energy at the outlet with minimum losses. Show for this meter with no losses that



Apply Bernoulli equation between sections ① and ②

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{--- (I)}$$

substitute $V_2 = V_1 A_1 / A_2$ (from continuity eqn) to (I)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{[V_1 (A_1 / A_2)]^2}{2g}$$

$$\Rightarrow \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{V_1^2}{2g} \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right] \quad \text{--- (II)}$$

Manometers give:

$$p_1 - \gamma(h + h_1) = p_2 - \gamma h_1 = p_{\text{atm}}$$

$$\Rightarrow p_1 - p_2 = \gamma h$$

$$\Rightarrow \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = h \quad \text{substitute in (II)}$$

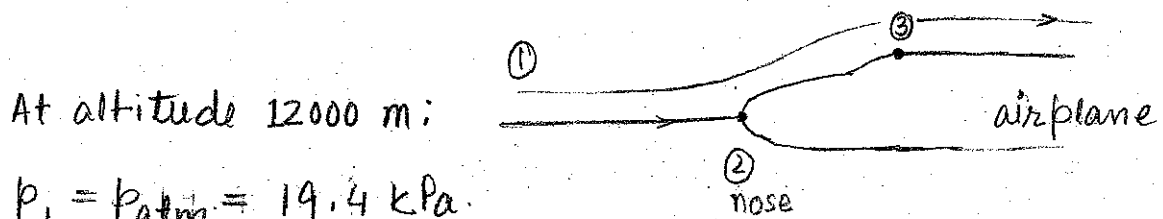
$$\frac{V_1^2}{2g} \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right] = h$$

$$V_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2} \right)^2 - 1}}$$

$$Q = A_1 V_1 = A_1 \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2} \right)^2 - 1}} \quad \text{Ans}$$

#

An airplane is flying at an altitude of 12000 m. Determine the gage pressure at the stagnation point on the nose of the plane if the speed of the plane is 200 km/h. What would be the gage pressure at a point on the surface of plane where the relative wind speed is 250 km/h?



$$p_1 = p_{atm} = 19.4 \text{ kPa.}$$

$$\rho_1 = 0.312 \text{ kg/m}^3$$

Apply Bernoulli relation between free stream ① and nose ② where the flow stagnates

$$p_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

$p_{\text{gage}} = 0$

$$\frac{1}{2} \times 0.312 \times \left(\frac{200 \times 10^3}{3600} \right)^2 = p_{2\text{-gage}}$$

$$p_{2\text{-gage}} = \frac{1}{2} \times 0.312 \times (55.55)^2 = 481.5 \text{ Pa.} \quad \underline{\text{Ans}}$$

At point ③, $V_3 = \frac{250 \times 10^3}{3600} = 69.44 \text{ m/s}$

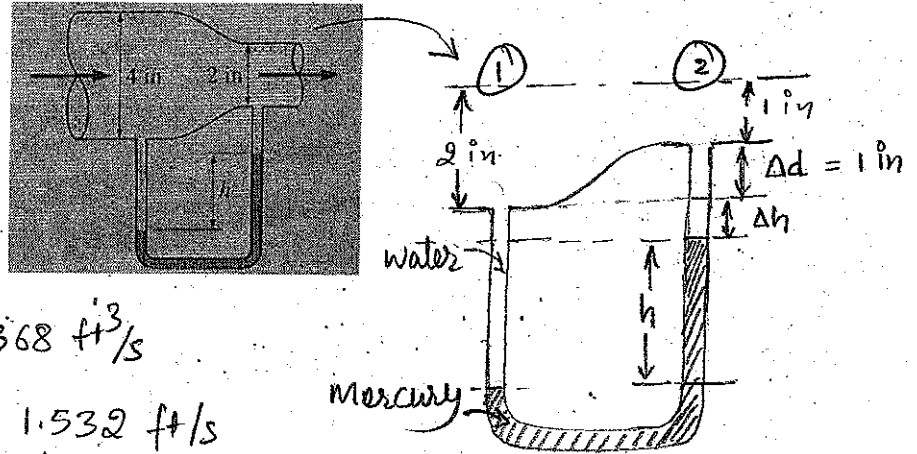
$$p_1 + \frac{1}{2} \rho V_1^2 = p_3 + \frac{1}{2} \rho V_3^2$$

$$p_{3\text{-gage}} = \frac{1}{2} \rho (V_1^2 - V_3^2) = \frac{1}{2} \times 0.312 \times (55.55^2 - 69.44^2)$$

$$= -271 \text{ Pa.} \quad \underline{\text{Ans}}$$

#

Water flows through a horizontal pipe at a rate of 1 gal/s. The pipe consists of two sections of diameters 4 in and 2 in with a smooth reducing section. The pressure difference between the two pipe sections is measured by a mercury manometer. Neglecting the frictional effects, determine the differential height of mercury between the two pipe sections.



Given :

$$Q = 1 \text{ gal/s} = 0.13368 \text{ ft}^3/\text{s}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4} \left(\frac{4}{12}\right)^2} = 1.532 \text{ ft/s}$$

$$V_2 = \frac{Q}{A_2} = 6.1265 \text{ ft/s}$$

Applying Bernoulli relation between ① and ②

$$p_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

$$\Rightarrow p_1 - p_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) = \frac{1}{2} \times 1.94 \times (6.126^2 - 1.532^2)$$

$$= 34.1312 \text{ lbf/ft}^2 \quad \text{--- ①}$$

From hydrostatics :

$$p_1 + \gamma_w (h + \Delta d) = p_2 + \gamma_w (\Delta d) + \gamma_m h$$

$$p_1 - p_2 = (\gamma_m - \gamma_w) h + \gamma_w \Delta d$$

$$= (846.54 - 62.4) h + 62.4 \times \frac{1}{12}$$

$$= 784.14 h + 5.2 \quad \text{--- ②}$$

Equating ① and ②

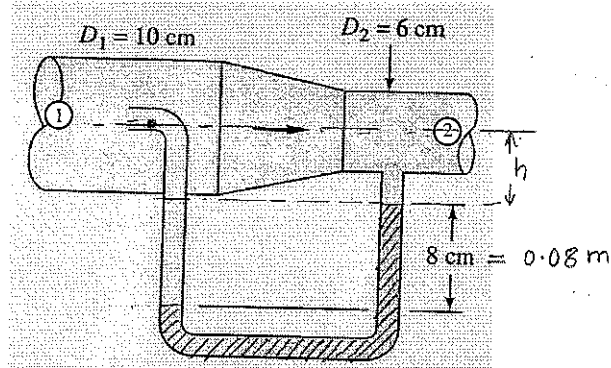
$$784.14 h + 5.2 = 34.1312$$

$$h = 0.0369 \text{ ft} = 0.443 \text{ inches} \quad \text{Ans}$$

$$\text{Ignoring } \Delta d, \quad h = \frac{34.1312}{784.14} = 0.0435 \text{ ft} = 0.522 \text{ inches}$$

#11.

In Fig. given the flowing fluid is CO_2 . Neglecting losses, if the total pressure at 1, p_{01} , is 170 kPa and the manometer fluid is Meriam red oil (SG = 0.827), estimate (a) p_2 and (b) the gas flow rate in m^3/h .



For CO_2 : $\rho_{\text{CO}_2} = 1.83 \text{ kgm}^{-3}$, $\gamma_{\text{CO}_2} = 18 \text{ Nm}^{-3}$

Since there are no losses, $p_{01} = p_{02}$

$$\Rightarrow p_1 + \frac{1}{2} \rho_1 V_1^2 = p_2 + \frac{1}{2} \rho_2 V_2^2 \quad \text{--- (I)}$$

From manometer :

$$p_{01} + (h + 0.08) \gamma_{\text{CO}_2} = p_2 + h \gamma_{\text{CO}_2} + 0.827 \times \gamma_w \times 0.08$$

$$p_1 + \frac{1}{2} \rho_1 V_1^2 = p_2 + (0.827 \times \gamma_w - \gamma_{\text{CO}_2}) 0.08 \quad \text{--- (II)}$$

Equating (I) and (II)

$$p_2 + \frac{1}{2} \rho_2 V_2^2 = p_2 + (0.827 \times 9790 - 18) \times 0.08$$

$$\frac{1}{2} \rho_2 V_2^2 = 646.3$$

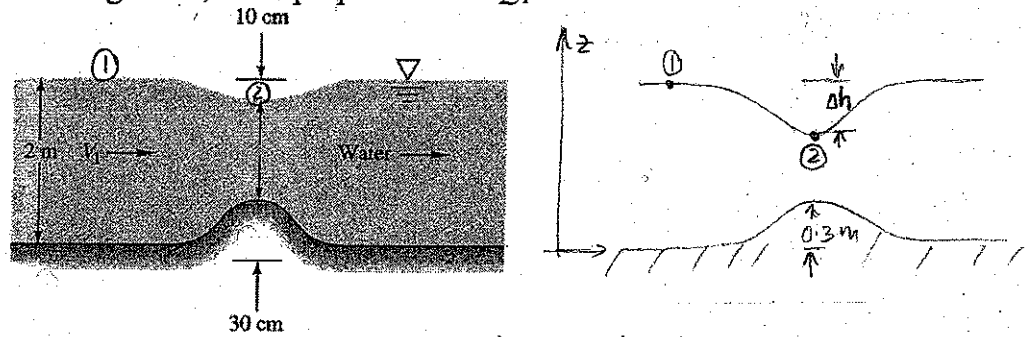
$$\Rightarrow V_2 = \sqrt{\frac{2 \times 646.3}{1.83}} = 26.58 \text{ ms}^{-1}$$

(a) $p_2 = p_{02} - \frac{1}{2} \rho_2 V_2^2 = 170000 - 646.3 = 169354 \text{ Nm}^{-2}$ Ans

(b) $Q = V_2 A_2 = 26.58 \times \frac{\pi}{4} \times (0.06)^2 = 0.07514 \text{ m}^3/\text{s}$
 $= 270.5 \text{ m}^3/\text{h}$ Ans

#

If the approach velocity is not too high, a hump in the bottom of a water channel causes a dip Δh in the water level, which can serve as a flow measurement. If, as shown in the Fig., $\Delta h = 10 \text{ cm}$ when the bump is 30 cm high, what is the volume flow Q_1 per unit width, assuming no losses? In general, is Δh proportional to Q_1 ?



continuity eqn: (Assume unit width)

$$A_1 V_1 = A_2 V_2$$

$$2V_1 = (1.7 - \Delta h)V_2$$

$$V_2 = \left(\frac{2}{1.7 - \Delta h} \right) V_1 = 1.25 V_1 \quad \text{--- (1)}$$

Apply Bernoulli Eqn. between ① and ②

$$\cancel{p}_1 + \frac{1}{2} \rho V_1^2 + \rho g z = \cancel{p}_2 + \frac{1}{2} \rho V_2^2 + \rho g (z - \Delta h)$$

$$\text{so } \cancel{p} g \Delta h = \frac{1}{2} \rho (V_2^2 - V_1^2)$$

$$\Rightarrow 2g\Delta h = V_2^2 - V_1^2 = \left[\left(\frac{2}{1.7 - \Delta h} \right)^2 - 1 \right] V_1^2$$

$$\Rightarrow V_1 = \frac{\sqrt{2g\Delta h}}{\sqrt{\left[\left(\frac{2}{1.7 - \Delta h} \right)^2 - 1 \right]}} = \sqrt{\frac{2g\Delta h (1.7 - \Delta h)^2}{4 - (1.7 - \Delta h)^2}}$$

$$\text{so } V_1 \propto \sqrt{\Delta h}$$

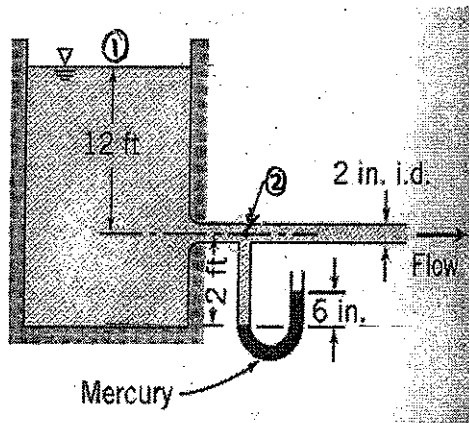
$$\text{or } \Delta h \propto V_1^2$$

Ans

$$Q_1 = A_1 V_1 = 2 \sqrt{\frac{2g\Delta h}{\left[\left(\frac{2}{1.7 - \Delta h} \right)^2 - 1 \right]}} = 3.73 \text{ m}^3/\text{s} \quad \text{--- Ans}$$

#6.

Water flows from a very large tank through a 2 in. diameter tube. The dark liquid in the manometer is mercury. Estimate the velocity in the pipe and the rate of discharge from the tank.



Bernoulli's eq. between ① and ②

$$P_{atm} + \frac{1}{2} \rho V_1^2 + \gamma_w \times 14 = P_2 + \frac{1}{2} \rho_w V_2^2 + \gamma_w \times 2$$

$$P_2 = P_{atm} + \gamma_w (14 - 2) - \frac{1}{2} \rho_w V_2^2 \quad \text{--- (I)}$$

Also from manometer :

$$P_{atm} + \gamma_m \cdot 6/12 - \gamma_w \times 2 \quad \text{--- (II)}$$

$$P_{atm} + \gamma_w \times 12 - \frac{1}{2} \rho_w V_2^2 = P_{atm} + 0.5 \times \gamma_m - 2 \gamma_w$$

$$\gamma_w \times (12 + 2) - 0.5 \times \gamma_m = \frac{1}{2} \rho_w V_2^2$$

$$62.4 \times 14 - 0.5 \times 13.56 \times 62.4 = 0.5 \times 1.938 \times V_2^2$$

$$V_2 = \sqrt{\frac{873.6 - 423.69}{0.9689}} = 21.55 \text{ ft/s}^{-1}$$

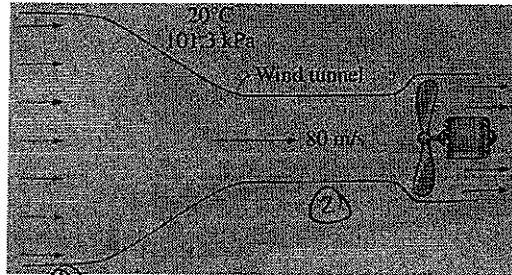
\dot{Q} = Rate of Discharge = Volume flow rate = AV

$$= \pi \times \left(\frac{1}{12}\right)^2 \times 21.55$$

$$\dot{Q} = 0.47 \frac{\text{ft}^3}{\text{s}}$$

#

A wind tunnel draws atmospheric air at 20°C and 101.3 kPa by a large fan located near the exit of the tunnel. If the air velocity in the tunnel is 80 m/s , determine the pressure in the tunnel.



$$p_1 = p_{\text{atm}} = 101330 \text{ N/m}^2$$

$$V_1 \approx 0$$

$$p_2 = ?$$

$$V_2 = 80 \text{ m/s}$$

Apply Bernoulli eqn between (1) and (2)

$$p_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

$$p_2 = p_1 + \frac{1}{2} \rho (V_1^2 - V_2^2)$$

$$= 101330 + \frac{1}{2} \times 1.23 \times (0 - 80^2)$$

$$= 101330 - 3936$$

$$= 97394 \text{ N/m}^2$$

$$p_2 = 97.394 \text{ kPa.}$$

Ans

#

The incompressible flow form of Bernoulli's relation,

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = \text{const}, \text{ is accurate only for Mach numbers less than } 0.3.$$

At higher speeds, variable density must be accounted for. The most common assumption for compressible fluid is *isentropic flow of an ideal gas*, or $p = C\rho^k$, where $k = c_p/c_v$. Substitute this relation into $\frac{dp}{\rho} + VdV + gdz = 0$, integrate, and eliminate the constant C . Compare your compressible result with first equation and comment.

General Bernoulli eqn :

$$VdV + \frac{dp}{\rho} + gdz = 0$$

integrating :

$$\frac{V^2}{2} + \int \frac{dp}{\rho} + gz = \text{const} \quad \text{--- (I)}$$

For isentropic flow ; $p = C\rho^k$

$$\Rightarrow \frac{dp}{d\rho} = Ck\rho^{k-1}$$

$$\Rightarrow dp = \frac{Ck\rho^{k-1}d\rho}{\rho}$$

Substitute this in (I)

$$\frac{V^2}{2} + \int \frac{Ck\rho^{k-1}d\rho}{\rho} + gz = \text{const}$$

$$\frac{V^2}{2} + Ck \int \rho^{k-2}d\rho + gz = \text{const}$$

$$\frac{V^2}{2} + Ck \frac{\rho^{k-1}}{k-1} + gz = \text{const}$$

$$\frac{V^2}{2} + \frac{k}{k-1} \frac{C\rho^k}{\rho} + gz = \frac{V^2}{2} + \frac{k}{k-1} \frac{p}{\rho} + gz = \text{const}$$

$$\frac{V^2}{2} + \frac{kRT}{k-1} + gz = \frac{V^2}{2} + c_p T + gz = \text{const}$$

$$\left[\because \frac{kR}{k-1} = c_p \right]$$

Ans.

#16

Consider an airfoil in a flow of air, where far ahead of the airfoil (the free stream), the pressure, velocity and density are 2116 lb/ft^2 , 500 mi/h and $0.002377 \text{ slug/ft}^3$ respectively. At a given point A on the airfoil, the pressure is 1497 lb/ft^2 . What is the velocity at point A ? Assume isentropic flow and $c_p = 6006 \text{ ft-lb/(slug)(}^\circ\text{R)}$.

$$\text{Given } V_\infty = 500 \text{ mph.}$$
$$= \frac{500 \times 5280}{3600} \text{ ft/s}$$

$$= 733.3 \text{ ft/s}$$

$$a_\infty = \sqrt{\gamma p_\infty / \rho_\infty} = \sqrt{\frac{1.4 \times 2116}{0.002377}} = 1116.4 \text{ ft/s}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{733.3}{1116.4} = 0.656$$

Since $M_\infty > 0.3$ flow is compressible.

$$T_\infty = \frac{p_\infty}{\rho_\infty R} = \frac{2116}{0.002377 \times 1716} = 519^\circ\text{R.}$$

Using isentropic relation to calculate T_A :

$$\frac{T_A}{T_\infty} = \left(\frac{p_A}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1497}{2116}\right)^{\frac{0.5}{1.4}} = 0.906$$

$$\therefore T_A = 0.906 T_\infty = 0.906 \times 519 = 470^\circ\text{R.}$$

Apply compressible Bernoulli equation between A and

free stream:

$$c_p T_\infty + \frac{V_\infty^2}{2} + g z_\infty = c_p T_A + \frac{V_A^2}{2} + g z_A$$

$$\Rightarrow V_A = \sqrt{2c_p(T_\infty - T_A) + V_\infty^2}$$
$$= \sqrt{2 \times 6006 \times (519 - 470) + 733.3^2}$$

$$V_A = 1061 \text{ ft/s} \quad \underline{\text{Ans}}$$

