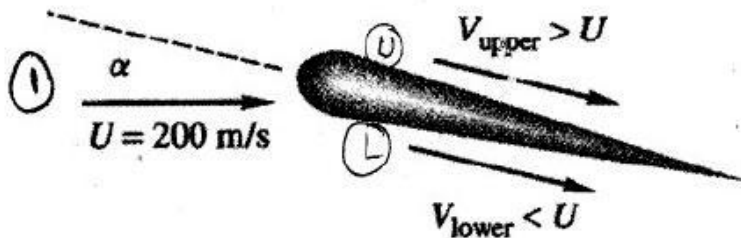


#1.

An airfoil at an angle of attack α , as shown in the Fig., provides lift by Bernoulli effect, because of lower surface slows the flow (high pressure) and the upper surface speeds up the flow (low pressure). If the foil is 1.5 m long and 18 m wide into the paper, and the ambient air is 5000 m standard atmosphere, estimate the total lift if the average velocities on upper and lower surfaces are 215 m/s and 185 m/s, respectively. Neglect gravity.

Note: For this case, the angle α is approximately 3° .



Wing Area, $A = 1.5 \times 18 = 27 \text{ m}^2$.

① 5000 m, $p_{\text{atm}} = p_1 = \frac{5.405 \times 10^4 \text{ N}}{\text{m}^2}$ } from Tables
 $T_1 = 255.6 \text{ K}$
 $\rho_1 = 0.7364 \text{ kg/m}^3$

Applying Bernoulli's eq between ① and U, ① and L

$$p_1 + \frac{1}{2} \rho_1 v_1^2 = p_U + \frac{1}{2} \rho_U v_U^2 = p_L + \frac{1}{2} \rho_L v_L^2 \quad [p_U = p_L = p_1 \text{ incomp flow}]$$

so we get:

$$p_U - p_1 = \frac{1}{2} \rho_1 (v_1^2 - v_U^2) = \frac{1}{2} \times 0.7364 \times (200^2 - 215^2) = -2292.045 \text{ Nm}^{-2}$$

$$p_L - p_1 = \frac{1}{2} \rho_1 (v_1^2 - v_L^2) = \frac{1}{2} \times 0.7364 \times (200^2 - 185^2) = 2126.35 \text{ Nm}^{-2}$$

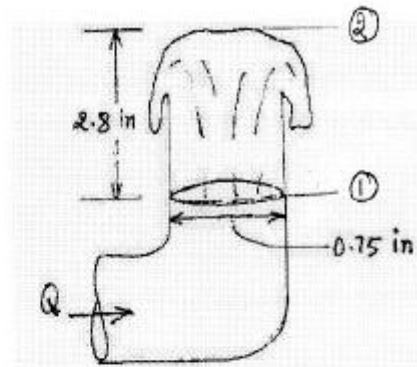
$$dp = p_L - p_U = p_L - p_1 - p_U + p_1 = 2126.35 - (-2292.045) = 4418.4 \text{ N/m}^2$$

$$F = \Delta p \times A = 4418.4 \times 27 = 119247 \text{ N}$$

$$L = F \cos \alpha = 119247 \cos 3^\circ = 119 \text{ kN}$$

#2.

Water flowing from the 0.75 inch-dia. outlet rises 2.8 inches above the outlet. Determine the flow rate.



Apply Bernoulli relation between ① and ②

$$p_1 + \frac{1}{2} \rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \gamma z_2$$

$V_2 = 0$ and $p_1 = p_2 = p_{atm}$ so they cancel out.

$$\frac{1}{2} \rho V_1^2 = \gamma (z_2 - z_1)$$

$$V_1 = \sqrt{\frac{2 \gamma (z_2 - z_1)}{\rho}}$$

$$= \sqrt{2g(z_2 - z_1)}$$

$$V_1 = \sqrt{2 \times 32.2 \times (2.8/12)} = 3.88 \text{ ft/s}$$

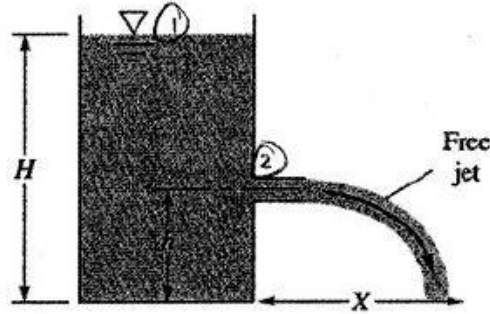
$$A_1 = \pi \frac{D_1^2}{4} = \frac{\pi (0.75)^2}{4} = 3.068 \times 10^{-3} \text{ ft}^2$$

$$\text{Flow rate, } Q = A_1 V_1 = 3.068 \times 10^{-3} \times 3.88$$

$$= 0.0119 \text{ ft}^3/\text{s} \quad \underline{\text{Ans}}$$

#3.

For the container shown, use Bernoulli's equation to derive a formula for the distance X where the free jet leaving horizontally will strike the floor, as a function of h and H . For what ratio h/H will X be maximum? Sketch the three trajectories for $h/H = 0.4$, 0.5 , and 0.6 .



Apply Bernoulli Eqn between ① and ②

$$p_1 + \frac{1}{2} \rho_w v_1^2 + \rho_w g H = p_2 + \frac{1}{2} \rho_w v_2^2 + \rho_w g h$$

$$\Rightarrow v_2 = \sqrt{2g(H-h)}$$

$$[p_1 = p_2 = p_{atm}]$$

For vertical motion of jet:

$$v^2 - u^2 = 2as \text{ gives}$$

$$v^2 - 0 = 2gh \Rightarrow v = \sqrt{2gh}$$

$$\text{and } v = u + at \Rightarrow t = \sqrt{2h/g}$$

Now for Horizontal motion of jet:

$$X = v_2 \cdot t = \sqrt{2g(H-h)} \cdot \sqrt{\frac{2h}{g}}$$

$$X = \sqrt{4h(H-h)} \quad \underline{\text{Ans}}$$

$$\frac{\partial X}{\partial h} = 0 \Rightarrow \frac{1}{2} (4h(H-h))^{-1/2} \cdot 4(H-2h) = 0$$

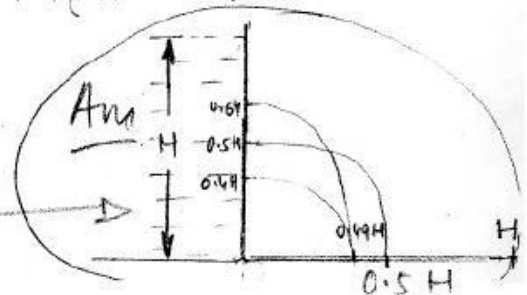
$$\Rightarrow H - 2h = 0$$

$$\Rightarrow h = \frac{H}{2}$$

$$X(0.4H) = 0.49H$$

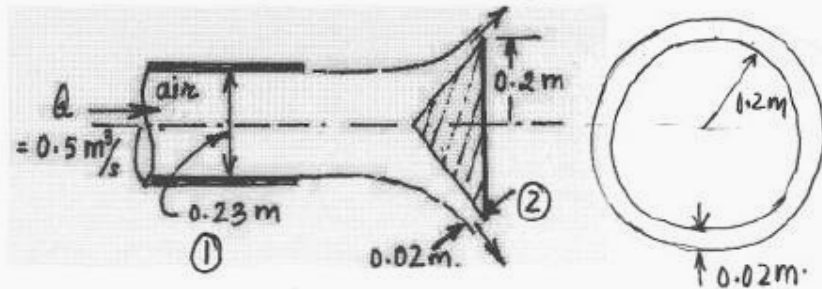
$$X(0.5H) = 0.5H$$

$$X(0.6H) = 0.49H$$



#4.

A conical plug is used to regulate the air flow from the pipe shown. The air leaves the edge of the cone with a uniform thickness of 0.02 m. If viscous effects are negligible and the flow rate is $0.50 \text{ m}^3/\text{s}$, determine the pressure within the pipe.



$$Q = 0.5 \text{ m}^3/\text{s}$$

$$A_1 = \frac{\pi}{4} (0.23)^2 = 0.04155 \text{ m}^2$$

$$A_2 = 2 \times \pi \times 0.2 \times 0.02 = 0.025133 \text{ m}^2$$

$$V_1 = Q/A_1 = \frac{0.5}{0.04155} = 12.034 \text{ m/s}$$

$$\text{Similarly: } V_2 = Q/A_2 = \frac{0.5}{0.025133} = 19.9 \text{ m/s}$$

$$p_2 = p_{\text{atm}} = 101330 \text{ Pa}, \quad p_1 = ?$$

Applying Bernoulli relation between ① and ②

$$p_1 + \frac{1}{2} \rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \gamma z_2 \quad [\because \text{Avg } z_2 = z_1]$$

$$p_1 = p_2 + \frac{1}{2} \rho (V_2^2 - V_1^2)$$

$$= 101330 + 0.5 \times 1.23 \times (19.9^2 - 12.034^2)$$

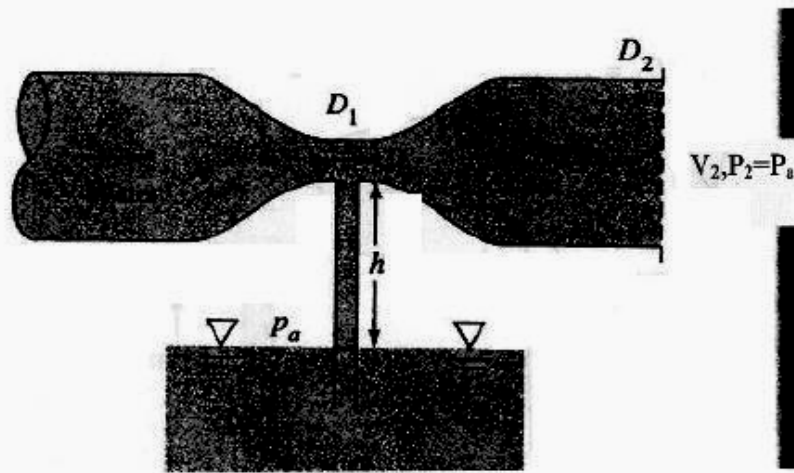
$$= 101330 + 154.5 = 101484.5$$

$$p_1 = 101.48 \text{ kPa.} \quad \underline{\text{Ans}}$$

$$\text{or } p_{1, \text{gage}} = 154.5 \text{ Pa} \quad \underline{\text{Ans}}$$

#5.

A necked-down section in a pipe flow, called a *venturi*, develops a low pressure that can aspirate fluid upward from a reservoir, as in Fig given. Using Bernoulli's equation, derive an expression for the velocity V_1 that is just sufficient to bring reservoir fluid into the throat.



Bernoulli's Eq. between 1 and 2. :

$$P_1 + \frac{1}{2} \rho V_1^2 = P_a + \frac{1}{2} \rho V_2^2$$

$$P_a - P_1 = \frac{1}{2} \rho (V_1^2 - V_2^2) = \Delta p \quad \text{--- I}$$

$$\text{Also } \Delta p = \gamma_w h. \quad \text{--- II}$$

continuity Equation : $A_1 V_1 = A_2 V_2$

$$\Rightarrow \frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2$$

$$\Rightarrow V_2 = V_1 \left(\frac{D_1}{D_2} \right)^2$$

Equation ① and ②

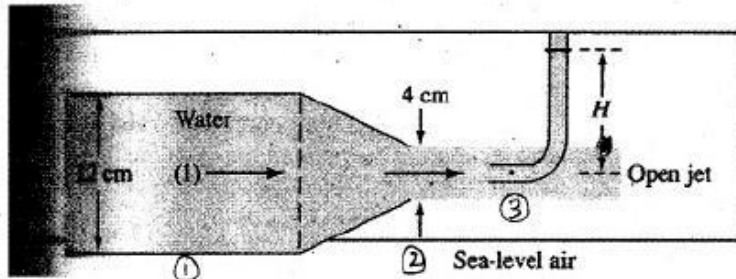
$$\frac{1}{2} \rho (V_1^2 - V_2^2) = \gamma_w h.$$

$$\Rightarrow V_1^2 \left[1 - \left(\frac{D_1}{D_2} \right)^4 \right] = \frac{2 \gamma_w h}{\rho_w} = 2gh.$$

$$\text{so } V_1 = \sqrt{\frac{2gh}{1 - (D_1/D_2)^4}} \quad \text{Ans}$$

#6.

A free jet of water exits a nozzle into sea-level air and strikes a stagnation tube as shown. If the pressure at the centerline at section 1 is 110 kPa, and losses are neglected, estimate (a) the mass flow in kg/s and (b) the height H of the fluid in the stagnation tube.



Continuity between ① and ② means

$$A_1 V_1 = A_2 V_2 \Rightarrow V_1 = \frac{16}{144} V_2 = \frac{V_2}{9}$$

Bernoulli eq $\Rightarrow p_1 + \frac{1}{2} \rho V_1^2 = p_{atm} + \frac{1}{2} \rho V_2^2$

$$p_1 - p_{atm} = 110000 - 101325 = 8675 \text{ Pa} = \frac{1}{2} \rho_w (V_2^2 - V_1^2)$$

$$\Rightarrow \frac{1}{2} \rho_w V_2^2 \left(1 - \frac{1}{81}\right) = 8675$$

$$\Rightarrow V_2 = \sqrt{\frac{2 \times 8675 \times 81}{1000 \times 80}} = 4.2 \text{ m/s}$$

(a) mass flow rate, $\dot{m} = \rho_w Q = \rho_w A_2 V_2 = 1000 \times \pi \times 0.02^2 \times 4.2$
 $= 5.27 \text{ kg/s}$ Ans

(b) Apply Bernoulli eqn between ② and ③

$$p_2 + \frac{1}{2} \rho V_2^2 = p_3 + \frac{1}{2} \rho V_3^2$$

$$\Rightarrow p_3 = p_{atm} + \frac{1}{2} \rho_w V_2^2 \quad \text{--- (I)}$$

Also $p_3 = p_{atm} + \gamma_w H \quad \text{--- (II)}$

Equating (I) and (II) we get: $\gamma_w H = \frac{1}{2} \rho_w V_2^2$

$$H = \frac{\frac{1}{2} \rho_w V_2^2}{\gamma_w} = \frac{\frac{1}{2} V_2^2}{2 \times 9.8} = \frac{(4.2)^2}{2 \times 9.8} = 0.896 \text{ m}$$
 Ans

#7.

A *venturi meter*, shown in Fig., is a carefully designed constriction whose pressure difference is a measure of the flow rate in a pipe. Using Bernoulli's equation for steady incompressible flow, show that the flow rate Q is related to the manometer reading h by

$$Q = \frac{A_2}{\sqrt{1 - (D_2/D_1)^4}} \sqrt{\frac{2gh(\rho_M - \rho)}{\rho}}$$

continuity Eqn

$$\frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2$$

$$\Rightarrow V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

Bernoulli eqn:

$$p_1 + \frac{1}{2} \rho V_1^2 = p_2 + \frac{1}{2} \rho V_2^2$$

$$p_1 - p_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) = \frac{1}{2} \rho V_2^2 \left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \quad \text{--- (I)}$$

$$\text{Also: } p_1 + \gamma h = p_2 + \gamma_m h$$

$$p_1 - p_2 = (\gamma_m - \gamma) h = gh(\rho_m - \rho) \quad \text{--- (II)}$$

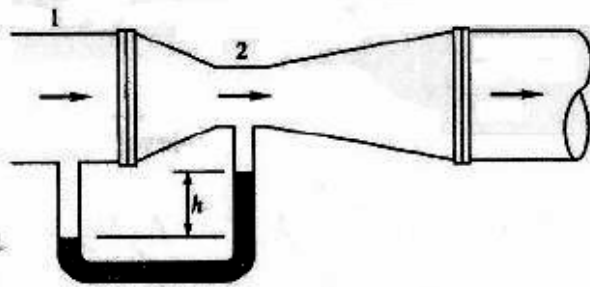
Equate (I) and (II)

$$\frac{1}{2} \rho V_2^2 \left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = gh(\rho_m - \rho)$$

$$V_2 = \sqrt{\frac{2gh(\rho_m - \rho)}{\rho \left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}}$$

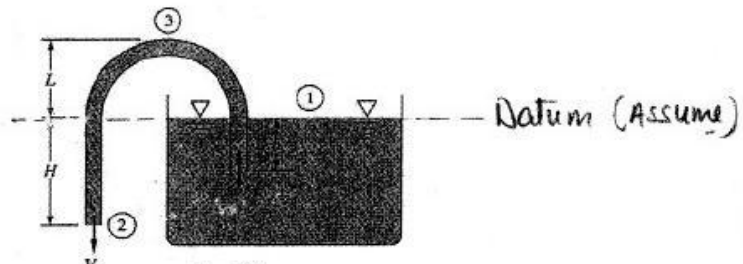
$$Q = V_2 A_2 = \frac{A_2}{\sqrt{1 - (D_2/D_1)^4}} \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}}$$

Ans



#8.

Once it has been started by sufficient suction, the siphon shown below will run continuously as long as reservoir fluid is available. Using Bernoulli's equation with no losses, show (a) that the exit velocity V_2 depends only on gravity and the distance H and (b) that the lowest (vacuum) pressure occurs at point 3 and depends on the distance $L+H$.



(a) Bernoulli eq between ① and ②

$$p_1 + \frac{1}{2} \rho V_1^2 + 0 = p_2 + \frac{1}{2} \rho V_2^2 - \gamma_w H \quad \left[\because p_1 = p_2 = p_{atm} \right]$$

so $\frac{1}{2} \rho V_2^2 = \gamma_w H$

$$V_2 = \sqrt{\frac{2 \gamma_w H}{\rho}} = \sqrt{2 g H} \quad \underline{\text{Ans}}$$

(b) Bernoulli eq. between ② and ③

$$p_2 + \frac{1}{2} \rho V_2^2 - \gamma_w H = p_3 + \frac{1}{2} \rho V_3^2 + \gamma_w L$$

$$p_{atm} - \gamma_w H = p_3 + \gamma_w L$$

$$\left[\begin{array}{l} \text{Since } A_2 = A_3 = A \\ \text{so } A_2 V_2 = A_3 V_3 \Rightarrow V_2 = V_3 \end{array} \right]$$

$$\underline{p_3 = p_{atm} - \gamma_w (H+L)}$$

Since $p_1 = p_2 = p_{atm}$ and $p_3 = p_{atm} - \gamma_w (H+L)$

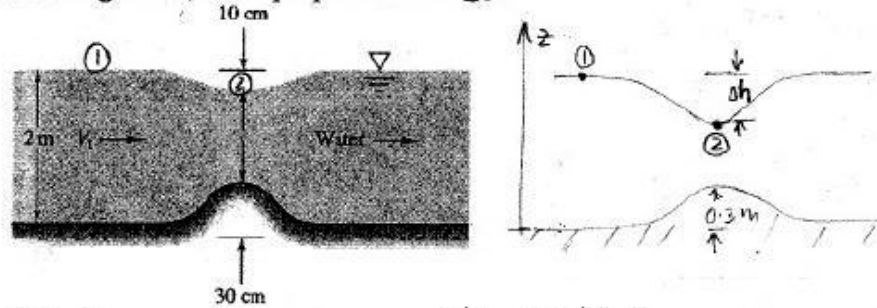
so $p_3 < p_1 = p_2$ and depends only on $L+H$ since $\gamma_w = \text{const}$

or if $p_3 = 0$ (minimum possible)

$$L+H = \frac{p_{atm}}{\gamma_w}$$

#9.

If the approach velocity is not too high, a hump in the bottom of a water channel causes a dip Δh in the water level, which can serve as a flow measurement. If, as shown in the Fig., $\Delta h = 10 \text{ cm}$ when the bump is 30 cm high, what is the volume flow Q_1 per unit width, assuming no losses? In general, is Δh proportional to Q_1 ?



continuity eqn: (Assume unit width)

$$A_1 v_1 = A_2 v_2$$

$$2v_1 = (1.7 - \Delta h)v_2$$

$$v_2 = \left(\frac{2}{1.7 - \Delta h} \right) v_1 = 1.25 v_1 \quad \text{--- (1)}$$

Apply Bernoulli Eqn. between ① and ②

$$\cancel{p}_1 + \frac{1}{2} \rho v_1^2 + \rho g z = \cancel{p}_2 + \frac{1}{2} \rho v_2^2 + \rho g (z - \Delta h)$$

$$\text{so } \rho g \Delta h = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\Rightarrow 2g\Delta h = v_2^2 - v_1^2 = \left[\left(\frac{2}{1.7 - \Delta h} \right)^2 - 1 \right] v_1^2$$

$$\Rightarrow v_1 = \frac{\sqrt{2g\Delta h}}{\sqrt{\left[\left(\frac{2}{1.7 - \Delta h} \right)^2 - 1 \right]}} = \sqrt{\frac{2g\Delta h (1.7 - \Delta h)^2}{4 - (1.7 - \Delta h)^2}}$$

$$\text{so } v_1 \propto \sqrt{\Delta h}$$

$$\text{or } \Delta h \propto v_1^2$$

Ans

$$Q_1 = A_1 v_1 = 2 \sqrt{\frac{2g\Delta h}{\left[\left(\frac{2}{1.7 - \Delta h} \right)^2 - 1 \right]}} = 3.73 \text{ m}^3/\text{s} \quad \text{--- Ans}$$

#10.

The incompressible flow form of Bernoulli's relation is accurate only for Mach numbers less than 0.3. At higher speeds, variable density must be accounted for. The most common assumption for compressible fluid is *isentropic flow of an ideal gas*. Show that the compressible isentropic Bernoulli relation can be written as

$$C_p T + \frac{1}{2} V^2 + gz = \text{const}$$

Applying Newton's 2nd law to fluid element in x direction gives:

$$\rho u \frac{\partial u}{\partial x} A \Delta x = - \frac{\partial p}{\partial x} A \Delta x - \frac{\partial (\rho g z)}{\partial x} A \Delta x$$

$$\Rightarrow u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} + g \frac{\partial z}{\partial x} = 0$$

$$u du + \frac{dp}{\rho} + g dz = 0 \quad (\text{Bernoulli eqn})$$

Integrating we get:

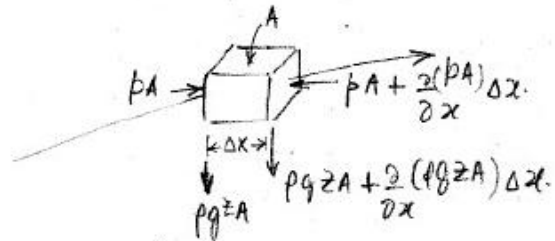
$$\frac{u^2}{2} + \int \frac{c \gamma p^{\gamma-1}}{\rho} dp + gz = \text{const.}$$

$$\frac{u^2}{2} + \frac{\gamma}{\gamma-1} \frac{c p^\gamma}{\rho} + gz = \text{const.}$$

$$\frac{u^2}{2} + \frac{\gamma}{\gamma-1} \frac{p}{\rho} + gz = \text{const.}$$

$$\frac{u^2}{2} + \frac{\gamma}{\gamma-1} RT + gz = \text{const.}$$

$$\frac{u^2}{2} + C_p T + gz = \text{const.}$$



$$\left[\begin{array}{l} \because \text{for isentropic flow} \\ p = c \rho^\gamma \\ \Rightarrow dp = \gamma c \rho^{\gamma-1} d\rho \end{array} \right]$$

$$\left[\because \frac{\gamma R}{\gamma-1} = C_p \right]$$

Ans