Solutions to HW#9 SP07

#1

Air at standard conditions enters a compressor at 75 m/s and leaves at an absolute pressure and temperature of 200 kPa and 345 K, respectively and speed V = 125 m/s. The flow rate is 1 kg/s. The cooling water circulating around the compressor casing removes 18 kJ/kg of air. Determine the power required by the compressor.

For air : $c_v = 716 \text{ J kg}^{-1}\text{K}^{-1}$ (sp. heat at constt. Volume)

$$\rho_1 = \frac{p_1}{RT_1} = \frac{101325}{287 \times (273+15)} = 1.225 \text{ kgm}^{-3}$$

Energy Equation :

$$(\check{u}_1 - \check{u}_2) + \left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right) + \left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) = W_{sh} + \text{losses} + \dot{q}_{net}$$

$$\Rightarrow c_v(T_1 - T_2) + R(T_1 - T_2) + \frac{1}{2}(V_1^2 - V_2^2) = W_{sh} + \underbrace{\cancel{\text{losses}}}_{0} + \dot{q}_{net}$$

$$716(288-345) + 287(288-345) + \frac{1}{2}(75^2 - 125^2) = W_{sh} + 180000$$

$$-40812 - 16359 - 5000 = W_{sh} + 18000$$

$$W_{sh} = -62171 - 18000$$

$$= -80171 \text{ J kg}^{-1}$$

$$\text{Power} = W_{sh} \times \dot{m} = -80171 \times 1 \text{ J/s}$$

$$= -80171 \text{ W}$$

$$= -80.171 \text{ kW}$$ Ans

2.

Air is drawn from the atmosphere into a turbomachine. At the exit, conditions are 500kPa (gage) and 130°C. The exit speed is 100 m/s and the mass flow rate is 0.8 kg/s. Flow is steady and there is no heat transfer. Compute the shaft work done.

Air :

$c_v = 716 \ J kg^{-1} K^{-1}$ (sp. heat at constt Volume)

$p_1 = 101325 \ Pa$ $\quad T_1 = 15\,^{\circ}C = 288 \ K$

$T_2 = 130\,^{\circ}C = 403 \ K$

Energy Eqn :

$$(\check{u}_1 - \check{u}_2) + \left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right) + \left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) = W_{sh} + \underbrace{losses}_{0} + \underbrace{q_{net}}_{0}$$

$$c_v(T_1 - T_2) + R(T_1 - T_2) + \frac{1}{2}(V_1^2 - V_2^2) = W_{sh}$$

$$716(288-403) + 287(288-403) + \frac{1}{2}(0-(100)^2) = W_{sh}.$$

$$82340 - 33120 - 5000 = W_{sh}.$$

$$W_{sh} = -120460 \ J kg^{-1}$$

$$\text{Power}, P = W_{sh} \times \dot{m}$$

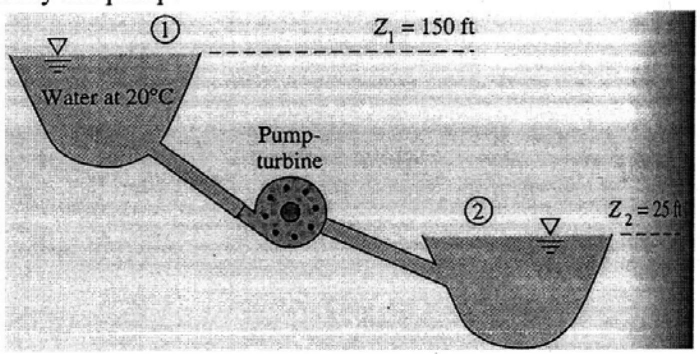
$$= -120460 \times 0.8 \ J s^{-1}$$

$$= -96368 \ W$$

$$P = -96.37 \ kW$$ Ans

#3.

The *pump-turbine* system shown draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower to upper reservoirs to restore the situation. For a design flow rate of 15,000 gal/min in either direction, the friction head loss is 17 ft. Estimate the power in kW *(a)* extracted by the turbine and *(b)* delivered by the pump.



a) Turbine: $Q = 15000 \frac{gal}{min} \cdot \frac{1}{7.48} \frac{ft^3}{gal} \cdot \frac{1 \, min}{60 \, s} = 33.4 \, ft^3/s$

Head loss $= 17$ ft $\Rightarrow \frac{loss}{mass} = 17 \times g = 547.4 \, \frac{ft\,lb}{slug}$.

Energy eqn:

$$\cancel{\frac{p_1}{\rho_1}} - \cancel{\frac{p_2}{\rho_2}} + \cancel{\frac{V_1^2}{2}} - \cancel{\frac{V_2^2}{2}} + g(z_1 - z_2) = W_{sh} + \cancel{q_{net}} + \text{losses} \quad \text{—(I)}$$

$$0 \qquad\qquad 0 \qquad\qquad 0$$

$$W_{sh} = g(z_1 - z_2) - \text{losses} = 32.2(150-25) - 547.4$$

$$W_{sh} = 3477.6 \, \frac{ft\,lb}{slug}.$$

$$P = W_{sh} \times \dot{m} = W_{sh} \times \rho Q = 3477.6 \times 1.94 \times 33.4 = 410 \text{ hp}.$$

$$= 410 \times 745.7 = 305.7 \text{ kW}$$

Ans

b) Pump: Eqn (I) for this case is:

$$g(z_2 - z_1) = W_{sh} + \text{losses}$$

so $W_{sh} = 32.2 \times -125 - 547.4 = -4572 \, \frac{ft\,lb}{slug}$.

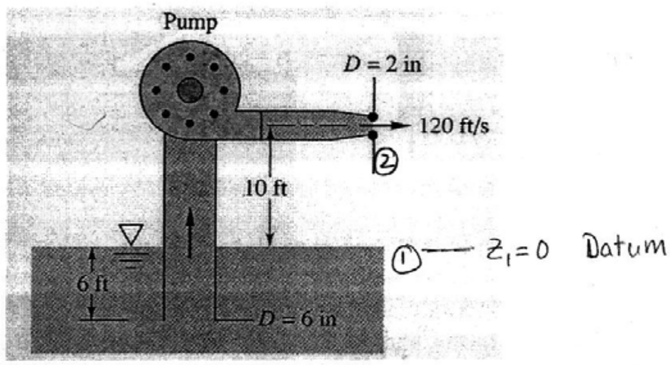
$$P = \rho Q W_{sh} = -1.94 \times 33.4 \times 4572 = -296000 \, \frac{ft\,lb}{s} = -538 \text{ hp}$$

$$= -401.2 \text{ kW}$$

Ans

4.

The fireboat draws seawater (SG = 1.025) from a submerged pipe and discharges it through a nozzle, as shown. The total head loss is 6.5 ft. If the pump efficiency is 75 percent, what horsepower motor is required to drive it?



$$\gamma = 1.025 \times 1.94 \times 32.2 = 64.03 \text{ lbf ft}^{-3}$$

Energy eqn. per unit wt:

$$\text{Loss} = (\check{u}_2 - \check{u}_1 - q_{net,in})$$

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \cancel{\frac{p_2}{\rho g}} + \frac{V_2^2}{2g} + z_2 + \frac{\text{loss}}{g} - \frac{W_{sh}}{g}$$

$$\frac{W_{sh}}{g} = \frac{V_2^2}{2g} + z_2 + \frac{\text{loss}}{g} = \frac{(120)^2}{2 \times 32.2} + 10 + 6.5$$

$$\frac{W_{sh}}{g} = 240 \text{ ft}$$

$$Q = A_2 V_2 = \frac{\pi}{4} \cdot \left(\frac{2}{12}\right)^2 \times 120 = 2.62 \text{ ft}^3/\text{s}$$

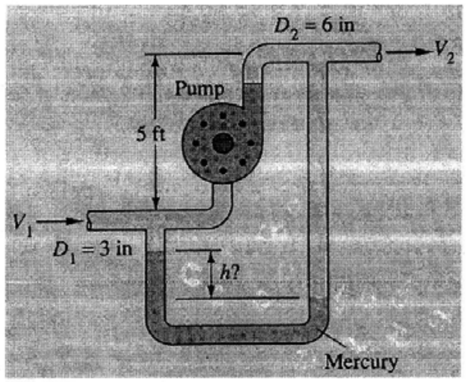
$$P_{pump} = \frac{\dot{m} W_{sh}}{\eta} = \frac{\rho Q W_{sh}}{\eta} = \frac{\rho \times 2.62 \times 240 \times g}{0.75}$$

$$= \rho g \frac{\times 2.62 \times 240}{0.75} = \frac{64.03 \times 2.62 \times 240}{0.75}$$

$$= 53683 \frac{\text{ft lb}}{\text{s}} \cong 97 \text{ hp} \quad \text{Ans.}$$

5.

Kerosene at 20°C flows through the pump shown at 2.3 ft^3/s. Head losses between 1 and 2 are 8 ft, and the pump delivers 8 hp to the flow. What should the mercury manometer reading *h* ft be?



$\rho_k = 804 \text{ kg m}^{-3}$
$= 1.56 \text{ slug ft}^{-3}$

$\gamma_k = \rho_k g$
$= 50.2 \text{ lbft}^{-3}$

$\gamma_M = 846 \text{ lb ft}^{-3}$

$A_1 = \frac{\pi}{4} \times \left(\frac{3}{12}\right)^2 = 0.049 \text{ ft}^2.$

$V_1 = Q/A_1 = 2.3/0.049 = 46.9 \text{ fts}^{-1}$

$V_2 = \frac{A_1}{A_2} V_1 = \frac{1}{4} \times 46.9 = 11.7 \text{ fts}^{-1}$

From manometer:

$p_2 - p_1 = (\gamma_M - \gamma_k)h - \gamma_k(z_2 - z_1) = (846 - 50.2)h - 50.2 \times 5 = 796h - 251$ —— (I)

Energy Eqn;

$\text{loss} = (\check{u}_2 - \check{u}_1 - q_{net,in})$

$\frac{p_1}{\gamma_k} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_k} + \frac{V_2^2}{2g} + z_2 + \frac{\text{loss}}{g} - \frac{W_{sh}}{g}$ —— (II)

$P = W_{sh} \times \dot{m} = W_{sh} \times \rho_k Q$

$\Rightarrow W_{sh} = \frac{P}{\rho_k Q}$

and $\frac{W_{sh}}{g} = \frac{P}{\rho_k g Q} = \frac{P}{\gamma_k Q} = \frac{8 \times 550}{50.2 \times 2.3} = 38.1 \text{ ft.}$ $\left[\because 1\text{hp} = 550 \frac{\text{ft lb}}{\text{s}}\right]$

substitute in eqn (II)

$\frac{p_1}{50.2} + \frac{(46.9)^2}{2 \times 32.2} + 0 = \frac{p_2}{50.2} + \frac{(11.7)^2}{2 \times 32.2} + 5 + 8 - 38.1$

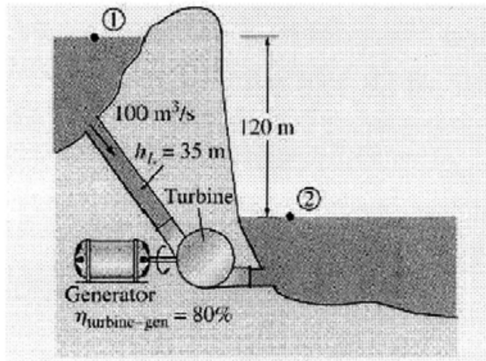
$\Rightarrow p_2 - p_1 = 2866 \text{ lb ft}^{-3}$ —— (III)

(I) = (III) means

$796h - 251 = 2866 \Rightarrow h = 3.92 \text{ ft}$ Ans

#6.

In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 *m* to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 *m*. If the overall efficiency of the turbine generator is 80%, estimate the electric power output.



Energy eqn:

$$\check{u}_1 - \check{u}_2 + \cancel{\frac{p_1}{\rho_1}} - \cancel{\frac{p_2}{\rho_2}} + \cancel{\frac{V_1^2 - V_2^2}{2}} + g(z_1 - z_2) = W_{sh_{net}} + q_{net}$$

0   0

$$g(z_1 - z_2) = W_{sh\,net} + \check{u}_2 - \check{u}_1 - q_{net\,in}$$

$$\left[\begin{array}{l} q_{net} < 0 \text{ if } q_{net} = q_{net,in} \\ \phantom{q_{net}} > 0 \text{ if } q_{net} = q_{net,out} \end{array} \right]$$

$$z_1 - z_2 = \frac{W_{sh\,net}}{g} + \frac{\check{u}_2 - \check{u}_1 - q_{net\,in}}{g}$$

$$= \frac{W_{sh\,net}}{g} + \text{Irrev. head loss.}$$

$$120 = \frac{W_{sh\,net}}{g} + 35$$

$$\Rightarrow \frac{W_{sh,net}}{g} = 120 - 35 = 85 \text{ m}$$

$$W_{sh,net} = 85 \times g = 85 \times 9.8 = 833.0 \ m^2 s^{-2}$$

$$\text{Power} = \dot{m} W_{sh} = \rho Q W_{sh} = 1000 \times 100 \times 833 = 83.3 \times 10^6 \ J s^{-1}$$

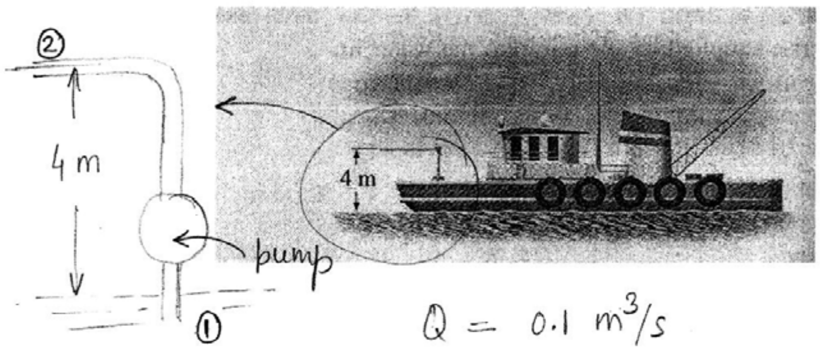
or Power = 83.3 MW

Electric Power Output = η Power = $0.8 \times 83.3 = 66.7$ MW

Ans

#7.

A fireboat is to fight fires at coastal areas by drawing seawater with a density of 1030 kg/m^3 through a 20 *cm*-diameter pipe at a rate of 0.1 m^3/s and discharging it through a hose nozzle with an exit diameter of 5 *cm*. The total irreversible head loss of the system is 3 *m*, and the position of the nozzle is 4 *m* above sea level. For a pump efficiency of 70%, determine the required shaft power input into the pump and the water discharge velocity.



$$Q = 0.1 \ m^3/s$$

$$A_1 = \pi \times \left(\frac{0.2}{2}\right)^2 = 0.03142 \ m^2$$

$$V_1 = Q/A_1 = 3.183 \ m/s$$

$$A_2 = \pi \times (0.05/2)^2 = 0.00196 \ m^2.$$

$$V_2 = Q/A_2 = 50.9 \ m/s \quad \underline{\text{Ans}}$$

Irrev. head losses = 3 m.

Energy eqn. in terms of (head) :

$$\frac{(\cancel{p_1 - p_2})}{\cancel{\rho g}}_{0} + \frac{(V_1^2 - V_2^2)}{2g} + (z_1 - z_2) = \frac{\text{Irrev losses}}{g} + \frac{W_{sh}}{g}$$

$$\frac{(3.183)^2 - (50.9)^2}{2 \times 9.8} - 4 = 3 + \frac{W_{sh}}{g}$$

$$(-131.8 - 4 - 3) \times 9.8 = W_{sh}.$$

used shaft power $= \rho Q W_{sh} = 1030 \times 0.1 \times -138.82 \times 9.8$

$$= -140.13 \ kW.$$

Required Sh. power $= \frac{-140.13}{\eta} = \frac{-140.13}{0.7} = -200 \ kW$

$\underline{\text{Ans}}$