MAE 2314 FLUID MECHANICS

SUMMER 2009

DEPARTMENT OF MECHANICAL
AND
AEROSPACE ENGINEERING

UNIVERSITY OF TEXAS AT ARLINGTON

EXAM #2

CLOSED BOOK AND NO NOTES
Only Nonprogrammable Calculators are allowed

AUGUST 5, 2009
Time Limit : 1 hr 45 min

This exam has 7 pages.

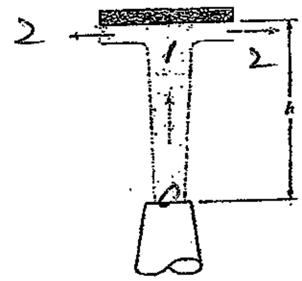
LAST NAME : Solution

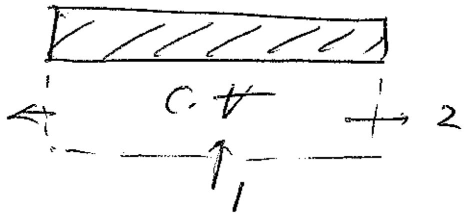
FIRST NAME : ____________________

Announcement on typos

$\forall \rho = 10^3$ kg/m^3
unless otherwise stated.

1.(20pts) A vertical jet of water leaves a nozzle at a speed of 10m/s and a diameter of 20 mm. It suspends a plate having a mass of 1.5 kg as indicated. What is the vertical distance h? ($\rho = 999\ kg/m^3$; $g = 9.81\ m/s^2$)





$W = \dot{m} V_1 \Rightarrow V_1 = \frac{W}{\dot{m}} = \frac{mg}{\dot{m}}$

$= \frac{(1.5\ kg)(9.8\ \frac{m}{s^2})}{3.14\ kg/s}$

$= 4.68\ m/s$

$\dot{m} = \rho A V$

$= (999\ \frac{kg}{m^3})$

$(\frac{\pi}{4}\ 0.02^2\ m^2)$

$10\ m/s$

$= 3.14\ kg/s$

Applying Bernoulli's Eq. between "0" & "1"

$\frac{P_0}{\rho} + \frac{V_0^2}{2} + g z_0 = \frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1$

$h = (z_1 - z_0) = \frac{V_0^2 - V_1^2}{2g}$

$= \frac{(10^2 - 4.68^2)\ m^2/s^2}{2\ (9.8)\ m/s^2} = \boxed{3.98\ m}$

2a.(5pts) The velocity components in a steady, incompressible, two-dimensional flow field are: u = 2x; v = -2y

For this flow field find the equation of the streamline through the point (1,1).

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-2y}{2x} = -\frac{y}{x}$$

$$\Rightarrow \frac{dy}{y} = -\frac{dx}{x} \Rightarrow \ln y = -\ln x + \ln c$$

$$\Rightarrow \cancel{\frac{y}{x} = c} \quad xy = c$$

To find C the streamline passes (1,1)

$$\Rightarrow C = 1 \Rightarrow \boxed{xy = 1}$$

2b.(5pts) An incompressible velocity field is given by $u = a(x^2-y^2)$, v unknown, w = b, where a and b are constants. What must the form of the velocity component v be?

$$\left.\begin{array}{l} u = a(x^2-y^2) \\ v = ? \\ w = b \end{array}\right\} \quad \text{Incompressible flow} \Rightarrow \nabla \cdot \vec{V} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow 2ax + \frac{\partial v}{\partial y} + 0 = 0 \Rightarrow \boxed{v = -2axy + f(x,z,t)}$$

2c.(5pts) The stream function for a two-dimensional, incompressible flow field is given by $\psi = x^2 + y^2$. Is this an irrotational flow field? Explain.

$$\left.\begin{array}{l} u = \dfrac{\partial \psi}{\partial y} = 2y \\ v = -\dfrac{\partial \psi}{\partial x} = -2x \end{array}\right\} \Rightarrow \zeta_z = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$$

$$= 2 - (-2)$$

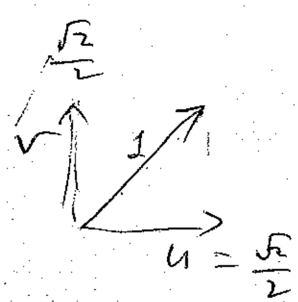
$$= 4 \Rightarrow \boxed{\text{Rotational}}$$

3~~4~~ (15pts) The transient temperature distribution in a fluid is given by T=(10x+5y)(1+t), where x and y are the horizontal and vertical coordinates in meters, T in degrees centigrade and t is time in seconds. Determine the time rate of change of temperature of a fluid particle located at (1,2) at t=5:

(i) travelling horizontally in the x-direction (i.e. θ = 0°) at 1m/s.

$$\frac{DT}{Dt} = \underbrace{\frac{\partial T}{\partial t}}_{(10x+5y)\,=\,20} + \underbrace{u}_{1}\underbrace{\frac{\partial T}{\partial x}}_{10(1+t)\,=\,60} = \underline{80\ ^\circ c/s}$$

(ii) travelling diagonally (i.e. θ = 45°) at 1m/s.



$$\frac{DT}{Dt} = \underbrace{\frac{\partial T}{\partial t}}_{20} + \underbrace{u}_{\frac{\sqrt{2}}{2}}\underbrace{\frac{\partial T}{\partial x}}_{10(1+t)\,=\,60} + \underbrace{v}_{\frac{\sqrt{2}}{2}}\underbrace{\frac{\partial T}{\partial y}}_{5(1+t)\,=\,30}$$

$$= 20 + 30\sqrt{2} + 15\sqrt{2}$$

$$= 20 + 45\sqrt{2} = \boxed{83.64\ ^\circ c/s}$$

(iii) staying stationary.

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} = \boxed{20\ ^\circ c/s}$$

4a(10pts). Determine the $\vec{a}$ acceleration (a_x,a_y,a_z) of a particle at (1,2,3) at t=4 in the velocity field $V = 3t\vec{i} + xz\vec{j} + ty^2\vec{k}$.

$u = 3t,\quad v = xz,\quad w = ty^2$

$$a_x = \underbrace{\frac{\partial u}{\partial t}}_{3} + \underbrace{u}_{3t}\underbrace{\frac{\partial u}{\partial x}}_{0} + \underbrace{v}_{xz}\underbrace{\frac{\partial u}{\partial y}}_{0} + \underbrace{w}_{ty^2}\underbrace{\frac{\partial u}{\partial z}}_{0} = \boxed{3}$$

$$a_y = \underbrace{\frac{\partial v}{\partial t}}_{0} + \underbrace{u}_{3t}\underbrace{\frac{\partial v}{\partial x}}_{z} + \underbrace{v}_{xz}\underbrace{\frac{\partial v}{\partial y}}_{0} + \underbrace{w}_{ty^2}\underbrace{\frac{\partial v}{\partial z}}_{x} = 3tz + txy^2$$

$$= \boxed{52}$$

$$a_z = \underbrace{\frac{\partial w}{\partial t}}_{y^2} + \underbrace{u}_{3t}\underbrace{\frac{\partial w}{\partial x}}_{0} + \underbrace{v}_{xz}\underbrace{\frac{\partial w}{\partial y}}_{2ty} + \underbrace{w}_{ty^2}\underbrace{\frac{\partial w}{\partial z}}_{0} = y^2 + 2txyz$$

$$= \boxed{52}$$

4b.(15pts) An incompressible viscous fluid is placed between two large parallel plates. The bottom plate is fixed and the upper plate moves with a constant velocity, U. For these conditions the velocity distribution between the plates is linear, and can be expressed as u = U y/b. Determine: (a) the volumetric dilatation rate, (b) the vorticity, and (c) the rate of angular deformation.

(a) Volumetric dilatation rate

$$= \nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Longrightarrow \text{Incompressible}$$

(b) Vorticity

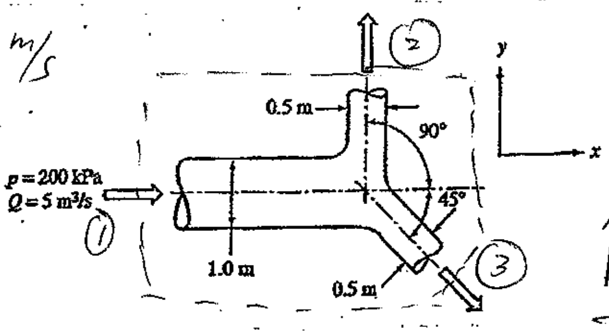
$$\zeta_z = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \frac{U}{b}$$

(c) Rate of angular deformation

$$\dot{\gamma} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{U}{b}$$

$$\rho = 10^3\ kg/m^3$$

5.(25pts) A horizontal flow with an inlet diameter of 1 m is divided and has equal flow rates from each outlet. The outlets have diameter of 0.5 m. The inlet gage pressure is 200 kPa and the inlet flow rate is 5 m^3/s. Determine the reaction force on the divided flow that must be absorbed by a support system. ***Define your control volume and label your nodes.***



$$V_1 = \frac{\dot{V}_1}{A_1} = \frac{5\ m^3/s}{\frac{\pi}{4} 1^2\ m^2} = 6.37\ m/s$$

$$V_2 = \frac{\dot{V}_2}{A_2} = \frac{2.5\ m^3/s}{\frac{\pi}{4} 0.5^2\ m^2} = 12.73\ m/s$$

$$V_3 = V_2$$

$\uparrow F_y$ $\rightarrow F_x$

Bernoulli's Eqn:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{P_3}{\rho} + \frac{V_3^2}{2} + g z_3$$

(neglect change in z)

$$\Rightarrow P_2 = P_3 = P_1 + \rho \frac{(V_1^2 - V_2^2)}{2}$$

$200\ kPa$ $+ (10^3\ \frac{kg}{m^3})(\frac{6.37^2 - 12.73^2}{2})\frac{m^2}{s^2} \cdot \frac{kN}{10^3\ kg \cdot m/s^2}$

$$= 139.3\ kPa$$

Force Balance:

$$\Sigma F_x = P_1 A_1 - P_3 A_3 \cos 45° + F_x$$

$$= \dot{m}_3 V_3 \cos 45° - \dot{m}_1 V_1$$

$$\Rightarrow F_x = P_3 A_3 \cos 45° - P_1 A_1 + \dot{m}_3 V_3 \cos 45° - \dot{m}_1 V_1$$

$(200\ \frac{kN}{m^2})(\frac{\pi}{4} 1^2)m^2$

$(139.3\ \frac{kN}{m^2})(\frac{\pi}{4}(0.5)^2 m^2)\ \frac{\sqrt{2}}{2}$ $\quad (2.5\ \frac{m^3}{s})(10^3\ \frac{kg}{m^3})(12.73\ \frac{m}{s})\ \frac{\sqrt{2}}{2}$ $\quad 5(10^3)(6.37)$

$$= \boxed{-147.09\ kN}$$

$$\Sigma F_y = F_y + P_3 A_3 \cos 45° - P_2 A_2 = \dot{m}_2 V_2 - \dot{m}_3 V_3 \cos 45°$$

$$\Rightarrow \boxed{F_y = 17.33\ kN}$$