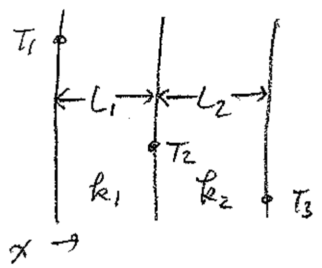
#1. composite wall

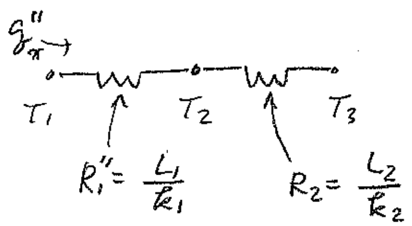


$T_2 = ?$

$L_1 = 0.2\,m$
$L_2 = 0.03\,m$
$T_1 = 1250\,K$
$T_3 = 310\,K$
$k_1 = 1.0\,W/m \cdot K$
$k_2 = 0.07\,W/m \cdot K$

Assumption

- long wall → 1-D conduction (x-dir)
- St. St process



$$q_x'' = \frac{T_1 - T_3}{R_1'' + R_2''} = \frac{T_1 - T_3}{\frac{L_1}{k_1} + \frac{L_2}{k_2}} = \frac{(1250-310)K}{\frac{0.2}{1.0} + \frac{0.03}{0.07}}$$

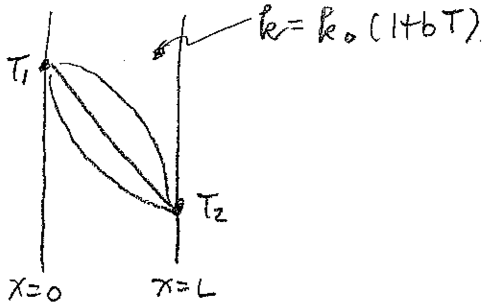
$$= 1495.45\ W/m^2$$

$$q_x'' = \frac{T_1 - T_2}{R_1''} \rightarrow T_2 = T_1 - q_x'' \cdot R_1''$$

$$= (1250\ K) - (1495.45\ W/m^2) \cdot \left(\frac{0.2}{1.0}\ m^2 \cdot K/W\right)$$

$$= \underline{950.91\ K}$$ //

\# 2.



Assumption

- 1-D (x-dir) conduction
- St. St
- no heat generation within the wall ($\dot{q}=0$)

heat equation

$$\nabla\cdot(k\nabla T)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

$$\frac{d}{dx}\left[k_o(1+bT)\cdot\frac{dT}{dx}\right]=0$$

$$k_o(1+bT)\frac{dT}{dx}=\text{const} \qquad (1+bT)\frac{dT}{dx}=C_1$$

$$\int(1+bT)\,dT=\int C_1\,dx$$

$$\boxed{T+\frac{b}{2}T^2=C_1x+C_2}$$ general solution

Two B.Cs

$$\begin{cases} x=0,\ T=T_1 \longrightarrow T_1+\frac{b}{2}T_1^2=C_2 \\ x=L,\ T=T_2 \longrightarrow T_2+\frac{b}{2}T_2^2=C_1L+C_2 \end{cases}$$

$$\therefore C_1=\frac{1}{L}\left\{(T_2-T_1)+\frac{b}{2}(T_2^2-T_1^2)\right\}$$

$$=\frac{1}{L}(T_2-T_1)\left(1+\frac{b}{2}(T_2+T_1)\right)$$

Temperature profile $T(x)$

$$T(x)+\frac{b}{2}T^2(x)=\frac{x}{L}(T_2-T_1)\left(1+\frac{b}{2}(T_2+T_1)\right)$$

$\longrightarrow$ (ii) for $b=0$, $\underline{T(x)=\frac{x}{L}(T_2-T_1)}$ $\therefore$ close to line (b)

for $b\neq 0$

$$q_x''=-k\frac{dT}{dx}=-k_o(1+bT)\frac{dT}{dx}$$

$$-\frac{q_x''}{k_o}=(1+bT)\frac{dT}{dx}$$

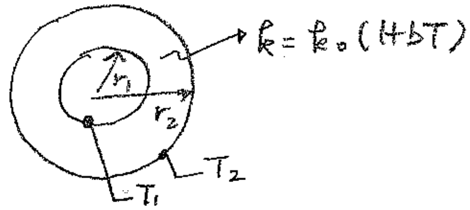
$$\underbrace{\frac{d}{dx}\left(-\frac{q_x''}{k_o}\right)}_{=0}=b\left(\frac{dT}{dx}\right)^2+(1+bT)\frac{d^2T}{dx^2}$$

$$\therefore \frac{d^2T}{dx^2}=-\left(\frac{b}{1+bT}\right)\cdot\left(\frac{dT}{dx}\right)^2 \Rightarrow$$

(i) for $b>0$, $\frac{d^2T}{dx^2}<0 \Rightarrow \therefore$ close to line (c)

(iii) for $b<0$, unable to define

#3.



From Fourier's law

$$q_r = -k \cdot A_r \cdot \frac{dT}{dr}$$

Where A_r is the area normal to r.
Substituting $k = k_o(1+bT)$ and $A_r = 2\pi r L$, where L is length of Tube,

$$q_r = -k_o(1+bT)\cdot(2\pi r L)\frac{dT}{dr}$$

$$\frac{q_r}{L} = -k_o(2\pi)(1+bT)r \cdot \frac{dT}{dr}$$

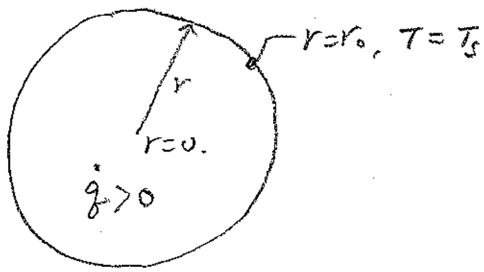
In the steady state, heat rate (q_r, thus q_r/L) is constant.

$$\frac{q_r}{L}\int_{r_1}^{r_2}\frac{dr}{r} = -k_o(2\pi)\int_{T_1}^{T_2}(1+bT)\,dT$$

$$\frac{q_r}{L}\ln\left(\frac{r_2}{r_1}\right) = -k_o(2\pi)\left[(T_2-T_1) + \frac{b}{2}(T_2^2-T_1^2)\right]$$

$$\therefore q_r/L = \frac{-k_o(2\pi)(T_2-T_1)\left(1+\frac{b}{2}(T_2+T_1)\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$ //

#4.



heat equation

$$\nabla \cdot (k \nabla T) + \dot{q} = \cancel{\rho c_p \frac{\partial T}{\partial t}} \xrightarrow{} \text{St, St}$$

for spherical coordination,

$$\frac{1}{r^2} \cdot \frac{d}{dr}\left(k\, r^2 \frac{dT}{dr}\right) + \dot{q} = 0$$

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = -\frac{\dot{q}}{k} r^2$$

$$r^2 \frac{dT}{dr} = -\frac{\dot{q}}{3k} r^3 + C_1$$

$$\frac{dT}{dr} = -\frac{\dot{q}}{3k} r + \frac{C_1}{r^2}$$

$$\therefore\ T(r) = -\frac{\dot{q}}{6k} r^2 - \frac{C_1}{r} + C_2$$

2 B.Cs

i) $r = 0$, $\left.\frac{dT}{dr}\right|_{r=0} = 0 \rightarrow C_1 = 0$. $\therefore\ T(r) = -\frac{\dot{q}}{6k} r^2 + C_2$

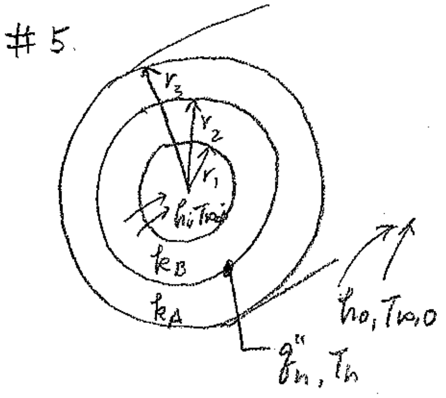
ii) $r = r_o$, $T = T_s \longrightarrow T_s = -\frac{\dot{q}}{6k} r_o^2 + C_2$ $\therefore\ C_2 = T_s + \frac{\dot{q}}{6k} r_o^2$

$\therefore$ Temperature profile within solid sphere

$$T(r) = -\frac{\dot{q}}{6k} r^2 + T_s + \frac{\dot{q}}{6k} r_o^2$$

$$= \frac{\dot{q} r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \quad //$$

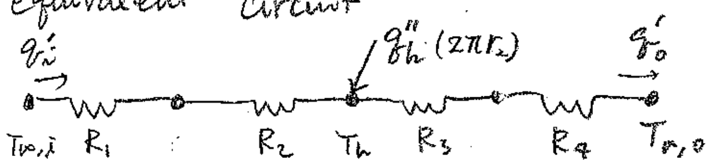
5.



Assumption

- 1-D (radial) heat transfer
- contact resistance of heater is negligible
- St. St

(a) equivalent circuit



$$R_1 = \frac{1}{h_i(2\pi r_1)} \qquad R_3 = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_A}$$

$$R_2 = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_B} \qquad R_4 = \frac{1}{h_o(2\pi r_3)}$$

(b) From the energy Balance $\dot{E}_{in} = \dot{E}_{out}$

$$q_i' + q_h''(2\pi r_2) = q_o'$$

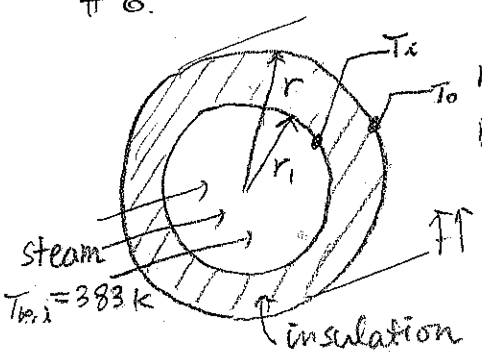
$$q_i' = \frac{T_h - T_{\infty,i}}{R_1 + R_2} = \frac{T_h - T_{\infty,i}}{\frac{1}{h_i(2\pi r_1)} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_B}}$$

$$q_o' = \frac{T_{\infty,o} - T_h}{R_3 + R_4} = \frac{T_{\infty,o} - T_h}{\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_A} + \frac{1}{h_o(2\pi r_3)}}$$

(c) $$\frac{q_o'}{q_i'} = \frac{T_{\infty,o} - T_h}{T_h - T_{\infty,i}} \cdot \frac{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_B} + \frac{1}{h_i(2\pi r_1)}}{\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_A} + \frac{1}{h_o(2\pi r_3)}}$$

To reduce q_o'/q_i', one could increase k_B, h_i, and $\frac{r_3}{r_2}$ while reducing k_A, h_o, $\frac{r_2}{r_1}$.

6.



$r_1 = 0.008\ m$

$r = ?$

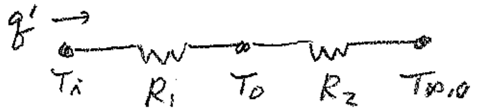
$h = 4.0\ W/m^2\cdot K$

$T_{\infty,o} = 298\ K$

Assumption

- 1-D (radial) conduction
- St. St.
- negligible contact resistance

Equivalent thermal circuit



$$R_1 = \frac{\ln\left(\frac{r}{r_1}\right)}{2\pi k}$$

$$R_2 = \frac{1}{h(2\pi r)}$$

$$q' = \frac{\Delta T}{R'_{tot}} = \frac{T_{\infty,o} - T_i}{\frac{\ln\left(\frac{r}{r_1}\right)}{2\pi k} + \frac{1}{h(2\pi r)}} = \frac{2\pi\left(T_{\infty,o} - T_i\right)}{\frac{\ln\left(\frac{r}{r_1}\right)}{k} + \frac{1}{hr}}$$

at critical thickness of insulation q' is maximum and R'_{tot} is minimum.

$$\frac{dR'_{tot}}{dr} = \frac{\frac{1}{r}}{2\pi k} - \frac{1}{2\pi h r^2} = \frac{1}{2\pi r}\left(\frac{1}{k} - \frac{1}{hr}\right) = 0$$

$$\therefore \frac{1}{k} = \frac{1}{hr} \qquad \therefore r = \frac{k}{h}$$

$$\therefore r_{crit} = \frac{k}{h} = \frac{0.071\ W/m\cdot K}{4.0\ W/m^2\cdot K} = 0.0178\ m$$