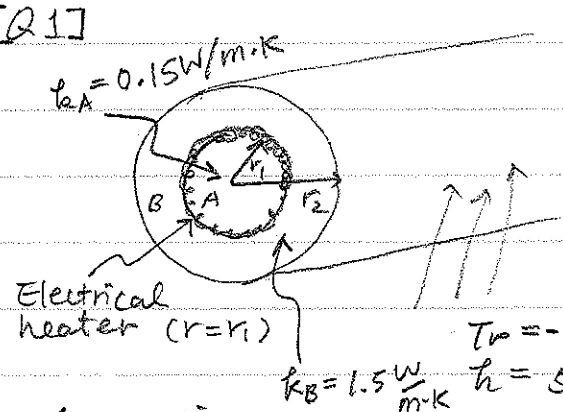
[Q 1]



$r_1 = 20$ mm
$r_2 = 40$ mm

Assumption

- St. St. 1-D (radial) conduction.
- Heater element has negligible thickness
- contact resistance between heater and cylinder is negligible
- constant properties
- No heat generation.

(a) $T(r_2) = T_s = 5°C$

energy balance $\underbrace{\dot{E}_{in}}_{q_{elec}} - \underbrace{\dot{E}_{out}}_{q_{conv}} + \dot{E}_{gen} = \dot{E}_{st}$ ($\dot{E}_{gen} = 0$, $\dot{E}_{st} = 0$)

$$\therefore q_{elec} = \underbrace{q_{conv} = h\cdot A\cdot (T_s - T_\infty)}_{\text{Newton's law}}$$

$$= h\cdot(2\pi r_2 L)(T_s - T_\infty)$$

for unit Length of cylinder ($\div L$)

$$q'_{elec} = q'_{conv} = h\cdot(2\pi r_2)(T_s - T_\infty)$$

$$= (50\ W/m^2\cdot K)(2\pi \times 0.04\ m)(5-(-15))^{K}$$

$$= 251\ W/m$$ //

(b) cylinder rod must be isothermal

$T(0) = T(r_1)$

equivalent thermal circuit

$$T(r_1) \xrightarrow[R'_B]{} T(r_2) = T_s \xrightarrow{q'} \qquad q' = \frac{T(r_1) - T_s}{R'_B} \quad (*)$$

For the cylinder, from equation 3.28

$$R_B = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_B}$$

for unit length of cylinder, $(\div L)$

$$R'_B = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_B}$$

From $(*)$

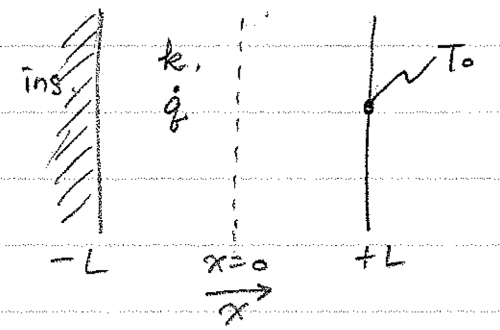
$$T(r_1) = T_s + q' \cdot R'_B$$

$$= 5^\circ C + \underbrace{\left(251 \frac{W}{m}\right)}_{\text{from (a)}} \cdot \left(\frac{\ln\left(\frac{40}{20}\right)}{2\pi \times 1.5 \frac{W}{m \cdot K}}\right)$$

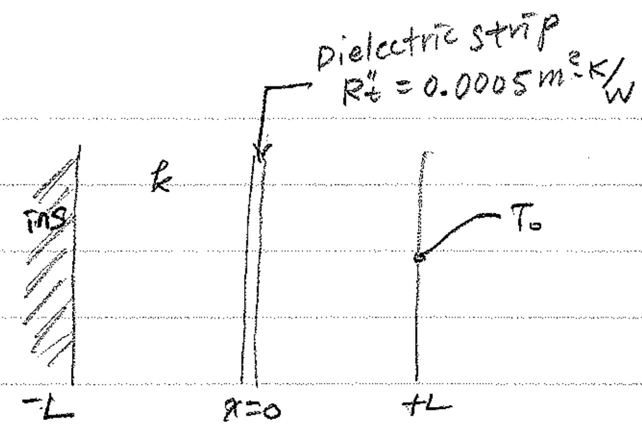
$$= \underline{23.5^\circ C} \;//$$

- Note that k_A has no influence on the temperature $T(0)$ (isothermal cylinder rod)

[Q2] plane wall

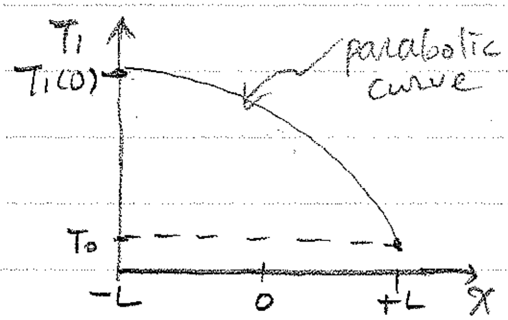


$k = 50 \frac{W}{m\cdot K}$
$L = 20mm$
$\dot{q} = 5\times10^6 \frac{W}{m^3}$
$T_o = 50^\circ C$



(case 1) (case 2)

(a) Case 1 is the same as fig 3.9 (c) (P127).
The temperature profile should be parabolic, and the gradient is zero at the insulated boundary ($x=-L$).
Maximum Temperature should be the temp. of insulated surface. ($x=-L$)



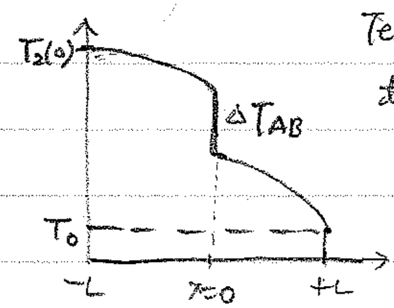
~~from eq. 3.43 (p 128)~~

$$\underbrace{T_1(L)}_{T_o} - \underbrace{T_1(+L)}_{\substack{T_s \\ \| \\ 50^\circ C}} = \frac{\dot{q}(2L)^2}{2k}$$

$$\therefore T_{max} = T_o = 50^\circ C + \frac{(5\times10^6 \frac{W}{m^3})\cdot(2\times0.02m)^2}{(2)\cdot(50\frac{W}{m\cdot K})}$$

$$= 130^\circ C$$

(b) case 2, the temperature distribution, $T_2(x)$ vs. x is piece-wise parabolic with zero gradient at $x=-L$ and a drop across the dielectric strip, ΔT_{AB}. The Temperature gradient at either side of the dielectric strip are equal.



(c) ΔT_{AB}?

$$q''_x(0) = \frac{\Delta T_{AB}}{R''_t} \qquad q''_x(0) = \dot{q} L \qquad \left(\begin{aligned} q_x(0) &= \dot{q} \cdot A \cdot L \\ q''_x(0) &= \dot{q} \cdot L \end{aligned} \right.$$

heat flux

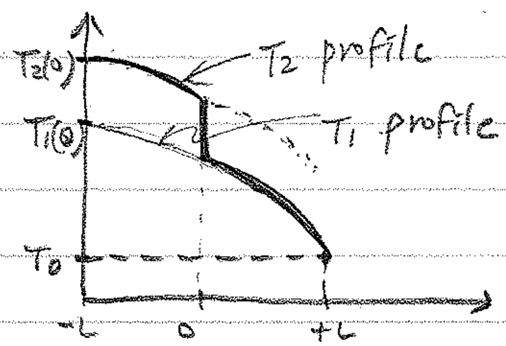
volumetric heat generation rate

$$\therefore \Delta T_{AB} = R''_t \cdot (\dot{q} L)$$
$$= \left(0.0005 \frac{m^2 \cdot K}{W}\right) \cdot \left(5 \times 10^6 \frac{W}{m^3}\right) \cdot (0.020\ m)$$
$$= \underline{50^\circ C}$$

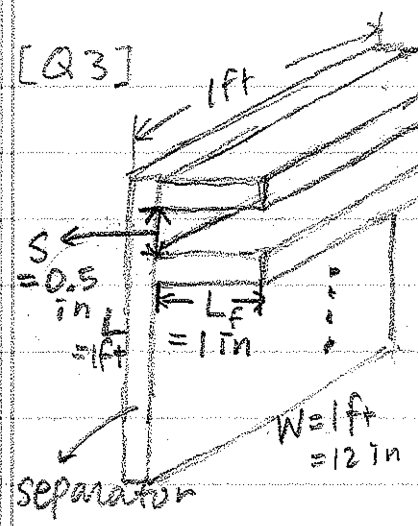
(d) T_{max} for case 2?

As shown in the graph in (b), the maximum temperature in the composite will occur at $x = -L$.

$$T_2(-L) = T_1(-L) + \Delta T_{AB}$$
$$= 130^\circ C + 50^\circ C = 180^\circ C$$



[Q3]



Let's use (1ft × 1ft) for separator.

\# of fins in (1ft × 1 ft) separator (= N)

: 22 Fins

Single Fin's Area

$$A_f = 2\times(L_f \cdot W) + (W \cdot t)$$
$$= 2\times(1\,in \times 12\,in) + (12\,in \times 0.05\,in)$$
$$= 24.6\,in^2 = 0.17\,ft^2$$

Exposed base Area

$$A_b = (L)(W) - N\cdot(W\cdot t) = (12\,in)(12\,in) - (22)\cdot(12\,in)(0.05\,in)$$
$$= 130.8\,in^2 = 0.91\,ft^2$$

Total convective area

$$A_t = N\cdot A_f + A_b = (22)\cdot(24.6) + (130.8)$$
$$= 672\,in^2 = 4.67\,ft^2$$

$q_t = \eta_o h A_t \cdot \Delta T$ (eq. 3.103, p154)

overall fin efficiency for array of fins

$$\eta_o = 1 - \frac{N A_f}{A_t}(1-\eta_f)$$

Method ① for η_f → Single Fin efficiency η_f can be found from Table 3.5 (p152)

For rectangular straight fin

$$\eta_f = \frac{\tanh mL_c}{mL_c} \qquad L_c = L_f + \frac{t}{2}, \quad m = (2h/kt)^{1/2}$$

- $L_c = L_f + \frac{t}{2} = 1\,in + \frac{0.05\,in}{2} = 1.0025\,in = 0.0835\,ft$

- $m = \left(\frac{2h}{kt}\right)^{1/2}$ $\qquad k_{steel} \approx \sim 25\,Btu/hr\cdot ft\cdot°F$

$$= \begin{cases} \left(\dfrac{(2)(2\,\frac{Btu}{hr\cdot ft^2\cdot F})}{(25\,\frac{Btu}{hr\cdot ft\cdot F})(0.05\,in)\frac{1ft}{12in}}\right)^{1/2} = 6.2/ft & \text{for air} \\ \left(\dfrac{(2)(45\,\frac{Btu}{hr\cdot ft^2\cdot F})}{(25\,\frac{Btu}{hr\cdot ft\cdot F})(0.05\,in)\left(\frac{1ft}{12in}\right)}\right)^{1/2} = 29.4\,ft & \text{for water} \end{cases}$$

$$mL_c = \begin{cases} (6.2/ft)\cdot(0.0835\,ft) = 0.5177 & \text{for air} \\ (29.4/ft)\cdot(0.0835\,ft) = 2.4549 & \text{for water} \end{cases}$$

$$\tanh mL_c = \begin{cases} 0.476 & \text{for air} \\ 0.985 & \text{for water} \end{cases}$$

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \begin{cases} 0.476/0.5177 = 0.9195 \\ 0.985/2.4549 = 0.4012 \end{cases}$$

Method (2) for η_f : using the graph (Fig 3.18, p

- $L_c = L_f + \frac{t}{2} = 0.0835\ ft$
- $A_p = L_c \cdot t = (1.0025\ in)\cdot(0.05)\ in^2$
  $= 0.05\ in^2$
  $= 3.47 \times 10^{-4}\ ft$
- $(L_c)^{3/2}\cdot\left(\frac{h}{k A_p}\right)^{1/2}$

$$= (0.0835\,ft)^{3/2}\cdot\left(\frac{2\ Btu/hr.ft^2\cdot F}{(25\ Btu/hr.ft\cdot F)(3.47\times10^{-4}\,ft)}\right)^{1/2}$$

15.18

$= 0.364$ (for air) (*)

$$(0.0835\,ft)^{3/2}\cdot\left(\frac{45\ Btu/hr.ft^2\cdot F}{(25\ Btu/hr.ft\cdot {}^\circ F)(3.47\times10^{-4}\,ft)}\right)^{1/2}$$

$= 1.725$ (for water) (**)

From the graph with (*) & (**)

$$\eta_f = \begin{cases} \sim 90\% = 0.9 & \text{for air} \\ \sim 40\% = 0.4 & \text{for water} \end{cases}$$

$\therefore$ Overall fin efficiency

$$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

$$= \begin{cases} 1 - \frac{(22)\cdot(0.17ft^2)}{4.67\ ft^2}\cdot(1-0.9) = 0.9199 & \text{(for air)} \\ 1 - \frac{(22)(0.17ft^2)}{4.67\ ft^2}(1-0.4) = 0.5195 & \text{(for water)} \end{cases}$$

$\therefore$ overall fin resistance

$$\frac{1}{R_{t,o}} = \eta_o h A_t$$

$$= \begin{cases} (0.9199)\cdot\left(2 \frac{Btu}{hr\cdot ft^2\cdot °F}\right)\cdot(4.67\ ft^2) \\ = 8.59 \left(\frac{Btu}{hr\cdot °F}\right) \quad \text{(For air)} \\ (0.5195)\left(45 \frac{Btu}{hr\cdot ft^2\cdot °F}\right)\cdot(4.67\ ft^2) \\ = 109.17 \left(\frac{Btu}{hr\cdot °F}\right) \quad \text{(For water)} \end{cases}$$

Without Fins (neglect plane conduction resistance)

$$q = \frac{T_{water} - T_{air}}{\frac{1}{h_{air}\cdot A} + \frac{1}{h_{water}\cdot A}} = \frac{\Delta T}{\frac{1}{2} + \frac{1}{45}} = 1.915\ \Delta T$$

$(\because A = 1 ft^2)$

277% increase

Fins on the air side

$$q = \frac{\Delta T}{\frac{1}{R_{t,o\,(air)}} + \frac{1}{h_{water}\cdot A}} = \frac{\Delta T}{\frac{1}{8.59} + \frac{1}{45}} = 7.23\ \Delta T$$

2.0% increase

Fins on the water side

$$q = \frac{\Delta T}{\frac{1}{R_{t,o\,(water)}} + \frac{1}{h_{air}\cdot A}} = \frac{\Delta T}{\frac{1}{109.17} + \frac{1}{2}} = 1.96\ \Delta T$$

316% increase

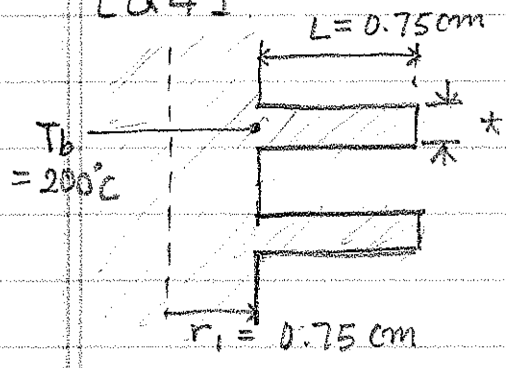
Fins on Both sides

$$q = \frac{\Delta T}{\frac{1}{R_{t,o\,(air)}} + \frac{1}{R_{t,o\,(water)}}} = \frac{\Delta T}{\frac{1}{8.59} + \frac{1}{109.17}} = 7.962\ \Delta T$$

It is apparent that, when fins are added to the air side, a greater increase occurs than for fins added to the water side alone.

HW#3 (8)

[Q4] Annular, copper fins.



For copper, $k = 480$ W/m·K @ 100°C

$T_\infty = 50°C$

$h = 75\ W/m^2 \cdot K$

(P.150)

The Fin parameters for use with Fig 3.19 are

$r_{2c} = r_2 + \frac{t}{2} = 1.5 + \frac{0.15}{2} = 1.575\ cm$

$= 0.01575\ m$

$L_c = L + \frac{t}{2} = 0.75 + \frac{0.15}{2} = 0.825\ cm$

$= 0.00825\ m$

$\frac{r_{2c}}{r_1} = \frac{1.575\ cm}{0.75\ cm} = 2.1$

$A_p = L_c \cdot t = (8.25 \times 10^{-3}) \cdot (1.5 \times 10^{-3}) = 1.2375 \times 10^{-5}\ m^2$

$L_c^{3/2} \cdot \left(\frac{h}{k A_p}\right)^{1/2} = (0.00825\ m)^{3/2} \cdot \left(\frac{75\ W/m^2 \cdot K}{480\ W/m \cdot K \quad 1.2375 \times 10^{-5}\ m^2}\right)^{1/2}$

$= 0.0841$

From the graph (Fig 3.19)

$\eta_f \approx 98\%$ ← (b)

$q_f = \eta_f \cdot q_{max}$ (eq. 3.86)

$= \eta_f \cdot (h \cdot A_f \cdot \theta_b) = \eta_f \cdot h \cdot [2(\pi r_2^2 - \pi r_1^2) + 2\pi r_2 t] \cdot \theta_b$

$= \eta_f \cdot h \cdot 2\pi (r_2^2 - r_1^2 + r_2 t) \cdot \theta_b$

$= (0.98) \cdot (75\ W/m^2 \cdot K) \cdot (2\pi) (0.015^2 - 0.0075^2 + 0.015 \cdot 0.0015)\ m^2$

0.000191

$(200 - 50)\ K$

$= 13.22\ W.$ ← (a)

From eq. 3.81 (p147)

$$\varepsilon_f = \frac{q_f}{h \cdot A_{c,b} \cdot \theta_b} \qquad A_{c,b} = 2\pi r_1 t$$

$$= \frac{13.22\ W}{\left(\frac{75\ W}{m^2 \cdot K}\right) \cdot (2\pi \cdot 0.0075 \cdot 0.0015\ m^2) \cdot (150\ K)}$$

$$= 16.63 \longleftarrow (b)$$

(c) The rate of heat transfer per unit length

$$q_t' = \underbrace{N q_f'}_{\eta_f h' A_f \theta_b} + h \underbrace{A_b'}_{\text{exposed base area per unit length } (=1\ m)} \theta_b \qquad \text{(eq. 3.100) p 154.}$$

$$= N q_f' + h\,(1 - N \cdot t) \cdot (2\pi r_1)\,\theta_b$$

$$= (100) \cdot (13.22\ W/m) + (75\ W/m^2 \cdot K)\,(\overset{m}{1} - \underbrace{100 \cdot 0.0015}_{0.15}) \cdot (2\pi)(0.0075) \cdot (150\ K)$$

450.4 W/m

$$= (1322\ W/m) + (450.4\ W/m)$$

$$= 1722.4\ W/m = \underline{1.72\ kW/m}.$$