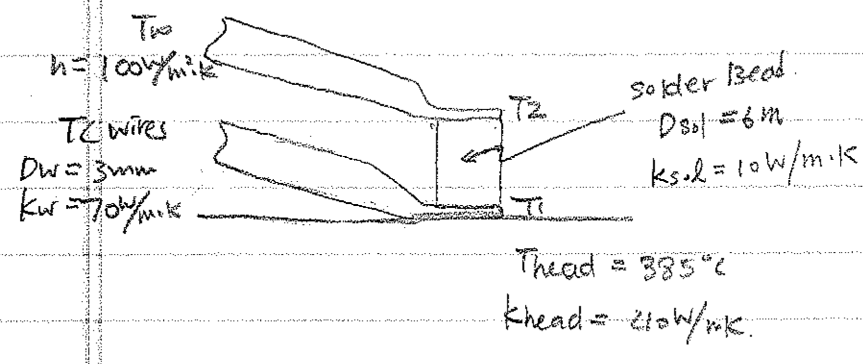
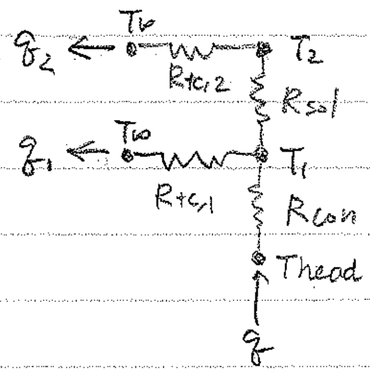
[Q1] Textbook 3.107 (P189)



(a) The equivalent thermal circuit



• construction resistance

$R_{con} = 1/(2k_{head} \cdot D_{sol}) = 1/(2 \times 40 W/m \cdot K \times 0.006 m) = 2.08 KW$

(you also can see Table 4.1, case 10 (P210))

• TC wires. Infinetely long fins (Eq. 3.80)

$R_{tc,1} = R_{tc,2} = R_{fin} = \sqrt{hPkA_c}$ Here, $P = \pi D_w$, $A_c = \pi D_w^2/4$

$= [(100 W/m^2 \cdot K) \cdot (\pi \cdot (0.003 m)^2)(70 W/m \cdot K) \cdot (\pi \cdot (0.003 m)^2/4)]^{1/2}$

$= 46.31 K/W$

• Solder bead cylinder

$R_{sol} = \frac{L_{sol}}{k_{sol} A_{sol}}$ $(A_{sol} = \pi D_{sol}^2/4)$

$= \frac{0.01 m}{(10 W/m \cdot K) \cdot (\pi \cdot (0.006 m)^2/4)}$

$= 35.37 K/W$

(b) Perform energy balances on the 1 & 2 nodes, Solve the equations to find T_1 & T_2 from which $(T_1 - T_2)$ can be determined.

- node 1 (T_1 node)

$$\frac{T_2 - T_1}{R_{sol}} + \frac{T_{head} - T_1}{R_{con}} + \frac{T_\infty - T_1}{R_{tc,1}} = 0$$

- node 2 (T_2 node)

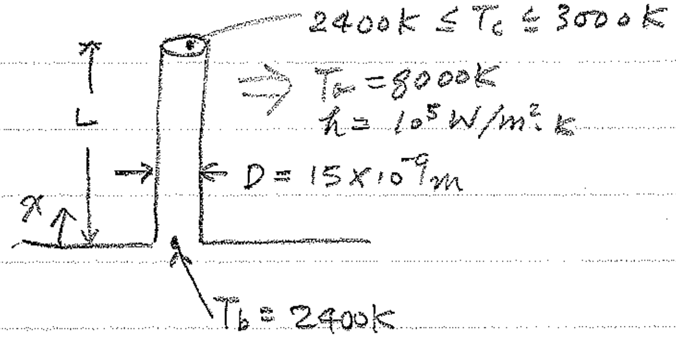
$$\frac{T_\infty - T_2}{R_{tc2}} + \frac{T_1 - T_2}{R_{sol}} = 0.$$

Simultaneously solving node 1 & node 2 equations results

$$\begin{cases} T_1 = 359^\circ C \\ T_2 = 199.2^\circ C \end{cases}$$

$\therefore T_1 - T_2 = 160^\circ C$ //

[Q2] Textbook 3.109 (P190)



<Assumption>

① NW stops growing when $T_c = T(x=L) = 3000K$

② St. St, one-dimensional.

③ convection from the tip.

- ~~k (silicon carbide) @ 1500K = 30 W/m·K~~
- The tip of nanowire is initially at T = 2400K, and increases in temperature as the nanowire becomes longer. At St. St. the tip reaches T = 3000K.
- The temperature distribution at St-St : eq 3.70 (P 144)

Case A : convection heat transfer

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL} \quad (*)$$

$$m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left(\frac{h\cdot\pi D}{k\cdot\frac{\pi D^2}{4}}\right)^{1/2} = \left(\frac{4h}{kD}\right)^{1/2} = \left(\frac{4\times10^5\ W/m^2\cdot K}{30\ W/m\cdot K\times 15\times10^{-9}\ m}\right)^{1/2}$$

$$= 943\times10^3\ m^{-1}$$

$$\frac{h}{mk} = \frac{(10^5\ W/m^2\cdot K)}{(943\times10^3\ m^{-1})(30\ W/m\cdot K)} = 3.53\times10^{-3}$$

(*), evaluated at x = L, is

$\theta = T - T_\infty$

$$\frac{\theta}{\theta_b} = \frac{(3000-8000)K}{(2400-8000)K} = 0.893$$

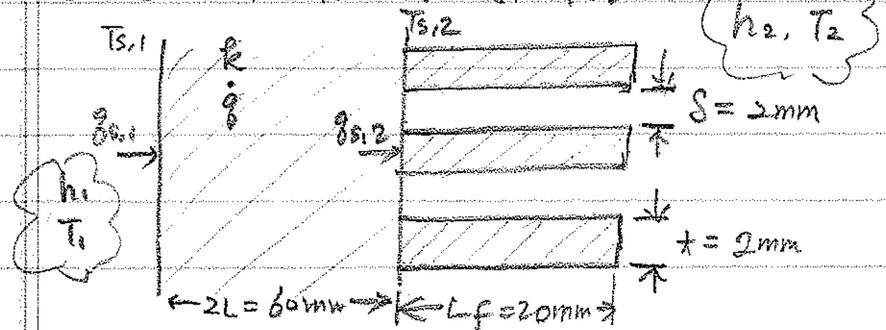
$$= \frac{1}{\cosh(943\times10^3\times L) + 3.53\times10^{-3}\sinh(943\times10^3\times L)}$$

A trial and error solution yields

$L \doteqdot 510\times10^{-9}$ m

$= 510$ nm

[Q3] Textbook 3.142 (P 198)



From eq 3.41 (p128) : the temp. distribution in a wall with uniform volumetric heat generation and specified Temp. boundary condition

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2} \cdot \frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \quad (1)$$

The heat transfer rates at the two surfaces, for a wall (area A), can be found from Fourier's Law:

$$q_{s,1} = -k \cdot A \left.\frac{dT}{dx}\right|_{x=-L} = -\dot{q}LA - kA\frac{T_{s,2} - T_{s,1}}{2L} \quad (2)$$

$$q_{s,2} = -k \cdot A \cdot \left.\frac{dT}{dx}\right|_{x=L} = \dot{q}LA - kA\frac{T_{s,2} - T_{s,1}}{2L} \quad (3)$$

we can express these same heat transfer rates alternatively as follows:

$$q_{s,1} = h_1 A (T_1 - T_{s,1}) \quad (4)$$

$$q_{s,2} = h_2 A_t (T_{s,2} - T_2)\eta_o \quad (5)$$

where

$$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta) \quad \text{(eg. 3.102, p154)}.$$

Equating (2) & (4) , (3) & (5) and solving $T_{s,1}$ and $T_{s,2}$

$$T_{s,1} = \frac{\left(\frac{k}{2L} + h_2\bar{A}\right)h_1T_1 + \frac{k}{2L}h_2\bar{A}T_2 + \left(\frac{k}{L} + h_2\bar{A}\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\bar{A} + \frac{kh_2\bar{A}}{2L}}$$

$$T_{s,2} = \frac{\left(\frac{k}{2L}h_1T_1\right) + \left(\frac{k}{2L} + h_1\right)\cdot h_2\bar{A}T_2 + \left(\frac{k}{L} + h_1\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\bar{A} + \frac{kh_2\bar{A}}{2L}}$$

where,

$$\bar{A} = \frac{A_t\eta_o}{A} = \frac{A_t}{A} - \frac{NA_f}{A}(1-\eta_f)$$

Performing the calculations :

$$\eta_f = \frac{\tanh(mL_f)}{mL_f} \quad \text{(eq 3.87)}$$

$$m = \left(\frac{h_2P}{k_fA_c}\right)^{1/2} = \left(\frac{2h_2}{k_ft}\right) = \left(\frac{2\times 12\ W/m^2\cdot K}{250\ W/m\cdot K\times 0.002m}\right)^{1/2} = 6.9\ m^{-1}$$

$\therefore\ \eta_f = 0.994$

$$\frac{NA_f}{A} = \frac{N\cdot 2wL_f}{(\delta+t)N\cdot w} = \frac{2L_f}{\delta+t} = \frac{2\times 0.02m}{0.004m} = 10.0$$

$$\frac{A_t}{A} = \frac{NA_f}{A} + \frac{A_b}{A} = \frac{NA_f}{A} + \frac{\delta Nw}{(\delta+t)Nw} = \frac{NA_f}{A} + \frac{\delta}{\delta+t}$$

$$= 10 + \frac{0.002m}{0.004m} = 10.5$$

$\bar{A} = 10.5 - 10\cdot(1-0.994) = 10.4$

$h_2\bar{A} = 12\ W/m^2\cdot K \times 10.4 = 125\ W/m^2\cdot K$

$$\frac{k}{2L} = \frac{25\ W/m\cdot K}{0.06m} = 417\ W/m^2\cdot K$$

Thus,

$T_{s,1} = 92.7°C$

$T_{s,2} = 85.8°C$

The location of the maximum temperature in the wall can be found by setting the gradient of the temperature (from (1) page ④) to zero;

$$\frac{dT}{dx} = -\frac{\dot{q}x}{k} + \frac{T_{s,2}-T_{s,1}}{2L} = 0$$

$$\therefore X_{max} = k\frac{T_{s,2}-T_{s,1}}{2L\dot{q}}$$

$$T_{max} = T(X_{max})$$

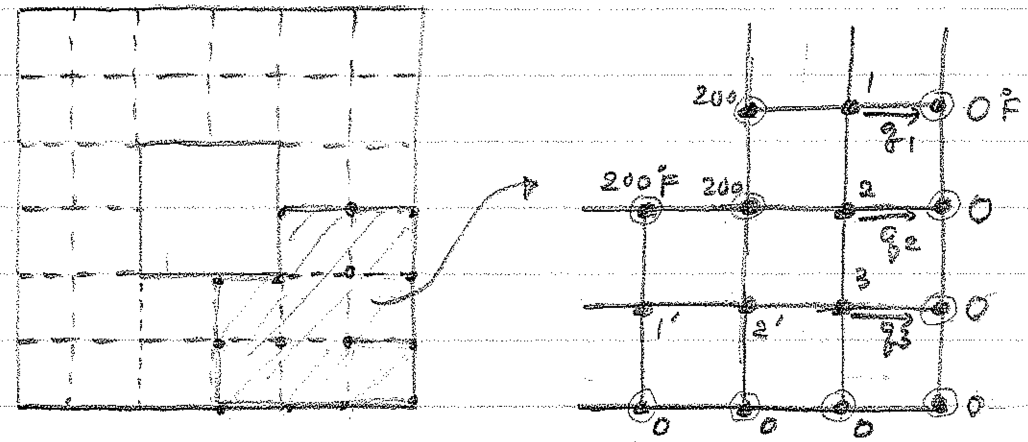
$$= \frac{\dot{q}L^2}{2k} + \frac{k(T_{s,2}-T_{s,1})^2}{8L^2\dot{q}} + \frac{T_{s,1}+T_{s,2}}{2}$$

$$= 93.7^\circ C$$

[Q4]

(a)

1.5ft grid.



Nodes 2 & 3 (2') : Case 1, interior node, table 4.2 (p218)

$$T_2 = \frac{200 + 0 + T_1 + T_3}{4} \quad (= T_2'), \qquad T_3 = \frac{T_2 + T_2' + 0 + 0}{4}$$

Node 1 (1') : case 5 with $q'' = 0$
(Symmetry line ⇒ adiabatic ⇒ $q'' = 0$)

$$T_1 = T_1' = \frac{2T_2 + 200 + 0}{4}$$

set of 3 equations, 3 unknown values may be solved.

$T_1 = 91.665°F$, $T_2 = 83.33°F$, $T_3 = 41.665°F$

heat flow from node 1, 2 and 3 to the cooler surface.

$$q_1' = k\cdot\left(\frac{\Delta y}{2}\cdot 1\right)\cdot\frac{T_1 - 0}{\Delta x} = \frac{k}{2}(91.67°F)/ft = 6.87 \text{ Btu/hr}\cdot\text{ft}$$

$\because \Delta x = \Delta y = 1.5ft$

$$q_2' = k(\Delta y \cdot 1)\cdot\frac{T_2 - 0}{\Delta x} = k(83.33°F)/ft = 12.50 \text{ Btu/hr.ft}$$

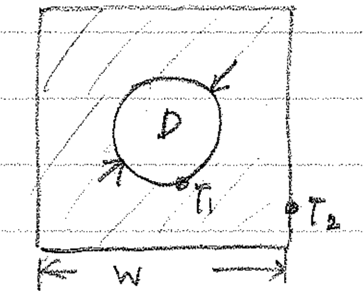
$$q_3' = k(\Delta y \cdot 1)\cdot\frac{T_3 - 0}{\Delta x} = k(41.67°F)/ft = 6.25 \text{ Btu/hr.ft}$$

$$q_t' = 8\times(q_1' + q_2' + q_3') = 8\times(25.62 \text{ Btu/hr.ft})$$

$$= 205 \text{ Btu/hr.ft}$$

This result will be ~~the~~ same when you do heat flow from hotter surface to the next nodes.

[Q5]



$D = 1\,ft$, $W = 2\,ft$

$T_1 = 400\,°F$ $k = 0.04\,Btu/hr \cdot ft^2 \cdot °F$

$L = 50\,ft$ $T_2 = 100\,°F$

Two Dimensional conduction

Case 6, Table 4.1 (P 209) $\left(\begin{matrix} W > D \\ L \gg W \end{matrix}\right)$

Shape factor

$$S = \frac{2\pi L}{\ln(1.08W/D)} = \frac{(2\pi)\cdot(50\,ft)}{\ln(1.08 \times 2\,ft/1\,ft)}$$

$$= 407.8\ (ft)$$

heat transfer rate per length

$$q' = S \cdot k \cdot \Delta T$$

$$= (407.8\,ft)\cdot(0.04\,Btu/hr \cdot ft^2 \cdot °F)\cdot(400 - 100\,°F)$$

$$= 4893.6\ Btu/hr \cdot ft$$

latent heat for condensation of steam = 1042 Btu/lb

total heat rate through the pipe (50-ft long)

$$q = q' \times 50\,ft = 244680\ Btu/hr$$

total amount of steam to be condensed

$$\frac{244680\ Btu/hr}{1042\ Btu/lb} = 235\ lb/hr$$ //