Homework #5

Question 1.

Known: initial temp., geometry > solid , convection condition of oil.

Find: Temperature $T(r, x, t)$ after 3 min.

(a) $T(0, 0, 3min)$ (b) $T(0, L, 3min)$ (c) $T(r_o, 0, 3min)$

Assumption: 2-dimensional conduction (x-dir, r-dir)

constant properties @ $T_f = (T_s + T_\infty)/2 = 450K$

Table A-1 (Stainless Steel AISI 304)

@ 450K, $\rho = 7900\ kg/m^3$

$c = 526\ J/kg \cdot K$

$k = 17.4\ W/m \cdot K$

$\alpha = k/(\rho \cdot c) = 4.19 \times 10^{-6}\ m^2/s$

Solution :

For short cylinder, we can approximate the temperature profile by superimposing solutions of infinite cylinder & plane plate

$$\therefore \frac{T(r, x, t) - T_\infty}{T_i - T_\infty} = P(x, t) \cdot C(r, t)$$

$$= \left.\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right|_{\text{plane wall}} \cdot \left.\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right|_{\text{infinite cylinder}}$$

For plane wall

$$Bi = \frac{h \cdot L}{k} = \frac{(500\ W/m^2 \cdot K)(0.03\ m)}{(17.4\ W/m \cdot K)} = 0.862$$

$$Fo = \frac{\alpha t}{L^2} = \frac{(4.19 \times 10^{-6}\ m^2/s) \cdot (3\ min)}{(0.03\ m)^2} = 0.84$$

From equation (5.41)

$$\theta_o^* = \frac{\theta_o}{\theta_i} = C_1 \exp(-\zeta_1^2 Fo)$$

From Table 5.1 with $Fo = 0.84 \rightarrow \begin{pmatrix} C_1 = 1.109 \\ \zeta_1 = 0.814 \end{pmatrix}$

$$\therefore \theta_o^* = \frac{\theta_o}{\theta_i} = \left.\frac{T(0, 3min) - T_\infty}{T_i - T_\infty}\right|_{\text{plane wall}} = 1.109 \exp\left(-(0.814)^2 \cdot 0.84\right) = 0.636$$

(For plane wall continued)..

From equation (5.40b)

$$\theta^* = \theta_o^* \cos(\xi_1 x^*) \qquad (x^* = \frac{x}{L}) \quad \text{when } x = L,\ x^* = 1$$

$$\frac{T(L, 3min) - T_\infty}{T_i - T_\infty} = \underbrace{\left(\frac{T(0, 3min) - T_\infty}{T_i - T_\infty}\right)}_{= 0.636} \cdot \underbrace{\cos(\xi_1 \times 1)}_{= 0.687\ (\text{when } \xi_1 = 0.814)}$$

$$= 0.437$$

For infinite cylinder

$$Bi = \frac{h \cdot r_o}{k} = \frac{(500\ W/m^2 \cdot K) \cdot (0.04\ m)}{17.4\ W/m \cdot K} = 1.149$$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{(4.19 \times 10^{-6}\ m^2/s)(3min)}{(0.04m)^2} = 0.47$$

From equation (5.49c)

$$\theta_o^* = C_1 \exp(-\xi_1^2 Fo)$$

Table 5.1 with $Fo = 0.47 \rightarrow C_1 = 1.227,\ \xi_1 = 1.307$

$$\therefore \theta_o^* = 1.227 \exp(-1.307^2 \times 0.47) = 0.550$$

$$\left(= \frac{T(0, 3min) - T_\infty}{T_i - T_\infty}\bigg|_{\substack{\text{infinite}\\ \text{cylinder}}}\right)$$

From equation (5.49b)

$$\theta^* = \theta_o^* \cdot J_o(\xi_1 r^*) \qquad (J_o(x) : \text{Bessel function}) \qquad r^* = \frac{r}{r_o}$$

Table B.4 $\rightarrow J_o(\xi_1 r^*) = J_o(1.307 \times 1) = 0.616$

when $r = r_o,\ r^* = 1$

$$\therefore \frac{T(r_o, 3min) - T_\infty}{T_i - T_\infty} = \underbrace{(0.550)}_{\theta_o^*} \times \underbrace{(0.616)}_{J_o(\xi_1 r^*)} = 0.339$$

(a) center of cylinder $T(0, 0, 3min)$

$$\frac{T(0, 0, 3min) - T_\infty}{T_i - T_\infty} = \frac{T(0, 3min) - T_\infty}{T_i - T_\infty}\bigg|_{\substack{\text{plane}\\ \text{wall}}} \times \frac{T(0, 3min) - T_\infty}{T_i - T_\infty}\bigg|_{\substack{\text{infinite}\\ \text{cylinder}}}$$

$$= \theta_o^*\big|_{\text{plane wall}} \times \theta_o^*\big|_{\text{infinite cylinder}}$$

$$= (0.636) \times (0.550) = 0.350 \rightarrow T(0, 0, 3min) = 405\ K$$

(b) Temperature @ center of cylinder face $T(0, L, 3\,min)$

$$\frac{T(0, L, 3min) - T_\infty}{T_i - T_\infty} = \left.\frac{T(L, 3min) - T_\infty}{T_i - T_\infty}\right|_{P.W} \times \left.\frac{T(0, 3min) - T_\infty}{T_i - T_\infty}\right|_{i.c}$$

$$= (0.437) \times (0.550) = 0.240$$

$$\therefore T(0, L, 3min) = 300K + (0.240)(600 - 300)K$$

$$= \underline{372\,K}$$ //

(c) Temp @ midheight of the side $T(r_o, 0, 3min)$

$$\frac{T(r_o, 0, 3min) - T_\infty}{T_i - T_\infty} = \left.\frac{T(0, 3min) - T_\infty}{T_i - T_\infty}\right|_{P.W} \times \left.\frac{T(r_o, 3min) - T_\infty}{T_i - T_\infty}\right|_{i.c}$$

$$= (0.636) \times (0.339)$$

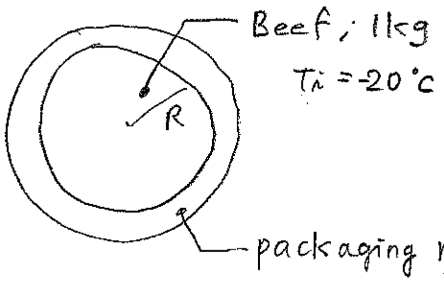
$$= 0.216$$

$$\therefore T(r_o, 0, 3min) = \underline{365\,K}$$ //

Question 2

Known: Mass, T_i, q, Power (P)

Find: time for beef adjacent to packaging to reach 0°C



Assumption
(1) Beef has properties of ice
(2) Constant properties
(3) packaging material has negligible heat capacity.
(4) Negligible Radiation & convection.

Solution

Neglecting radiation & convection losses, all the power absorbed in the packaging material conducts into the beef.

The surface heat flux (q_s'')

$$q_s'' = \frac{\dot{q}}{A_s} = \frac{0.5P}{4\pi R^2}$$

Radius of beef can be found using density & mass information.

$$\rho = \frac{m}{V} \qquad \underset{(\text{volume})}{V} = \frac{4}{3}\pi R^3 , \quad m = 1\text{kg}.$$

$$\therefore R = \left(\frac{3}{4\pi}\cdot\frac{m}{\rho}\right)^{1/3} = \left(\frac{3}{4\pi}\cdot\frac{1\text{kg}}{920\text{kg/m}^3}\right)^{1/3}$$

$$= 0.0638\text{ m}$$

Table A-3
ice @ 273 K
- $\rho = 920\text{ kg/m}^3$
- $k = 1.88\text{ W/m}\cdot\text{K}$
- $c = 2040$

$$\therefore q_s'' = \frac{0.5(1000\text{W})}{(4\pi)\cdot(0.0638\text{m})^2} = 9780\text{ W/m}^2$$

For constant surface heat flux, the relationship in Table 5.2b, Interior Case, Sphere, can be used.

dimensionless heat

$$q^* = \frac{q_s'' r_o}{k(T_s - T_i)} = \frac{(9780\text{ W/m}^2)\cdot(0.0638\text{ m})}{(1.88\text{ W/m}\cdot\text{K})\cdot(0°C - (-20°C))} = 16.6$$

We proceed to solve for F_o.

Assume that $F_o < 0.2$, from the Table 5.2b.

$$q^* \cong \frac{1}{2}\sqrt{\frac{\pi}{F_o} - \frac{\pi}{4}} \Rightarrow F_o = 0.0026 < 0.2$$

$\therefore$ The assumption was correct!

$$F_o = \frac{\alpha t}{r_o^2} \Rightarrow t = \frac{F_o \cdot r_o^2}{\alpha}$$

$$= \frac{F_o \cdot r_o^2}{\left(\frac{k}{\rho c}\right)} = \frac{(0.0026)\cdot(0.0638\,m)^2}{\left(\frac{1.88\ W/m\cdot K}{(920\ kg/m^3)\cdot(2040\ J/kg\cdot K)}\right)}$$

$$= \underline{10.6\,S}$$