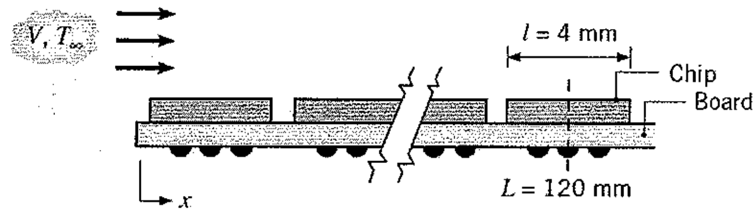
MAE 3314: Heat Transfer
Homework #6 (Due date – 11/1/07) Key Assignment

1. [10 pts] Forced air at T_∞=25°C and V=10m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4mm by 4mm, located 120mm from the leading edge of the board. A chip is dissipating 30 mW. Experiments have revealed that flow over the board is distributed by the elements and that convection heat transfer is correlated by the elements and that convection heat transfer is correlated by an expression of the form $Nu_x = 0.04\,\mathrm{Re}_x^{0.85}\,\mathrm{Pr}^{1/3}$.
The cooling condition listed above was designed at atmospheric pressure (= 1 atm).



(a) Estimate the surface temperature of the chip located 120 mm from the leading edge of the board when the board is operated at 76.5 kPa.

(b) It is desirable for the chip operating temperature to be independent of the location. What air velocity is required for operation at 76.5 kPa if the chip temperature is to be the same as at 1 atm?

* At 1 atm, 35°C (= T_f= (45°C+25°C)/2), air has values of: k=0.0269 W/m·K, ν=16.69x10^{-6} m^2/s, Pr=0.706 (From Table A.4)
* Assume that the average heat transfer coefficient for the chip surface is equivalent to the local value at x=L.
* Assume that air has ideal gas behavior.

273
66
339 340K

150°F → 66°C
$\nu = 20.78\times10^{-6}$ m²/s.

2. [10 pts] Nitrogen at 100 °F and 1 atm flows at a velocity of 10fps. A flat plate 6 in wide, at a temperature of 200 °F, is aligned parallel to the direction of flow. At a position 4 ft from the leading edge, determine the following:
(a) δ $\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}$
(b) δ_t
(c) $C_{f,x}$
(d) $\bar{C}_{f,L}$
(e) h_x
(f) $\bar{h}_L$
(g) total drag force
(h) total heat transfer

eq 7.10 → eq. 7.19

$Re_x = \frac{u_\infty \cdot L}{\nu} =$ ______

$\frac{\frac{10\,ft}{s}\cdot 4\,ft}{20.78\times10^{-6}\,\frac{m^2}{s}}\cdot\left(\frac{1m}{3.2808ft}\right)^2 = 0.178\times10^6$

$= 1.78\times10^5$

17.88×10^4
$17.88^{-1/2}\cdot10^{-2}$
0.236

$(17.87\times10^{-6})^{-1/2}$
$17.88^{-1/2}\cdot10^{3}$

0.3777

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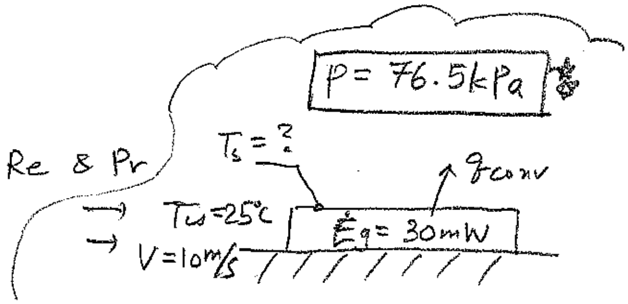
Homework 6

Question 1



Known:
- Nu_x expression interms of Re & Pr
- Flow condition
- $\dot{E}_g$ = 30mW.

Find: Surface temperature of chip surface (T_s) under atmospheric pressure of 76.5 kPa

Assumption:
(1) St. St. condition
(2) chip serface is isothermal
(3) The average heat transfer coefficient ($\bar{h}$) for the chip surface is equivalent to the local value at $x=L$ ($h_x(L)$)
(4) ideal gas behavior.

properties: Table A-4 air $p = 1\,atm$, $T_f = (45°C + 25°C)/2 = 35°C$
- $k = 0.0269\ W/m\cdot K$
- $\nu = 16.69 \times 10^{-6}\ m^2/s$
- $Pr = 0.706$

Solution

(a) From an energy balance on the chip

$$\underbrace{\cancel{\dot{E}_{in}}}_{\text{no q input}} - \underbrace{\dot{E}_{out}}_{q_{conv}} + \underbrace{\dot{E}_{gen}}_{30mW} = \underbrace{\cancel{\dot{E}_{st}}}_{St.St.}$$

$$\therefore q_{conv} = \dot{E}_g = 30mW.$$

$$= \bar{h}\, A_{chip}\, (T_s - T_\infty)$$

$$T_s = T_\infty + \frac{q_{conv}}{\bar{h}\cdot A_{chip}}$$

$A_{chip} = l^2$, $l = 4mm$.

from the Assumption $\bar{h} \approx h_x(L)$

$$Nu_x = \frac{h_x \cdot x}{k} = 0.04\, Re_x^{0.85}\, Pr^{1/3}$$

To calculate Re_x, we need ν (dynamic viscosity)

$$\nu = \frac{\mu}{\rho}$$

We assumed that air has ideal gas behavior,

$$\rho = \frac{P}{RT} \qquad \therefore \frac{\nu_1}{\nu_2} = \frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \qquad \therefore \nu_{at\,76.5kpa} = \nu_{1atm} \cdot \frac{1\,atm}{76.5kpa}$$

$\therefore \nu = (16.69\times10^{-6}\ m^2/s)\cdot\left(\dfrac{1\,atm = 101.3\,Pa}{76.5\ kPa}\right) = 22.10\times10^{-6}\ m^2/s$

$Pr = \dfrac{\nu}{\alpha} = \dfrac{\left(\frac{\mu}{\rho}\right)}{\left(\frac{k}{\rho c}\right)} = \dfrac{\mu\cdot c}{k} \Rightarrow$ independent of pressure change.

So at 76.5 kPa

$$\left.\begin{array}{l}\nu = 22.10\times10^{-6}\ m^2/s\\ k = 0.0269\ W/m\cdot K\end{array}\right\} \Rightarrow Re_x = \frac{U_\infty\cdot L}{\nu} = \frac{(10\,m/s)\cdot(120\,mm)}{(22.10\times10^{-6}\ m^2/s)}$$

$Pr = 0.706$ $\qquad = 5.43\times10^4$

$$Nu_x = \frac{h_x\cdot L}{k} = 0.04\ Re_x^{0.85}\cdot Pr^{1/3}$$

$$= (0.04)(5.43\times10^4)^{0.85}\cdot(0.706)^{1/3} = 376.73$$

$$\therefore h_x = (Nu_x)\cdot\left(\frac{k}{L}\right)$$

$$= (376.73)\cdot\left(\frac{0.0269\ W/m\cdot K}{120\ mm}\right)$$

$$= 84.45\ W/m^2\cdot K$$

$$\therefore T_s = T_\infty + \frac{q_{conv}}{\underset{(=h_x)}{h}\cdot A_{chip}}$$

$$= 25^\circ C + \frac{30\,mW}{(84.45\,\frac{W}{m^2\cdot K})\cdot(4\,mm)^2} = \underline{47.2\ ^\circ C}$$

(b) For 1 atm operation, $h_x = (0.04)\cdot\left(\dfrac{0.0269\ W/m\cdot K}{120\,mm}\right)\cdot\left(\dfrac{(10\,m/s)\cdot(120\,mm)}{16.69\times10^{-6}\ m^2/s}\right)^{0.85}\cdot(0.706)^{1/3}$

$= 107\ W/m^2\cdot K$ $\qquad$ ν at 1 atm

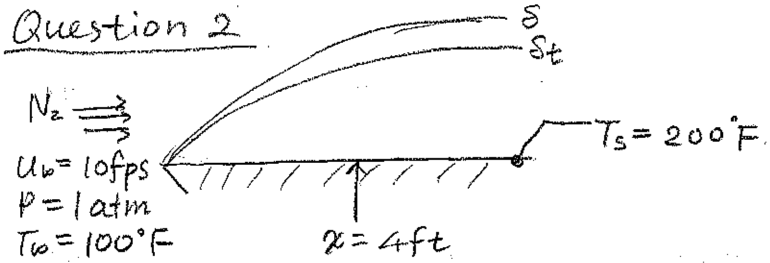
$\therefore$ in order to achieve this h value at 76.5 kPa,

$$h_x = 107\ W/m^2\cdot K = 0.04\left(\frac{0.0269\ W/m\cdot K}{120\ mm}\right)\cdot\left(\frac{V\cdot 120\,mm}{22.10\times10^{-6}\ m^2/s}\right)^{0.85}\cdot(0.706)^{1/3}$$

ν at 76.5 kPa

$\Rightarrow V = 13.2\ m/s$

Question 2



properties N_2 @ $p = 1atm$

$T_f = (100+200)°F/2 = 150°F = 338.56K$

Table A-4. @350K	Linear interpolation $\longrightarrow$	@ 338.56 K
• $\rho = 0.9625\ kg/m^3$		$\rho = 0.9993\ kg/m^3$
• $c = 1.042\ kJ/kg \cdot K$		$\nu = 19.65 \times 10^{-6}\ m^2/s$
• $\mu = 200.0 \times 10^{-7}\ N \cdot S/m^2$		$k = 28.52 \times 10^{-3}\ W/m \cdot K$
• $\nu = 20.78 \times 10^{-6}\ m^2/s$		$Pr = 0.712$
• $k = 29.3 \times 10^{-3}\ W/m \cdot K$		↓ unit conversion.
• $\alpha = 29.2 \times 10^{-6}\ m^2/s$		$\rho = 0.0620\ lbm/ft^3$
• $P_r = 0.711$		$\nu = 211.506 \times 10^{-6}\ ft^2/s$
		$k = 16.478 \times 10^{-3}\ Btu/hr \cdot ft \cdot °F$

at $x = 4ft$

$$Re_x = \frac{u_\infty \cdot L}{\nu} = \frac{(10 ft/s) \cdot (4ft)}{211.506 \times 10^{-6}\ ft^2/s} = 1.8912 \times 10^5 < Re_{x,c} = 5 \times 10^5$$

$\therefore$ flow is fully laminar till $x = 4ft$

Rember the flow condition is "Laminar Flow over isothermal Plate"
Textbook 7.2.1.

(a) equation (7.19)

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5 \cdot x}{\sqrt{Re_x}} = \frac{(5) \cdot (4ft)}{\sqrt{1.8912 \times 10^5}} = 0.0460\ ft$$

(b) equation (7.24)

$$\frac{\delta}{\delta_T} \approx Pr^{1/3} \quad \therefore \delta_T \approx \frac{\delta}{Pr^{1/3}} = (0.0460\ ft) \cdot (0.712)^{-1/3}$$
$$= 0.05152\ ft$$

(c) equation (7.20)

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\rho u_\infty^2/2} = 0.664\ Re_x^{-1/2} = (0.664) \cdot (1.8912 \times 10^5)^{-1/2}$$
$$= 0.001526 = 1.526 \times 10^{-3}$$

(d) equation (7.29)

$$\bar{C}_{f,x} = 1.328 Re_x^{-1/2} = 2 \cdot C_{f,x}$$
$$= 2 \times 0.001526$$
$$= 0.003053 \quad = 3.053 \times 10^{-3}$$

(e) equation (7.23)

$$Nu_x \equiv \frac{h_x \cdot x}{k} = 0.332\ Re_x^{1/2} \cdot Pr^{1/3} \quad \text{for } Pr = 0.711 \geq 0.6$$

$$\therefore\ h_x(L) = \frac{Nu \cdot k}{x} = \left(\frac{k}{L}\right) \cdot (0.332)\ Re_x^{1/2}\ Pr^{1/3}$$

1926.49

$$= \left(\frac{16.478 \times 10^{-3}\ Btu/hr \cdot ft\ {}^\circ F}{4\,ft}\right) \cdot (0.332)(1.8912 \times 10^{5})^{1/2}(0.712)^{1/3}$$

$$= 0.5313\ Btu/hr \cdot ft^2 \cdot {}^\circ F$$

(f) equation (7.30)

$$\overline{Nu}_x \equiv \frac{\bar{h}_x \cdot x}{k} = 0.664\ Re_x^{1/2}\ Pr^{1/3}$$
$$= 2 \cdot Nu_x$$

$$\therefore\ \bar{h}_L = 2 \cdot h_x(L) = 1.0626\ Btu/hr \cdot ft^2 \cdot {}^\circ F$$

(g) Total drag force (F_D) $= \bar{\tau}_{s,x} \cdot A_s$

equation (7.28)

$$\bar{C}_{f,x} \equiv \frac{\bar{\tau}_{s,x}}{\rho u_\infty^2/2} \longrightarrow F_D = \bar{\tau}_{s,x} \cdot A_s$$

$$= \left(\bar{C}_{f,x} \cdot \frac{\rho u_\infty^2}{2}\right)(A_s)$$

$$= (3.053 \times 10^{-3}) \cdot \left(\frac{(0.0620\ lbm/ft^3) \cdot (10\ ft/s)^2}{2}\right) \cdot \left(4ft \times 0.5\,ft\right)$$

(=6 in)

$$= 18.929 \times 10^{-3}\ lbm \cdot ft/s^2$$

(h) Total heat transfer

$$q = \bar{h} \cdot A \cdot (T_s - T_\infty)$$

$$= (1.0626 \text{ Btu/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}) \cdot (4\text{ft} \times 6\text{in}) \cdot (200^\circ\text{F} - 100^\circ\text{F})$$

$$= 212.52 \text{ Btu/hr}$$