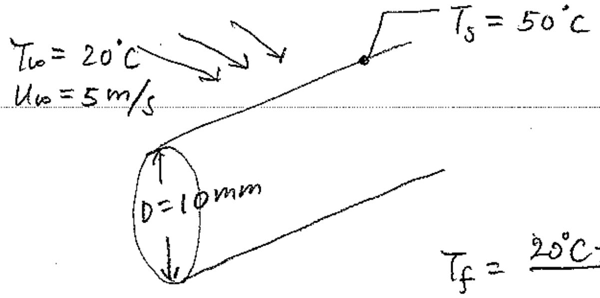
Homework # 7

[1]



$$Re_D = \frac{\rho u_\infty D}{\mu} = \frac{u_\infty D}{\nu}$$

$$T_f = \frac{20^\circ C + 50^\circ C}{2} = 35^\circ C = 308 K$$

(a) air

Table A-4

@ 300K		@ 308 K
$\nu = 15.89 \times 10^{-6}\ m^2/s$	interpolation	$\nu = 16.69 \times 10^{-6}\ m^2/s$
$P_r = 0.707$	$\longrightarrow$	$P_r = 0.7059$
$\rho = 1.1614\ kg/m^3$		$k = 0.0269$
$k =$		

$$Re_D = \frac{u_\infty D}{\nu} = \frac{(5 m/s)\cdot(0.01 m)}{16.69 \times 10^{-6} m^2/s} = 2.9958 \times 10^3$$

(7.54)
$$\overline{Nu}_D = 0.3 + \frac{0.62\, Re_D^{1/2}\, P_r^{1/3}}{[1 + (0.4/P_r)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 28.0546$$

$$\bar{h} = \overline{Nu}_D \cdot \frac{k}{D} = (28.0546) \times \frac{(0.0269\ W/m\cdot K)}{(0.01\ m)} = 76.789\ W/m^2\cdot K$$

$$q' = \bar{h} \cdot \underbrace{A_s}_{=\pi D} \cdot (T_s - T_\infty) = (76.789\ W/m^2\cdot K)\cdot(\pi)(0.01 m)\cdot(50 - 20\ K)$$
$$= 72.372\ W/m$$

(b) water

Table A-6, $T_f = 308 K$.

$\rho = 994\ kg/m^3$

$\mu = 725 \times 10^{-6}\ N\cdot S/m^2$

$k = 0.625\ W/m\cdot K$

$Pr = 4.85$

$$Re_D = \frac{(5 m/s)\cdot(0.01 m)}{\frac{725 \times 10^{-6}\ N\cdot S/m^2}{994\ kg/m^3}} = 6.855 \times 10^4$$

(7.54) $\overline{Nu}_D = 347.37$

$\bar{h} = \overline{Nu}_D \cdot \frac{k}{D} = 21703.74\ W/m^2\cdot K$ $\qquad \therefore q' = 20.46\ KW/m$

(c) oil @ $T_f = 308K$

$\nu = 340 \times 10^{-6} m^2/s$

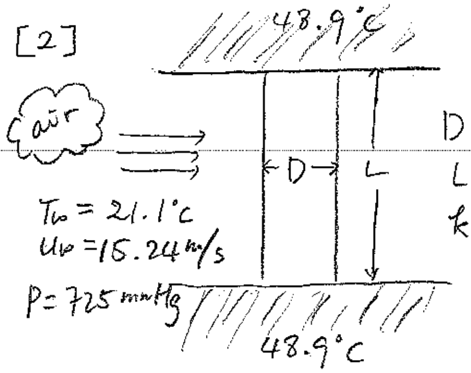
$k = 0.145 W/m \cdot K$

$Pr = 4000$

$Re_D = 147$, $\overline{Nu}_D = 120.358$

$\bar{h} = 1745.19 W/m^2 \cdot K$ $q' = 1644.8 W/m$

[2]



D = 1.27 cm
L = 60.96 cm
k = 53 W/m·K

Assumption

(1) St. St condition
~~(2) 1-D conduction through cylinder~~
(3) constant properties
(4) air behaves as ideal gas

$T_f \sim 35°C = 308\,K$

Table A-4, Air @ 308 K (use the interpolated value from question 1)

$\nu = 16.69 \times 10^{-6}\ m^2/s$
$Pr = 0.7059$
) @ 1 atm = 760 mmHg

$k = 26.892 \times 10^{-3}\ W/m\cdot K$

ideal gas law $\rho = \frac{PV}{RT}$

$\therefore \frac{\rho_1}{\rho_2} = \frac{P_1}{P_2} = \frac{\nu_2}{\nu_1}$

$$\nu_{(at\ 725\ mmHg)} = \frac{760\ mmHg}{725\ mmHg} \cdot \nu_{(at\ 1\ atm)} = \frac{760}{725} \cdot (16.69 \times 10^{-6}) = 17.496 \times 10^{-6}\ m^2/s$$

$$Re_D = \frac{u_\infty D}{\nu} = \frac{(15.24\ m/s)\cdot(0.0127\ m)}{17.496 \times 10^{-6}\ m^2/s} = 1.106 \times 10^4$$

(7.54) Churchill & Bernstein correlation

$$\overline{Nu}_D = 0.3 + \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 56.5765$$

(a) $$\bar{h} = \overline{Nu}_D \cdot \frac{k}{D} = (56.5765) \cdot (26.892 \times 10^{-3}\ W/m\cdot K)/(0.0127\ m) = \underline{119.80\ W/m^2\cdot K}$$

(b) For the fin with (B) Adiabatic tip condition (Table 3.4)

- $M = (\bar{h} P k A_c)^{1/2}\, \theta_b = (\bar{h} \cdot \pi D \cdot \pi D^2/4 \cdot k)^{1/2}\, \theta_b = \frac{\pi}{2}(\bar{h} \cdot k \cdot D^3)^{1/2}\, \theta_b$

$$= (\pi/2)\sqrt{(119.8\ W/m^2\cdot K)(53\ W/m\cdot K)(0.0127\ m)^3} \cdot (27.8\ K) = 4.9776\ W$$

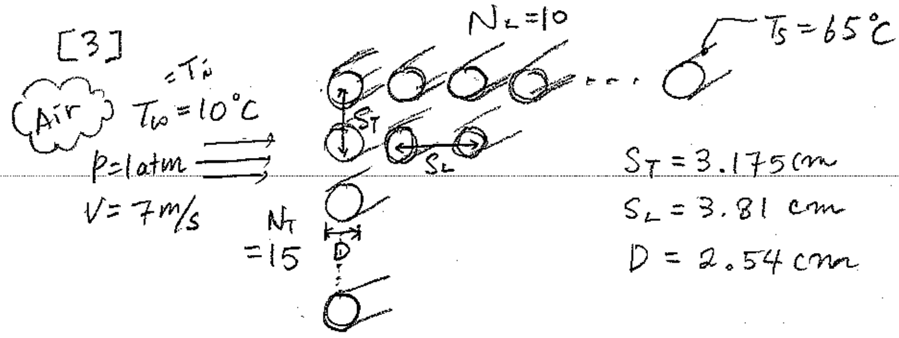
$P = \pi D$
$A_c = \pi D^2/4$
$\theta_b = 48.9 - 21.1 = 27.8\ °C = 27.8 K$

- $m = (\bar{h} \cdot P / k \cdot A_c)^{1/2} = (4\bar{h}/kD)^{1/2} = \left(\frac{4 \times 119.80\ W/m^2\cdot K}{53\ W/m\cdot K \cdot 0.0127\ m}\right)^{1/2} = 26.682\ m^{-1}$
- $mL = (26.682\ m^{-1}) \cdot (0.6096/2\ m) = 8.1327$

the fin heat rate $q_f = M \cdot \tanh mL = (4.9776\ W) \cdot \tanh(8.1327) = 4.9776\ W$

Total heat transfer from cylinder to air
$q = 2 \cdot q_f = 9.9552\ W$

[3]



$S_T = 3.175$ cm
$S_L = 3.81$ cm
$D = 2.54$ cm

$T_m \sim 35°C = 308K$
Air @ 308K
$\nu = 16.69 \times 10^{-6}\ m^2/s$
$Pr = 0.7059$
$k = 26.892 \times 10^{-3}$ W/m·K
$Pr_s = 0.7014$
(at 338K)

$$V_{max} = \frac{S_T}{S_T - D} \cdot V = \frac{3.175\,cm}{(3.175 - 2.54)\,cm} \cdot (7\,m/s) = 35\,m/s$$

$$Re_{D,max} = \frac{V_{max} \cdot D}{\nu} = \frac{(35\,m/s)\cdot(0.0254\,m)}{(16.69 \times 10^{-6}\,m^2/s)} = 5.3265 \times 10^4$$

Zukauskas' correlation (7.64)

$$\overline{Nu}_D = C\, Re_{D,max}^{m}\; Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

For Aligned configuration, $Re_{D,max} \sim 5 \times 10^4$, $C = 0.27$, $m = 0.63$
($S_T/S_L = 0.83 > 0.7$)

$$\therefore \overline{Nu}_D = (0.27)\cdot(5.3265 \times 10^4)^{0.63}(0.7059)^{0.36}\left(\frac{0.7059}{0.7014}\right)^{1/4}$$
$$= 226.602$$

$$\bar{h} = \overline{Nu}_D \cdot \frac{k}{D} = (226.602)\frac{26.892 \times 10^{-3}\,W/m\cdot K}{0.0254} = 239.913\,W/m^2\cdot K$$

(7.67)

$$\frac{T_s - T_o}{T_s - T_i} = exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T C_p}\right) = exp\left(-\frac{\pi \cdot (0.0254)(10 \times 15)(239.913\,W/m^2\cdot K)}{(1.130\,kg/m^3)\cdot(7\,m/s)(15)\cdot(0.03175\,m)(1007\,J/kg\cdot K)}\right)$$

$$\left(\begin{array}{l}\rho = 1.130\,kg/m^3 \\ C_p = 1007\,J/kg\cdot K\end{array}\right. \qquad = exp(-0.757) = 0.469$$

$$T_s - T_o = (T_s - T_i)\cdot(0.469) = (65-10)(0.469) = 25.795\,°C$$

$$\Rightarrow T_o = T_s - 25.795 = 39.205\,°C$$

(7.66)

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = \frac{(55) - (25.795)}{\ln\left(\frac{55}{25.795}\right)} = 38.572\,°C$$

(7.67)

$$q' = N \cdot (\bar{h} \cdot \pi \cdot D \cdot \Delta T_{lm})$$
$$= (15 \times 10)\left[(239.913\,W/m^2\cdot K)\cdot(\pi)\cdot(0.0254\,m)\cdot(38.572\,°C)\right]$$
$$= 110.764\,kW/m$$