@ 700 °F
$k = 25$ Btu/hr.ft. °F

@ 68 °F
$k = 81$ Btu/h.ft.°F

New PIIS 4.20, 4.22

1. [20 pts] A solid, 0.5% carbon steel, 2-in diameter by 2.5-in long cylinder, initially at 1200°F, is quenched during heat treatment in a fluid at 200°F. The surface heat transfer coefficient is 150 Btu/h·ft^2·°F.

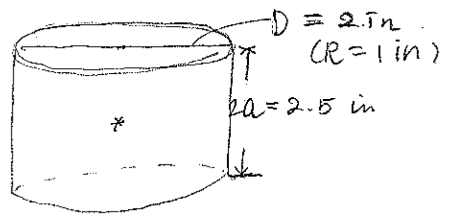
(i) Determine the centerline temperature at the midpoint of length 2.7 min after immersion in the fluid.

* short cylinder problem. ~~(2-D, transient)~~ prob)

(Infinite cylinder) x (semi Infinite slab)

~~$\alpha_{68} = 0.57$ ft²/h~~

$\alpha_{700} = 0.46$ ft²/h



[1] Checking for suitability of a Lumped analysis

$$L = \frac{V}{A_s} = \frac{\pi R^2 (2a)}{2\pi R(2a) + 2\pi R^2} = \frac{R(2a)}{2(2a)+2R}$$

characteristic length

$$= \frac{(1\,in)\cdot(2.5\,in)}{2\cdot(2.5\,in) + 2\cdot(1\,in)} = 0.36\ in$$

$$B_i = \frac{\bar{h}\cdot L}{k} = \frac{(150\ Btu/h\cdot ft^2\cdot{}^\circ F)(0.36/12)ft}{25\ Btu/h\cdot ft\ {}^\circ F} = 0.18$$

∴ Lumped Parameter analysis is not suitable.

$T(0, 0, 2.7\ min)$?

⇓

$$\left(\frac{T-T_\infty}{T_i-T_\infty}\right) = \left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{I.C} \times \left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{P.W}$$

[2] plane wall subproblem.

$$B_i = \frac{\bar{h}\cdot a}{k} = \frac{(150\ Btu/h\cdot ft^2\cdot{}^\circ F)\cdot(1.25/12)ft}{25\ Btu/hr\cdot ft\ {}^\circ F} = 0.625$$

$$F_o = \frac{\alpha t}{a^2} = \frac{(0.46\ ft^2/h)(2.7/60)h}{(1.25/12)^2\ ft^2} = 1.92$$

1 term approx. solution

$$\theta_o^* = C_1 \exp(-\zeta_1^2 F_o) \quad (5.41)$$
$$= (1.0814)\exp(-0.7051^2 \times 1.92)$$
0.95456
$$= 0.4163$$

[3] cylindrical subproblem

$$B_i = \frac{\bar{h}\cdot R}{k} = \frac{(150\ Btu/h\cdot ft^2\cdot{}^\circ F)\cdot(1/12\ ft)}{(25\ Btu/h\cdot ft\cdot{}^\circ F)} = 0.5$$

$$F_o = \frac{\alpha\cdot t}{R^2} = \frac{(0.46\ ft^2/h)\cdot(2.7/60)h}{(1/12\ ft)^2} = 2.99$$

1 term approx. solution

$$\theta_o^* = C_1\cdot\exp(-\zeta_1^2 F_o) \quad (5.49c)$$
$$= (1.1143)\ \exp(-0.9408^2 \times 2.99)$$
$$= 0.0790$$

[4] complete solution.

$$\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{short\ cylinder} = (0.4163)\times(0.0790)$$
$$= 0.03289$$

$$\therefore T(0, 0, 2.7\ min) = 0.03289\,(T_i - T_\infty) + T_\infty$$
$$= (0.03289)(1200-200) + 200$$
$$= 232.89\ {}^\circ F$$

(ii) Determine the time required for the temperature at the center (radially and axially) of the solid cylinder to reach 205°F.

To find the time required to attain a given temperature in transient multidimensional problem, a trial-and-error solution is required. From 1-(i), we know that the time is greater than 2.7 min. A logical approach is to use this as a beginning point and to calculate T for several larger values of time.

$\boxed{1}$ Try $t = 3.5$ min

① plane wall solution

$Bi = 0.625$, $Fo = \alpha t/a^2 = \dfrac{(0.46)(3.5/60)}{(1.25/12)^2} = 2.473$

$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo)$

$= (1.0814)\exp(-0.7051^2 \times 2.473)$

$= 0.3162$

② Infinite cylinder solution

$Bi = 0.5$, $Fo = \alpha t/R^2 = \dfrac{(0.46)(3.5/60)}{(1/12)^2} = 3.864$

$\theta_o^* = (1.1143)\exp(-0.9408^2 \times 3.864)$

$= 0.03645$

③ complete solution.

$\dfrac{T - T_\infty}{T_i - T_\infty} = (0.3162) \times (0.03645)$

$= 0.0115$

$\therefore T = 200 + (0.0115)(1200-200)$

$= 211.5\ °F$

$\boxed{2}$ Try $t = 4.0$ min

① Plane wall

$Bi = 0.625$, $Fo = 2.82624$

$\theta_o^* = (1.0814)\exp(-0.7051^2 \times 2.82624)$

$= 0.2653$

② Infinite cylinder

$Bi = 0.5$ $Fo = 4.416$

$\theta_o^* = (1.1143)\exp(-0.9408^2 \times 4.416)$

$= 0.02236$

$T = 200 + (0.2653 * 0.02236)(1200-200)$

$= 205.932$

$\boxed{3}$ try $t = 4.05$ min

① plane wall

$Bi = 0.625$; $Fo =$

New P169. 6.16.

2. [20 pts] Castor oil at 38°C flow over a wide, 6 m long, heated plate at 0.06 m/s. For a surface temperature of 93°C, determine: (a) Film temperature to evaluate the fluid properties, (b) the hydrodynamic boundary layer thickness δ at the end of the plate, (c) the total drag on the surface per unit width, (d) the thermal boundary layer thickness δ_t at the end of the plate, (e) the local heat transfer coefficient h_x at the end of the plate, and (f) the total heat flux from the surface per unit width. Assume the thermal diffusivity α to be 7.22×10^{-8} m^2/s and the thermal conductivity k to be 0.213 W/m·K at the film temperature. $\nu = 6.0 \times 10^{-5}\ m^2/s$

$\rho = 1000\ kg/m^3$

(a) Film temperature $T_f = \dfrac{T_s + T_\infty}{2} = \dfrac{93+38}{2} = 65.5°C$

(b) $Re_L = \dfrac{u_\infty \cdot L}{\nu} = \dfrac{(0.06 m/s)\cdot(6m)}{(6.0\times10^{-5}\ m^2/s)} = 6000$

(7.19) $\delta = \dfrac{5.0\ L}{\sqrt{Re_L}} = \dfrac{(5.0)\ (6\ m)}{\sqrt{6000}} = 0.387\ m$

(c) $\overline{C_{f_L}} = 1.328\ Re_L^{-1/2} = \dfrac{1.328}{\sqrt{6000}} = 0.01714$
(7.29)

The drag force F_f → (7.20)

$F_f = (area)\cdot \tau_{s.L} = (area)\cdot\left[C_{f.L}\cdot \dfrac{\rho u_\infty^2}{2}\right] \cong (area)\cdot\left[0.664\ Re_L^{-1/2}\right]$

$= (6m^2/m)\cdot(0.01714)\cdot \dfrac{(1000 kg/m^3)\cdot(0.06 m/s)^2}{2}$

or $(6m^2/m)\cdot(0.664/\sqrt{6000})$

$= 0.185\ \dfrac{kg/m.s^2}{m} = 0.185\ N/m$

(d) $P_r = \dfrac{\nu}{\alpha} = \dfrac{6.0\times10^{-5}\ m^2/s}{7.22\times10^{-8}\ m^2/s} = 8.31\times10^2$

$\delta_t \cong \dfrac{\delta}{Pr^{1/3}} = \dfrac{0.387\ m}{(831)^{1/3}} = 0.041\ m$
(7.24)

(e) Local heat transfer coeff h_L
(7.23)
$Nu_x \equiv \dfrac{h_x \cdot x}{k} = 0.332\ Re_x^{1/2}\ P_r^{1/3}$

$\therefore h_L = \left(\dfrac{k}{L}\right)\cdot(0.332)\cdot Re_L^{1/2}\cdot P_r^{1/3}$

$= \left(\dfrac{0.213\ W/m.K}{6m}\right)(0.332)\cdot\sqrt{6000}\cdot 831^{1/3}$

$= 8.58\ W/m^2.K$

$\therefore q_s' = \bar{h}_L \cdot A_s \cdot (T_s - T_\infty)$
$= (17.16 W/m^2.K)\cdot(6m^2/m)\cdot(93-38°C)$
$= 5665\ W/m$

(f) Total heat flux.
(7.30)
$\overline{Nu_L} = \dfrac{\bar{h}_L \cdot L}{k} = 0.664\ Re_L^{1/2}\ P_r^{1/3} = 2 Nu_x \rightarrow \therefore \bar{h}_L = 2h_L = 17.16\ W/m^2.K$

P 176

New 6.24

$T_{m,i}$ $T_{m,o}$

3. [20 pts] For fully developed velocity profile, approximate the length of 0.10-in i.d. tube required to raise the bulk temperature of benzene from 60 °F to 100 °F. The tube wall temperature is constant at 150 °F, and the average velocity is 1.6 fps.

@ $\Delta T_{am} = 80°F$

$$\begin{cases} \rho = 54.6 \text{ lbm/ft}^3 \\ k = 0.092 \text{ Btu/h}\cdot\text{ft}\cdot°F \\ C_p = 0.42 \text{ Btu/lbm}\cdot°F \\ \mu_m = 3.96\times10^{-4} \text{ lbm/ft}\cdot s \\ Pr = 6.5 \end{cases}$$

$$\Delta T_{am} = \frac{T_{mi} + T_{m,o}}{2} = \frac{60+100}{2} = 80°F$$

$$Re_D = \frac{\rho \cdot V \cdot D}{\mu_m} = \frac{(54.6 \text{ lbm/ft}^3)\cdot(1.6 \text{ ft/s})\cdot(0.1/12 \text{ ft})}{3.96\times10^{-4} \text{ lbm/ft}\cdot s}$$

$= 1838 \rightarrow$ laminar flow.

For fully developed, internal, larminar flow (Textbook section 8.4)

Average Nusselt number $\overline{Nu}_D$ for isothermal pipe (T_s = constant)

(8.55) $\overline{Nu}_D = 3.66$

$$\therefore \bar{h}_D = (\overline{Nu}_D)\cdot\left(\frac{k}{D}\right) = (3.66)\cdot\left(\frac{0.092 \text{ Btu/h}\cdot\text{ft}\cdot°F}{(0.1/12)\text{ ft}}\right) = 40.406 \text{ Btu/h}\cdot\text{ft}^2\cdot°F$$

Another equation should be found by energy balance on the fluid

(8.43) $\longrightarrow$ $q = \dot{m} C_p \cdot \Delta T_m = \bar{h}\cdot A_s \,\Delta T_{lm} = \bar{h}\cdot(\pi D L)\cdot \Delta T_{lm}$

(8.44) $\longrightarrow$
$$\begin{cases} \Delta T_m = (T_{m,o} - T_{m,i}) = (100-60) = 40°F \\ \Delta T_{lm} \equiv \dfrac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)} = \dfrac{(150-100)-(150-60)}{\ln\left(\frac{50}{90}\right)} = 68.05°F \end{cases}$$

$$\therefore L = \frac{\dot{m} C_p \cdot \Delta T_m}{\bar{h}\cdot(\pi D)\cdot\Delta T_{lm}} = \frac{\rho\cdot V\cdot A_c\, C_p\, \Delta T_m}{\bar{h}\cdot(\pi D)\,\Delta T_{lm}} = \frac{\rho\cdot V\cdot D\cdot C_p\,\Delta T_m}{4\bar{h}\,\Delta T_{lm}}$$

$$= \frac{(54.6 \text{ lbm/ft}^3)\cdot(1.6 \text{ ft/s})(0.1/12 \text{ ft})(0.42 \text{ Btu/lbm}°F)\cdot(40°F)}{(4)(40.406 \text{ Btu/h}\cdot\text{ft}^2\cdot°F)\cdot(68.05°F)}\cdot\left(\frac{3600 s}{h}\right)$$

$= 4.003$ ft

p203

5.143

$N_L = 9$

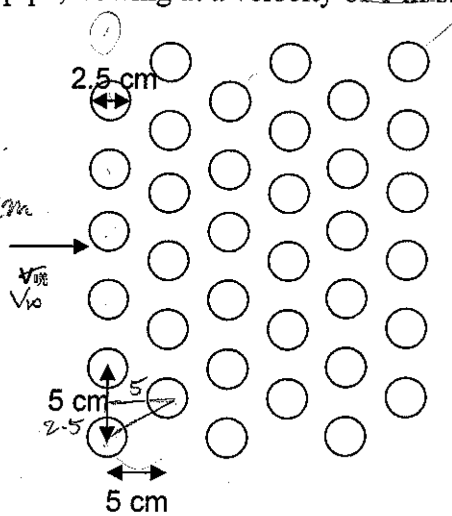
4. [20 pts] Pressurized liquid water at 40°C flows (without phase change) across a 46-cm-wide (tube-length direction) staggered tube bank (Fig.), carrying combustion gases which keep the tube surfaces at 120°C. For each 30 cm in height of tube bank, water is supplied in a 15-cm-ID pipe, flowing at a velocity of 1 m/s. $N_T = 6$

$D = 2.5\,cm$

$S_T = 5\,cm$

$S_L = 5\,cm$

$S_D = 5.59\,cm$



@ T_f: $\rho = 994.6\ kg/m^3$

$c_p = 4175.6\ J/kg\cdot K$

$\nu = 0.364\times10^{-6}\ m^2/s$

$k = 0.668\ W/m\cdot K$

$P_r = 2.22$

$P_{rs} = 1.34$

(i) What temperature would you use to evaluate fluid properties?

external flow : film temperature

$$T_f = \frac{T_s + T_\infty}{2} = \frac{120+40}{2} = 80^\circ C$$

except P_{rs} @ $T_s = 120^\circ c$

(ii) Assume that at the temperature you use in (i), $\nu = 0.364\text{x}10^{-6}\ m^2/s$, k=0.668 W/m·K, and **Pr**=2.22. Estimate the temperature of the water after passing through the tube bank.

Since $S_D = 5.59\ cm > \frac{S_T + D}{2} = 3.75\ cm$

$$\therefore V_{max} = \frac{S_T}{S_T - D}\cdot V = \frac{5}{(5-2.5)}\cdot(1\,m/s) = 2\,m/s$$

$$Re_{D,max} = \frac{\rho\cdot V_{max} D}{\mu} = \frac{V_{max}\cdot D}{\nu} = \frac{(2\,m/s)\cdot(0.025\,m)}{0.364\times10^{-6}\,m^2/s} = 1.3736\times10^5$$

Since $Re_{,max} > 40{,}000$, eq.(7.60) may not be good fomular to use

(7.64) Zukauska's equation

$$\overline{Nu}_D = C\cdot Re_{D,max}^{m}\cdot P_r^{0.36}\left(\frac{P_r}{P_{rs}}\right)^{1/4} \qquad C = 0.35\left(\frac{S_T}{S_L}\right)^{1/5} = 0.35$$

$m = 0.60$

$$= (0.35)(1.3736\times10^5)^{0.6}\cdot(2.22)^{0.36}\,(2.22/1.34)^{1/4}$$

$$= 640.1408$$

$N_L < 20 \quad \therefore \quad \overline{Nu}_{D\,(N_L<20)} = \overset{0.97\ (\text{Table 7.8})}{C_2}\ \overline{Nu}_D$

$$= 620.9365$$

$$\therefore \bar{h}_o = \overline{Nu}_D\cdot\left(\frac{k}{D}\right) = (620.9365)\cdot\left(\frac{0.668\,W/m\cdot K}{0.025\,m}\right)$$

$$= 16591.42\ W/m^2 K$$

Those who used $N = 36$ will get full credit.

Space for question 4.

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right) \qquad (7.67)$$

0.05645

$$= \exp\left(-\frac{\pi (0.025\,m)(54)\cdot(16591.42\ W/m^2\cdot K)}{(994.6\,kg/m^3)(1\ m/s)(6)(0.05\,m)\cdot(4175.6\,J/kg\cdot K)}\right)$$

$$= 0.9451$$

$$T_o = T_s - (0.9451)\cdot(T_s - T_i)$$

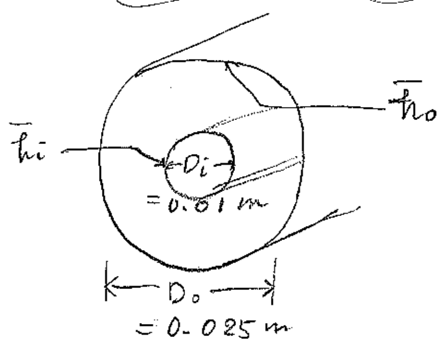
$$= 120 - (0.9451)\cdot(120 - 40) = 44.39\ ^\circ C$$

$\dot{m} = \rho \cdot V \cdot A_c$

$\therefore V = \dfrac{\dot{m}}{\rho \cdot A_c}$

0228|63

5. [20 pts] Saturated liquid water at T_m=20°C flows inside the annular region formed by two concentric circular tubes. The outer tube has ID=25mm, while the inner tube has OD=10mm. The mass flow is 0.02 kg/s. The outer surface is insulated and the inner surface is kept at constant $T_{s,i}$=50°C. Determine the average ~~heat transfer coefficient~~ from the inner surface to the water, h_i, for fully developed velocity and temperature flow. Saturated water at T_m has ν=1.006x10^{-6} m2/s, ρ=1000.5 kg/m^3, k=0.597 W/m·K.



hydraulic diameter

$$D_h \equiv \frac{4(A_c)}{P} = \frac{(4)\cdot\frac{\pi}{4}(D_o^2 - D_i^2)}{\pi(D_o + D_i)} = D_o - D_i = 0.015\text{m}$$

$$Re_{,D} = \frac{V \cdot D_h}{\nu} = \frac{\dot{m} \cdot D_h}{\rho \cdot \left(\frac{\pi}{4}\right)(D_o^2 - D_i^2)\nu} = \frac{4\dot{m}}{\rho \cdot \pi (D_o + D_i)\nu}$$

$$\left(\begin{matrix}\dot{m} = \rho \cdot V \cdot A_c \\ \rightarrow V = \frac{\dot{m}}{\rho \cdot A_c}\end{matrix}\right)$$

$$= \frac{(4)(0.02\text{kg/s})}{(1000.5\text{kg/m}^3)\cdot(\pi)\cdot(0.035\text{m})\cdot(1.006\times10^{-6}\ \text{m}^2/\text{s})}$$

$= 723 \rightarrow$ Laminar flow!

$$\overline{Nu}_i \equiv \frac{\bar{h}_i D_h}{k}, \quad \overline{Nu}_o \equiv \frac{\bar{h}_o D_h}{k}$$

For Laminar, fully developed (velocity & Temperature) flow, we can use Table 8.2

one surface insulated
the other at const T_s

$$\frac{D_i}{D_o} = \frac{0.01}{0.025} = 0.4$$

Interpolation in table 8.2 yields $\overline{Nu}_i \cong 6.39$

$\overline{Nu}_o = 0$ (insulated surface!!)

$$\therefore \bar{h}_i = \overline{Nu}_i \frac{k}{D_h} = (6.39)\cdot\left(\frac{0.597\text{ W/m}\cdot\text{K}}{0.015\text{ m}}\right) = 254\text{ W/m}\cdot\text{K}$$

$\bar{h}_o = 0$

0231169

6. [Optional, 20 pts] In a laboratory demonstration ethylene glycol, at T_m=60°C and mass flow 0.045kg/s, flows through a triangular duct having equal sides s=20mm. The flow is fully developed with respect to velocity and temperature. What is the average heat transfer coefficient if the duct surfaces are maintained at a constant T_s=80°C? At T_m, ethylene glycol has v=4.747x10^{-6} m2/s, ρ=1087.6 kg/m^3, k=0.0.259 W/m·K.

$s = 0.02\,m$

$s^2 - \frac{s^2}{4} = \frac{3}{4}$

$$D_h = \frac{4A_c}{P} = \frac{(4)\cdot(\frac{1}{2})\cdot(s)\cdot(\frac{\sqrt{3}}{2}s)}{3s} = \frac{s}{\sqrt{3}} = 0.01155\,m$$

$$Re_{D_h} = \frac{V\cdot D_h}{\nu} = \frac{\dot{m}\,D_h}{\rho\cdot A_c\cdot\nu} = \frac{\dot{m}\,(\frac{s}{\sqrt{3}})}{\rho\cdot(\frac{\sqrt{3}}{4}s^2)\nu} = \frac{4\dot{m}}{3\cdot\rho\cdot s\cdot\nu}$$

$$= \frac{(4)\cdot(0.045\,kg/s)}{(3)\cdot(1087.6\,kg/m^3)\cdot(0.02\,m)\cdot(4.747\times10^{-6}\,m^2/s)}$$

$$= 581 \rightarrow \text{Laminar flow!}$$

Table 8.1 → Nu for fully developed, laminar flow at constant Ts, in triangular tube.

$$Nu_D = \frac{hD_h}{k} = 2.49$$

$$\therefore h_D = (2.49)\cdot\left(\frac{k}{D_h}\right) = (2.49)\cdot\frac{0.0259\,W/m\cdot K}{0.01155\,m}$$

$$= 5.5836\,W/m^2\cdot K$$

If people used $k = 0.259\,W/m\cdot K$

h_D will be $55.836\,W/m^2\cdot K$