Problem 8.5

It is desired to place a satellite into a circular orbit such that it will always remain directly above the same point on the earth's equator. Determine the velocity and altitude that the satellite must have at the instant of burn-out or orbit insertion to accomplish this.

The radius of the earth is 6.4×10^6 m.

The satellite for this problem must have a orbital period of 24 hours.

24 hours = $8.64.10^4$ sec. $\qquad k^2 = GM = 3.956 \times 10^{14}\ m^3/s^2$

From Kepler's third law of planetary motion,

$$\tau^2 = \left(\frac{4\pi^2}{k^2}\right) a^3 = \left(\frac{4\pi^2}{k^2}\right) r^3 \qquad r = \left(\frac{k^2}{4\pi^2}\right)^{1/3} \tau^{2/3}$$

$$r = \left(\frac{3.956 \times 10^{14}}{4\pi^2}\right)^{1/3} (8.64 \times 10^4)^{2/3} = 4.21 \times 10^7\ \text{m}$$

$$h_G = 4.21 \times 10^7 - 6.4 \times 10^6 = 3.57 \times 10^7\ \text{m} = 35{,}700\ \text{km}$$

$$V = \sqrt{\frac{k^2}{r}} = \sqrt{\frac{3.956 \times 10^{14}}{4.21 \times 10^7}} = 3065\ \text{m/sec}$$

Problem 8.6

Consider a solid iron sphere of 1.6 m in diameter entering the earth's atmosphere at 8 km/hr and at an angle of 30° below the local horizontal. Determine a) the altitude at which maximum deceleration occurs, b) the velocity at which the sphere would impact the earth's surface.

The mass and cross sectional area of the sphere can be calculated.

$$m = \rho v = \rho\left(\frac{4}{3}\pi\, r^3\right) = (6963)\left(\frac{4}{3}\pi\right)(0.8)^3 = 1.4933 \text{ x } 10^4 \text{ kg}$$

$$S = \pi\, r^2 = \pi\,(0.8)^2 = 2.01 \text{ m}^2$$

The ballistic parameter (or ballistic coefficient) and Z are thus,

$$\frac{m}{C_D S} = \frac{1.4933 \text{ x } 10^4}{(1)(2.01)} = 7429 \text{ kg/m}^3$$

$$Z = g_o/RT = 9.81/287(288) = 1.18 \text{ x } 10^{-4} \text{ m}^{-1}$$

The air density and altitude for maximum deceleration can be determined from

$$\rho = \frac{m}{C_D S}\, Z \sin\theta = (7429)(0.000118) \sin 30^\circ = 0.4383\ \frac{\text{kg}}{\text{m}^3}$$

$$h = -\frac{1}{Z}\,\ell n\,(\rho/\rho_o) = -\frac{1}{0.000118}\,\ell n\left(\frac{0.4383}{1.225}\right) = 8710 \text{ m}$$

The maximum deceleration is therefore,

$$\left|\frac{dV}{dt}\right|_{max} = \frac{V_E^2\, Z \sin\theta}{2e} = \frac{(8000)^2(0.000118)(\sin\ 30^\circ)}{2e} = 694.6 \text{ m/sec}^2$$

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e =2.718

$$\left|\frac{dV}{dt}\right|_{max} = \frac{694.6}{9.8} = 70.88 \text{ g's}$$

b) The velocity of the sphere at impact is:

$$\frac{V}{V_E} = e^{-\rho/2[m/(C_D S)]Z \sin\theta}$$

$$\frac{V}{V_E} = e^{-\left[\frac{(1.225)}{2(7429)(0.000118)\sin 30°}\right]} = 0.247$$

$$V = 0.247\ V_E = 0.247\ (8000) = 1978 \text{ m/sec}$$