Mechanical and Aerospace Engineering Department
University of Texas at Arlington
Introduction to Robotics - ME 5337
Homework # 2
Due Date: Third class meeting from day of assignment

Textbook Problems

2.1
2.5
2.12
2.13
2.27 through 2.34

Problem 2.18 (Spong and Vidyasagar)

Consider the diagram of the following Figure. A robot is set up 1 meter from a table, two of whose legs are on the y_0 axis as shown. The tabletop is 1 meter high and 1 meter square. A frame $o_1\ x_1\ y_1\ z_1$ is fixed to the edge of the table as shown. A cube measuring 20 cm on a side is placed in the center of the table with frame $o_2\ x_2\ y_2\ z_2$ established at the center of the cube as shown. A camera is situated directly above the center of the block 2m above the tabletop with frame $o_3\ x_3\ y_3\ z_3$ attached as shown. Find the homogenous transformations relating each of these frames to the base frame $o_0\ x_0\ y_0\ z_0$. Find the homogeneous transformation relating the frame $o_2\ x_2\ y_2\ z_2$ to the camera frame $o_3\ x_3\ y_3\ z_3$.

Problem 2.19

In Problem 2-18, suppose that, after the camera is calibrated, it is rotated 90^0 about the axis z_3 to $o_3\ X_3'\ Y_3'\ Z_3'$. Re-compute the above coordinate transformations.

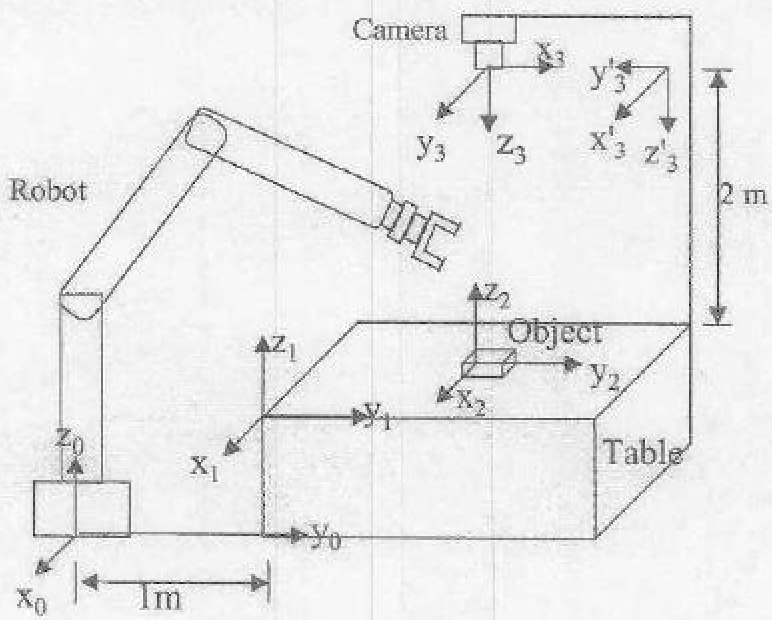

FIGURE 2-11

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2.1. Vector ${}^{A}p$ rotated about $\hat{Z}_A$ by $\hat{\theta}$ degrees and is subsequently rotated about $\hat{X}_A$ by ϕ degrees.

Give the rotation matrix which accomplishes these rotations in the given order.

$$R = Rot(\hat{x}, \phi)\, Rot(\hat{z}, \theta)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta & -s\theta & 0 \\ c\phi s\theta & c\phi c\theta & -s\phi \\ s\phi s\theta & s\phi c\theta & c\phi \end{bmatrix}$$

2.5 ${}^{A}_{B}R$ is a 3x3 matrix w/ eigenvalues $1, e^{\alpha j}, e^{-\alpha j}$. What is the physical meaning of the eigenvector of ${}^{A}_{B}R$ associated w/ eigenvalue 1?

→ If V_i is an eigenvector of R, (associated with eigenvalue λ) then

$$\left.\begin{aligned} RV_i &= \lambda V_i \\ \lambda &= 1 \end{aligned}\right\} \quad RV_i = V_i$$

Therefore, in order for vector V_i to be operated upon by R and not change, i.e. remain the same, then R must be the identity matrix. OR V_i could be the axis of rotation and R be a general matrix satisfying the eigenvalue conditions $1, e^{\pm \alpha j}$

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2.12. Velocity vector ${}^{B}V = [10, 20, 30]^T$

$$ {}^{A}_{B}T = \begin{bmatrix} 0.866 & -0.5 & 0.0 & 11.0 \\ 0.500 & 0.866 & 0.0 & -3.0 \\ 0 & 0 & 1.0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Compute ${}^{A}V$

${}^{A}V = {}^{A}_{B}T * {}^{B}V \Rightarrow$ This is what most of us will apply in order to compute ${}^{A}V$.

However, velocity vectors are what we called free vectors (as pure moment). Then, when we relate velocities in various frames, all we need is the relative ~~po~~ orientation (R) and NOT the relative position between these frames (see textbook JJ Craig → p 56, section 2.9).

$$ \therefore \quad {}^{A}V = {}^{A}_{B}R \, {}^{B}V = \begin{bmatrix} 0.866 & -0.5 & 0.0 \\ 0.500 & 0.866 & 0.0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 10 \\ 20 \\ 30 \end{Bmatrix} $$

$$ {}^{A}V = \begin{Bmatrix} -1.34 \\ 22.32 \\ 30.00 \end{Bmatrix} $$

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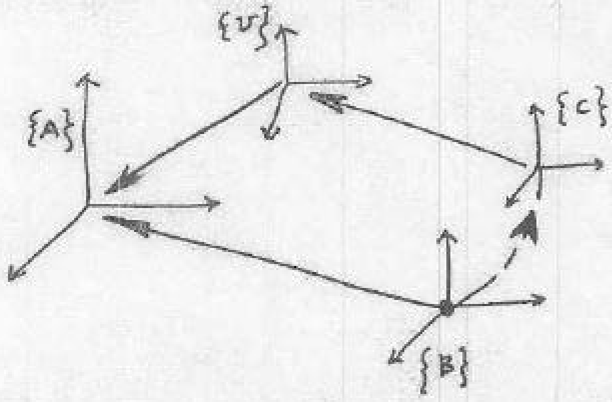
2.13. Given 3 frame definitions; ${}^{U}_{A}T$, ${}^{B}_{A}T$, ${}^{C}_{U}T$

- Draw a frame diagram that qualitatively shows their arrangement
- Solve for ${}^{B}_{C}T$



${}^{B}_{C}T = ?$

$${}^{B}_{C}T \; {}^{C}_{U}T \; {}^{U}_{A}T \; {}^{B}_{A}T^{-1} = I \qquad \Rightarrow {}^{B}_{C}T = {}^{B}_{A}T \; {}^{U}_{A}T^{-1} \; {}^{C}_{U}T^{-1}$$

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% Solution to problem 2.13 from J.J..Craig
% P. S. Shiakolas
% aut : transform of a relative to u
%
aut = [
   0.866, -0.500, 0.000, 11;
   0.500, 0.866, 0.000, -1.0;          U_A T
   0.000, 0.000, 1.000, 8.0;
   0, 0, 0, 1];
abt = [
   1.000, 0.0000, 0.000, 0.0;          B_A T
   0.000, 0.866, -0.500, 10.0;
   0.000, 0.500, 0.866, -20.0;
   0, 0, 0, 1];
uct = [
   0.866, -0.500, 0.000, -3.0;         C_U T
   0.433, 0.750, -0.500, -3.0;
   0.250, 0.433, 0.866, 3.0;
   0, 0, 0, 1];

% find cbt : c relative to b
cbt = abt * inv(aut) * inv(uct)

% matlab result
% » hw1q213
% cbt =
%
%     0.5000    0.7500    0.4330   -6.5754
%    -0.7500    0.6250   -0.2165   19.7877
%    -0.4330   -0.2165    0.8750  -28.3185
%          0         0         0    1.0000
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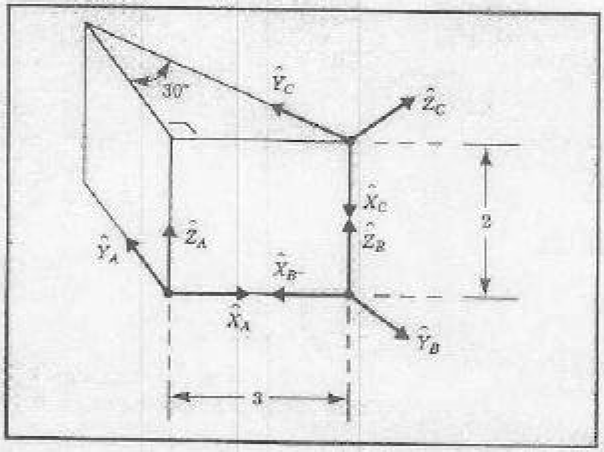
2.27, 2.28, 2.29, 2.30



$$ {}^{A}_{B}T = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{A}_{C}T = \begin{bmatrix} 0 & -0.5 & 0.866 & 3 \\ 0 & 0.866 & 0.5 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{B}_{C}T = \begin{bmatrix} 0 & 0.5 & -0.866 & 0 \\ 0 & -0.866 & -0.5 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{C}_{A}T = \begin{bmatrix} 0 & 0 & -1 & 2 \\ -0.5 & 0.866 & 0 & 1.5 \\ 0.866 & +0.5 & 0 & -2.56 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2.18 SPONG.

Need

^0_1T, ^0_2T, ^0_3T, ^3_2T

Block = 0.2 m each side

$$^0_1T = \begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&1\\0&0&0&1\end{bmatrix} ; \quad {}^0_2T = \left[\begin{array}{ccc|c} & I & & -0.5\\ & & & 1.5\\ & & & 1+0.1\\ 0&0&0&1\end{array}\right] = \left[\begin{array}{ccc|c} & I & & -0.5\\ & & & +1.5\\ & & & 1.1\\ 0&0&0&1\end{array}\right]$$

$$^0_3T = \left[\begin{array}{ccc|c}0&1&0&-0.5\\1&0&0&1.5\\0&0&-1&3\\0&0&0&1\end{array}\right] \qquad {}^3_2T = ? \qquad {}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T$$

$$^0_3T^{-1}\,{}^0_3T = {}^0_3T^{-1}\,{}^0_1T\,{}^1_2T\,{}^2_3T$$

$$\therefore \quad I = {}^0_3T^{-1}\,{}^0_1T\,{}^1_2T\,{}^2_3T$$

$$I\,{}^2_3T^{-1} = {}^0_3T^{-1}\,{}^0_1T\,{}^1_2T\,{}^2_3T\,{}^2_3T^{-1} = {}^0_3T^{-1}\,{}^0_1T\,{}^1_2T$$

$$^2_3T^{-1} = {}^3_2T = {}^0_3T^{-1}\,{}^0_1T\,{}^1_2T$$

Using Matlab for calculations:

or by inspection $^3_2T = \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&-1&1.9\\0&0&0&1\end{bmatrix}$

$$^{3'}_2T = \begin{bmatrix}0&1&0&0\\-1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&-1&1.9\\0&0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&1.9\\0&0&0&1\end{bmatrix}$$

2.19 SPONG

Camera rotated 90° about z_3

$$\Rightarrow {}^3_{3'}T = \begin{bmatrix}0&-1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$^{3'}_2T = {}^{3'}_3T\,{}^3_2T = {}^3_{3'}T^{-1}\,{}^3_2T$$