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MAE 3
MACHINE D

EXAM#2

1. A part has the combined stress state and strengths given below. Choose an appropriate failure theory based on the given data and find the effective stress and factor of safety against static failure.

   Stresses: σ_x=10 ksi, σ_y=5 ksi, τ_{xy}=4.5 ksi
   Strengths: S_y=18 ksi, S_{ut}=20 ksi, S_{uc}=80 ksi

2. A movie scene calls for a stuntman to hang from a rope that is suspended 3 m above a pit of poisonous snakes. The rope is attached to a glass sheet that is 3000 mm long by 100 mm wide and 1.27 mm thick. The stuntman (who had successfully completed Machine Design I class at UTA) knows that the glass sheet contains a central crack with a total length of 16.2 mm that is oriented parallel to the ground. The fracture toughness of the glass is 0.83 MPa-m$^{0.5}$. Should he do the stunt? Support your answer with relevant assumptions and calculations.

$\sigma_x = 10$ ksi

$\sigma_y = 5$ ksi

$\tau_{xy} = 4.5$ ksi

$S_y = 18$ ksi

$S_{ut} = 20$ ksi

$S_{uc} = 80$ ksi

$\}$ since $S_{uc} > S_{ut} \rightarrow$ Brittle Material

Q: effective stress = ?

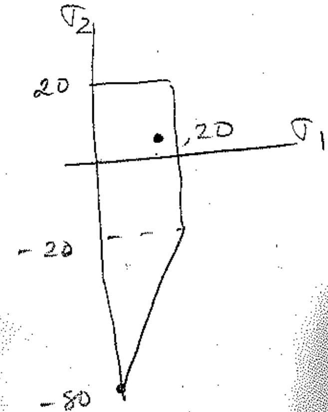
N = ?

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10-5}{2}\right)^2 + (4.5)^2} = 5.148 \text{ ksi}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_2}{2} + \tau_{max} = \frac{10+5}{2} + 5.148 = 12.65 \text{ ksi}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \tau_{max} = \frac{10+5}{2} - 5.148 = 2.352 \text{ ksi}$$

$$\sigma_3 = 0$$



$$C_1 = \frac{1}{2}\left[|\sigma_1 - \sigma_2| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_1 + \sigma_2)\right]$$

$$= \frac{1}{2}\left[|12.65 - 2.352| + \frac{2(20)-80}{-80}(12.65 + 2.352)\right]$$

$$= 8.8995$$

$$C_2 = \frac{1}{2}\left[|\sigma_2 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_2 + \sigma_3)\right]$$

$$= \frac{1}{2}\left[2.352 + \frac{2(20)-80}{-80}(2.352)\right]$$

$$= 1.764$$

$$C_3 = \frac{1}{2}\left[|\sigma_3 - \sigma_1| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_3 + \sigma_1)\right]$$

$$= \frac{1}{2}\left[12.65 + \frac{2(20)-80}{-80}(12.65)\right]$$

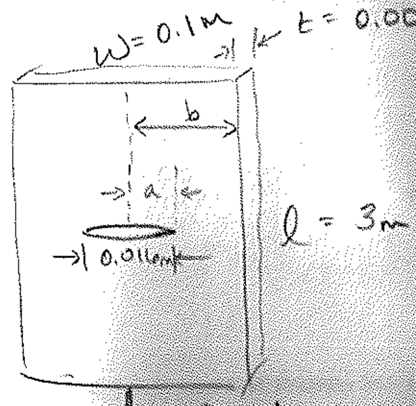
$$= 9.4875$$

$$\therefore \tilde{\sigma} = \mathrm{MAX}\left[C_1, C_2, C_3, \sigma_1, \sigma_2, \sigma_3\right]$$

$$\tilde{\sigma} = \sigma_1$$

$$N = \frac{S_{ut}}{\tilde{\sigma}} = \frac{20}{12.65} = \boxed{1.58}$$

2.)



$P = 900N$

$K_c = 0.83\ MPa\text{-}m^{1/2}$

$a = 0.008m$

$b = 0.05m$

center-crack

$$\beta = \sqrt{\sec\left(\frac{\pi a}{2b}\right)} = \sqrt{\sec\left(\frac{\pi \cdot 0.008}{2(0.05)}\right)} = 1.016$$

$$K = \beta\, \sigma_{nom} \sqrt{\pi a}$$

$$\sigma_{nom} = \frac{P}{A} = \frac{900N}{(0.00127)(0.1)} = 7{,}086.6\ MPa$$

$$K = 1.016\,(7{,}086.6)\sqrt{\pi(0.008)} = 1141.4\ MPa\sqrt{m}$$

Since $K > K_c$, I would not suggest to perform the stunt.

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