

Homework #10  
Due April 15, 2008  
NO LATE HOMEWORK ACCEPTED

Chapter 9  
Problems 4, 6, 12, 14, 20, 23, 27, 38, D6.

9.4 In order to determine whether or not this ceramic material will fail, we must compute its theoretical fracture (or cohesive) strength; if the maximum strength at the tip of the most severe flaw is greater than this value then fracture will occur--if less than, then there will be no fracture. The theoretical fracture strength is just  $E/10$  or 25 GPa ( $3.63 \times 10^6$  psi), inasmuch as  $E = 250$  GPa ( $36.3 \times 10^6$  psi).

The magnitude of the stress at the most severe flaw may be determined using Equation 9.1b as

$$\begin{aligned}\sigma_m &= 2\sigma_0 \sqrt{\frac{a}{\rho_t}} \\ &= (2)(750 \text{ MPa}) \sqrt{\frac{0.20 \text{ mm}}{0.001 \text{ mm}}} = 15 \text{ GPa} \quad (2.2 \times 10^6 \text{ psi})\end{aligned}$$

Therefore, fracture will *not* occur since this value is less than  $E/10$  (i.e., 25 GPa).

9.6 The maximum allowable surface crack length for MgO may be determined using Equation 9.3; taking 225 GPa as the modulus of elasticity (Table 7.1), and solving for  $a$ , leads to

$$\begin{aligned}a &= \frac{2E\gamma_s}{\pi\sigma_c^2} = \frac{(2)(225 \times 10^9 \text{ N/m}^2)(1.0 \text{ N/m})}{(\pi)(13.5 \times 10^6 \text{ N/m}^2)^2} \\ &= 7.9 \times 10^{-4} \text{ m} = 0.79 \text{ mm} \quad (0.031 \text{ in.})\end{aligned}$$

9.12 This problem asks us to determine whether or not the 4340 steel alloy specimen will fracture when exposed to a stress of 1030 MPa, given the values of  $K_{Ic}$ ,  $Y$ , and the largest value of  $a$  in the material. This requires that we solve for  $\sigma_c$  from Equation 9.14. Thus

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{54.8 \text{ MPa}\sqrt{\text{m}}}{(1)\sqrt{(\pi)(0.5 \times 10^{-3} \text{ m})}} = 1380 \text{ MPa} \quad (199,500 \text{ psi})$$

Therefore, fracture will *not* occur because this specimen will tolerate a stress of 1380 MPa (199,500 psi) before fracture, which is greater than the applied stress of 1030 MPa (150,000 psi).

9.14 This problem asks us to determine the stress level at which an aircraft wing component on an aircraft will fracture for a given fracture toughness ( $26 \text{ MPa}\sqrt{\text{m}}$ ) and maximum internal crack length (6.0 mm), given that fracture occurs for the same component using the same alloy at one stress level (112 MPa) and another internal crack length (8.6 mm). It first becomes necessary to solve for the parameter  $Y$  for the conditions under which fracture occurred using Equation 9.11. Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{26 \text{ MPa}\sqrt{\text{m}}}{(112 \text{ MPa})\sqrt{(\pi)\left(\frac{8.6 \times 10^{-3} \text{ m}}{2}\right)}} = 2.0$$

Now we will solve for  $\sigma_c$  using Equation 9.13 as

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{26 \text{ MPa}\sqrt{\text{m}}}{(2.0)\sqrt{(\pi)\left(\frac{6 \times 10^{-3} \text{ m}}{2}\right)}} = 134 \text{ MPa} \quad (19,300 \text{ psi})$$

9.20 (a) There may be significant scatter in the fracture strength for some given ceramic material because the fracture strength depends on the probability of the existence of a flaw that is capable of initiating a crack; this probability varies from specimen to specimen of the same material.

(b) The fracture strength increases with decreasing specimen size because as specimen size decreases, the probability of the existence of a flaw of that is capable of initiating a crack diminishes.

9.23 For thermoplastic polymers, five factors that favor brittle fracture are as follows: (1) a reduction in temperature, (2) an increase in strain rate, (3) the presence of a sharp notch, (4) increased specimen thickness, and (5) modifications of the polymer structure.

9.27 This problem asks that we determine the minimum allowable bar diameter to ensure that fatigue failure will not occur for a 1045 steel that is subjected to cyclic loading for a load amplitude of 66,700 N (15,000 lb<sub>f</sub>). From Figure 9.63, the fatigue limit stress amplitude for this alloy is 310 MPa (45,000 psi). Stress is defined in

Equation 7.1 as  $\sigma = \frac{F}{A_0}$ . For a cylindrical bar

$$A_0 = \pi \left( \frac{d_0}{2} \right)^2$$

Now we may solve for  $d_0$  from these expressions, taking stress as the fatigue limit divided by the factor of safety.

Thus

$$d_0 = 2 \sqrt{\frac{F}{\pi \left( \frac{\sigma}{N} \right)}}$$

$$= (2) \sqrt{\frac{66,700 \text{ N}}{\pi \left( \frac{310 \times 10^6 \text{ N/m}^2}{2} \right)}} = 23.4 \times 10^{-3} \text{ m} = 23.4 \text{ mm} \quad (0.92 \text{ in.})$$

9.38 Four measures that may be taken to increase the fatigue resistance of a metal alloy are:

- (1) Polish the surface to remove stress amplification sites.
- (2) Reduce the number of internal defects (pores, etc.) by means of altering processing and fabrication techniques.
- (3) Modify the design to eliminate notches and sudden contour changes.
- (4) Harden the outer surface of the structure by case hardening (carburizing, nitriding) or shot peening.

9.D6 We are to determine the maximum load that may be applied without failure to a thin bar of rectangular cross-section that is loaded in three-point bending per Figure 9.13c. It first becomes necessary to

determine the value of  $Y$  for the given geometry, which is possible using this figure; however, this determination necessitates the computation of  $a/W$  and  $S/W$  ratios as

$$\frac{a}{W} = \frac{0.5 \text{ mm}}{2.5 \text{ mm}} = 0.20$$

$$\frac{S}{W} = \frac{10 \text{ mm}}{2.5 \text{ mm}} = 4$$

From Figure 9.13c,  $Y = 0.96$  from the  $S/W = 4$  curve and for  $a/W = 0.20$ . Now solving for the applied load  $F$  using the equation also provided in this figure

$$F = \frac{4K_{Ic} W^2 B}{3SY \sqrt{\pi a}}$$

$$= \frac{4(0.60 \text{ MPa}\sqrt{\text{m}})(2.5 \times 10^{-3} \text{ m})^2 (1.5 \times 10^{-3} \text{ m})}{3(10 \times 10^{-3} \text{ m})(0.96)\sqrt{(\pi)(0.5 \times 10^{-3} \text{ m})}}$$

$$= 1.97 \times 10^{-5} \text{ MN} = 19.7 \text{ N} \quad (4.69 \text{ lb}_f)$$