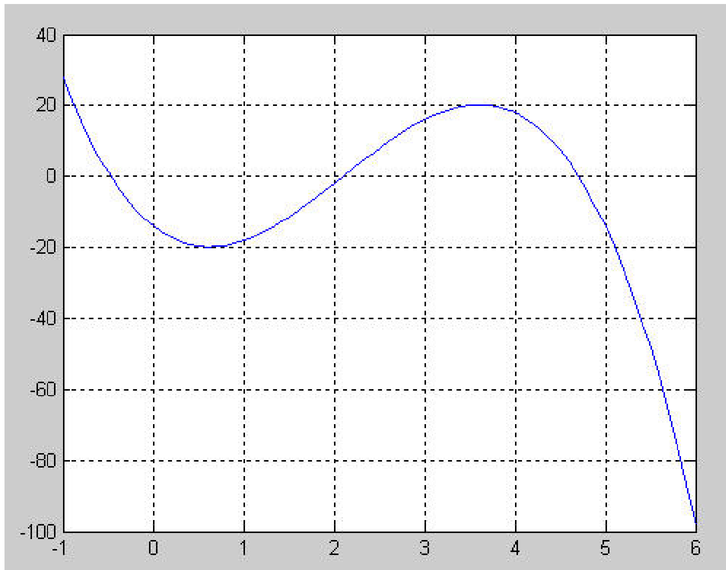
HOMEWORK 4– SOLUTIONS

5.6(a)

M-file:

```
x=[-1:0.1:6];
fx=-14-20*x+19*x.^2-3*x.^3;
plot(x,fx)
grid
```



(b) Bisection method:

1st iteration:
$x_r = (-1 + 0)/2 = -0.5$

f(-1)f(-0.5) = 28(1.125) = 31.5 > 0

Thus the root lies in the second interval and $x_l = -0.5$

$x_r = (-0.5 + 0)/2 = -0.25$

Ea = abs((-0.25-(-0.5))/(-0.25))*100 = 100%
f(-0.5)f(-0.25) = 1.125(-7.7653) = -8.73633 < 0

The root lies in the first interval and x_u =-0.25. The below table has the remaining iterations.

i	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	$f(x_l)\times f(x_r)$	$\|\varepsilon_a\|$
1	-1.00000	0.00000	-0.50000	28.00000	1.12500	31.50000	100.00%
2	-0.50000	0.00000	-0.25000	1.12500	-7.76563	-8.73633	100.00%
3	-0.50000	-0.25000	-0.37500	1.12500	-3.66992	-4.12866	33.33%
4	-0.50000	-0.37500	-0.43750	1.12500	-1.36206	-1.53232	14.29%
5	-0.50000	-0.43750	-0.46875	1.12500	-0.14120	-0.15886	6.67%
6	-0.50000	-0.46875	-0.48438	1.12500	0.48619	0.54697	3.23%
7	-0.48438	-0.46875	-0.47656	0.48619	0.17107	0.08317	1.64%
8	-0.47656	-0.46875	-0.47266	0.17107	0.01458	0.00249	0.83%

The root = -0.47266 and approximate error = 0.83%

(C) False position:

1st iteration:
x_r =0- (-14 (-1 -0))/(28-(-14)) = -0.33333

f(-1)f(-0.33333) = 28(-5.11111) = -143.11111< 0
The root lies in the first interval and x_u =-0.33333.

2nd iteration:

x_r =-0.33333- (-5.11111 (-1 –(-0.33333)))/(28-(-5.11111)) = -0.43624

Ea = abs((-0.43624-(-0.33333))/(-0.43624))*100 = 23.59%

f(-1)f(-0.43624) = 28(-1.41028) = -39.48785 < 0
The root lies in the first interval and x_u =-0.43624. The remaining iterations are

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$f(x_l)\times f(x_r)$	$\|\varepsilon_a\|$
1	-1	28.00000	0.00000	-14.00000	-0.33333	-5.11111	-143.11111	
2	-1	28.00000	-0.33333	-5.11111	-0.43624	-1.41028	-39.48785	23.590%
3	-1	28.00000	-0.43624	-1.41028	-0.46327	-0.35836	-10.03415	5.835%
4	-1	28.00000	-0.46327	-0.35836	-0.47006	-0.08914	-2.49593	1.443%
5	-1	28.00000	-0.47006	-0.08914	-0.47174	-0.02206	-0.61754	0.357%

Root = -0.47174 and approximate error = 0.357%.

6.2
(a) Fixed point iteration:

x_{i+1} = sqrt(1.8*x_i + 2.5)

The iterations are as follows:

i	x_i	ε_a
0	5	
1	3.391165	47.44%
2	2.933274	15.61%
3	2.789246	5.16%
4	2.742379	1.71%
5	2.726955	0.57%
6	2.721859	0.19%
7	2.720174	0.06%
8	2.719616	0.02%

The root estimate is 2.719616 and the approximate error is0.02%
Check:
f(2.719616) = $-(2.719616)^2$ + 1.8(2.719616) +2.5 = -0.001

(b) Newton-Raphson:

$x_{i+1} = x_i - (-x_i^2 + 1.8*x_i + 2.5)/(-2*x_i + 1.8)$

i	x_i	*f(x)*	*f'(x)*	ε_a
0	5	-13.5	-8.2	
1	3.353659	-2.71044	-4.90732	49.09%
2	2.801332	-0.30506	-3.80266	19.72%
3	2.721108	-0.00644	-3.64222	2.95%
4	2.719341	-3.1E-06	-3.63868	0.06%
5	2.719341	-7.4E-13	-3.63868	0.00%

The root estimate = 2.719341 with approximate error = 0.00%

Check:
$f(2.719616) = -(2.719616)^2 + 1.8(2.719616) + 2.5 = -7.36*10^{-13}$

6.26
Newton-Raphson method:

$x_{i+1} = x_i - (e^{-0.5xi}(4 - x_i) - 2) / (-e^{-0.5xi}(3-0.5x_i))$

(a)

i	*x*	*f(x)*	*f'(x)*
0	2	-1.26424	-0.73576
1	0.281718	1.229743	-2.48348
2	0.776887	0.18563	-1.77093
3	0.881708	0.006579	-1.64678
4	0.885703	9.13E-06	-1.64221
5	0.885709	1.77E-11	-1.6422
6	0.885709	0	-1.6422

(b) The derivative at xo = 6 is zero. Hence, this case does not work.

(c)

i	*x*	*f(x)*	*f'(x)*
0	8	-2.07326	0.018316
1	121.1963	-2	2.77E-25
2	7.21E+24	-2	0

The initial guess does not work as the positive slope sends the method away from the root.

6.27
The derivative of the function is
$f'(x) = -12x^5 - 6x^3 + 10$
The Newton-Raphson method is chosen as the function can be easily differentiated and it converges rapidly.

$x_{i+1} = x_i - (-12x^5 - 6x^3 + 10) / (-60x^4 - 18x^2)$

First iteration:

$x_1 = x_0 - (-12(1)^5 - 6(1)^3 + 10) / (-60(1)^4 - 18(1)^2) = 0.897436$

Ea = abs((0.897436 – 1)/0.897436) * 100 = 11.43%

Second iteration:

$x_1 = x_0 - (-12(0.897436)^5 - 6(0.897436)^3 + 10) / (-60(0.897436)^4 - 18(0.897436)^2) = 0.872682$

Ea = abs((0.872682– 1)/ 0.872682) * 100 = 2.84%

Ea< 5%, and the iterations can be stopped.

7.5

The golden ratio can be used to create the interior points,

D = (sqrt(5) – 1)*(2-0)/2 = 1.2361
X1 = 0 + 1.2361 = 1.2361
X2 = 2 – 1.2361 = 0.7639

The function at the interior points will be
f(X2) = f(0.7639) = 8.1879
f(X1) = f(1.2361) = 4.8142

Because *f*(*X2*) > *f*(X1), the maximum is in the interval defined by *X1, X2* and *X,* where *X2* is the optimum. The error at this point can be computed as

Ea = (1-0.61803) *abs((2-0)/0.7639)*100 = 100%

For the second iteration, Xl = 0 and Xu = 1.2361. The former *X2* value becomes the new *X1*, that is, *X1*= 0.7639 and *f*(*X1*) = 8.1879. The new values of *D* and *X2* can be computed as

D = (sqrt(5) – 1)*(1.2361-0)/2 = 0.7639
X2 = 1.2361-0.7639 = 0.4721

The function evaluation at *f*(*X2*) = 5.5496. Since this value is less than the function value at *X1*, the maximum is in the interval prescribed by *X2*, *X1* and *Xu*. The process can be repeated and three iterations are shown below

i	x_l	$f(x_l)$	x_2	$f(x_2)$	x_1	$f(x_1)$	x_u	$f(x_u)$	d	x_{opt}	ε_a
1	0.0000	0.0000	0.7639	8.1879	1.2361	4.8142	2.0000	-104.0000	1.2361	0.7639	100.00%
2	0.0000	0.0000	0.4721	5.5496	0.7639	8.1879	1.2361	4.8142	0.7639	0.7639	61.80%
3	0.4721	5.5496	0.7639	8.1879	0.9443	8.6778	1.2361	4.8142	0.4721	0.9443	30.90%

7.6

The function values at the initial values are calculated as follows:
$f(x_1) = f(0) = 0$
$f(x_2) = f(1) = 8.5$
$f(x_3) = f(2) = -104$
and using Eq. (7.10) ,

$x_4 = 1 - (1/2) * [(1-0)^2*(8.5+104) - (1-2)^2*(8.5-0)]/ [(1-0)*(8.5+104) - (1-2)*(8.5-0)]$
$= 0.570248$

which has a function value of $f(0.570248) = 6.5799$. Because the function value for the new point is lower than for the intermediate point (x_2) and the new x value is to the left of the intermediate point, the lower guess (x_1) is discarded. Therefore, for the following iteration,
$f(x_1) = f(0.570248) = 6.6799$
$f(x_2) = f(1) = 8.5$
$f(x_3) = f(2) = -104$
which can be substituted into Eq. (7.10) to give $x_4 = 0.812431$, which has a function value of $f(0.812431) = 8.446523$. The approximate error is

Ea = abs((0.81243 – 0.570248)/0.81243)*100 = 29.81%

The process is repeated and the results are as follows:

i	x_1	$f(x_1)$	x_2	$f(x_2)$	x_3	$f(x_3)$	x_4	$f(x_4)$	ε_a
1	0.00000	0.00000	1.00000	8.50000	2.0000	-104	0.57025	6.57991	
2	0.57025	6.57991	1.00000	8.50000	2.0000	-104	0.81243	8.44652	29.81%
3	0.81243	8.44652	1.00000	8.50000	2.0000	-104	0.90772	8.69575	10.50%

The result converges on the true value of f(x) = 8.69793 at x = 0.91692.