

HOMEWORK 3- SOLUTIONS

4.3

M-File:

```
e = 1;
while (1)

    if (e+1<=1)
        break;
    end
    e = e/2;
end
ep = 2*e;
```

Results:

```
>> ep
ans =
2.2204e-016
```

```
>> eps
ans =
2.2204e-016
```

4.9

(a) For this case, $x=0$ and $h=x$. Thus, the Taylor series will be of the form

$$f(x) = f(0) + f'(0)*x + (f''(0) * x^2) / 2! + (f^{(3)}(0) * x^3) / 3! + \dots \text{-----(1)}$$

$$f(x) = e^x$$

Thus, for $x=0$

$$f(0) = f'(0) = f''(0) = f^{(3)}(0) = 1$$

Substituting in (1),

$$f(x) = 1 + x + x^2/2! + x^3/3! + \dots$$

and this is the Maclaurin series.

(b) The true value is $e^{-1} = 0.367879$ and the step size is $h = x_{i+1} - x_i = 1 - 0.2 = 0.8$.

The Taylor series to the third-order term is

$$f(x_{i+1}) = e^{-x_i} - e^{-x_i} * h + e^{-x_i} * h^2/2! - e^{-x_i} * h^3/3!$$

Zero-order approximation:

$$f(1) = e^{-0.2} = 0.818731$$

$$Et = \text{abs}((0.367879 - 0.818731)/0.367879)*100 = 122.55\%$$

First-order approximation:

$$f(1) = 0.818731 - 0.818731(0.8) = 0.163746$$

$$Et = \text{abs}((0.367879 - 0.163746)/0.367879)*100 = 55.49\%$$

Second-order approximation:

$$f(1) = 0.818731 - 0.818731(0.8) + 0.818731*0.8^2/2 = 0.42574$$

$$Et = \text{abs}((0.367879 - 0.42574)/0.367879)*100 = 15.73\%$$

Second-order approximation:

$$f(1) = 0.818731 - 0.818731(0.8) + 0.818731*0.8^2/2 = 0.42574$$

$$Et = \text{abs}((0.367879 - 0.42574)/0.367879)*100 = 15.73\%$$

Third-order approximation:

$$f(1) = 0.818731 - 0.818731(0.8) + 0.818731*0.8^2/2 - 0.818731*0.8^3/6 = 0.355875$$

$$Et = \text{abs}((0.367879 - 0.355875)/0.367879)*100 = 3.26\%$$

4.10

Let $E_s = 0.5 * 10^{2-2} = 0.5\%$
True value = $\cos(\pi/3) = 0.5$

zero-order:

$\cos(\pi/3) \approx 1$

$E_t = \text{abs}((0.5 - 1)/0.5)*100 = 100\%$

first-order:

$\cos(\pi/3) \approx 1 - (\pi/3)^2/2 = 0.451689$

$E_t = \text{abs}((0.5 - 0.451689)/0.5)*100 = 9.6623\%$

$E_a = \text{abs}((0.451689-1)/ 0.451689)*100 = 121.39\%$

second-order:

$\cos(\pi/3) \approx 0.451689 - (\pi/3)^4/24 = 0.501796$

$E_t = \text{abs}((0.5 - 0.501796)/0.5)*100 = 0.359\%$

$E_a = \text{abs}((0.501796-0.451689)/ 0.501796)*100 = 9.986\%$

third-order:

$\cos(\pi/3) \approx 0.501796 - (\pi/3)^6/720 = 0.499965$

$E_t = \text{abs}((0.5 - 0.499965)/0.5)*100 = 0.007\%$

$E_a = \text{abs}((0.499965-0.501796)/ 0.499965)*100 = 0.366\%$

Since $E_a < 0.5\%$, the computation can be stopped.