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THE UNIVERSITY OF TEXAS AT ARLINGTON

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For Stress Relaxation

from Maxwell model:

$$\frac{d\epsilon}{dt} = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt}$$

boundary conditions for stress relaxation:

$$\epsilon = \epsilon_o, \quad \frac{d\epsilon}{dt} = 0$$

σ_o corresponds to initial stress when ϵ applied @ $t=0$

thus,

$$0 = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt}$$

rearranging,

$$\frac{d\sigma}{\sigma} = d\ln\sigma = -\frac{E}{\eta}dt$$

integrating from σ_0 @ $t=0$ to $\sigma(t)$ at time t:

$$\ln \sigma(t) = \ln \sigma_0 - \frac{Et}{\eta}$$

exponentiation yields:

$$\sigma(t) = \sigma_0 e^{-Et/\eta}$$

or letting $\tau = \frac{\eta}{E}$ and substituting:

$$\sigma(t) = \sigma_0 e^{-t/\tau}$$

where τ is defined as the relaxation time

τ - time required for $\frac{\sigma(t)}{\sigma_0}$ to decay to $\frac{1}{e}$ or 0.37

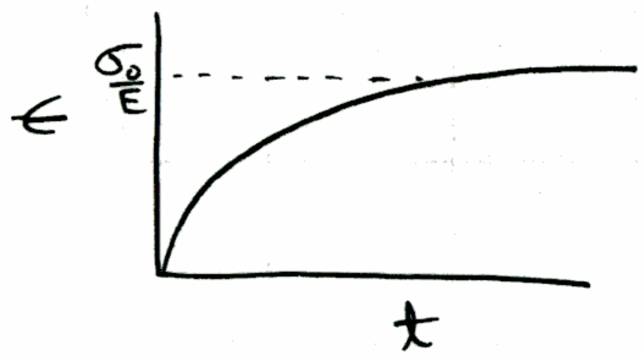
For Creep :

recalling Voigt equation:

$$\sigma_T(t) = E\epsilon + \eta \frac{d\epsilon}{dt}$$

for a creep test, $\sigma_T(t) = \sigma_0$, therefore:

$$\epsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-t/\tau}\right)$$



recalling the Maxwell equation:

$$\frac{d\epsilon}{dt} = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt}$$

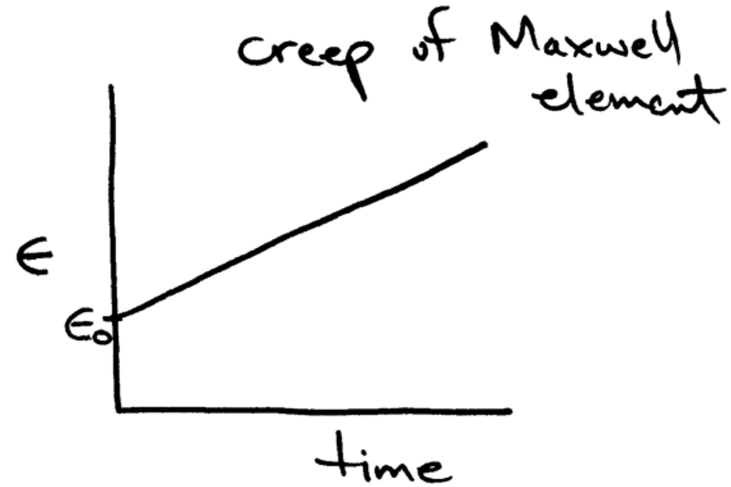
or:

$$\frac{d\sigma}{dt} = E\,\frac{d\epsilon}{dt} - \frac{E\sigma}{\eta}$$

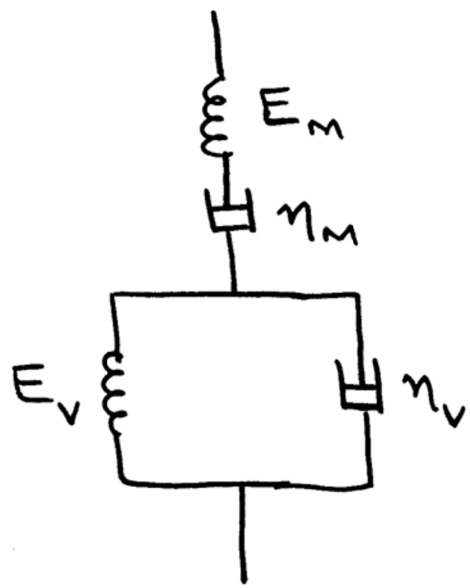
for creep $d\sigma/dt = 0$; therefore:

$$\epsilon = \epsilon_0 + \frac{\sigma t}{\eta}$$

$$\epsilon = \epsilon_0 \left(1 + \frac{t}{\tau}\right)$$



SERIES COMBINATIONS OF MAXWELL AND VOIGT MODELS



The total strain for creep conditions will be due to:

- an instantaneous elastic deformation (Maxwell spring element
- an irrecoverable viscous flow (Maxwell element dashpot)
- a recoverable retarded elastic deformation (Voigt element)

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Molecular mechanisms associated:

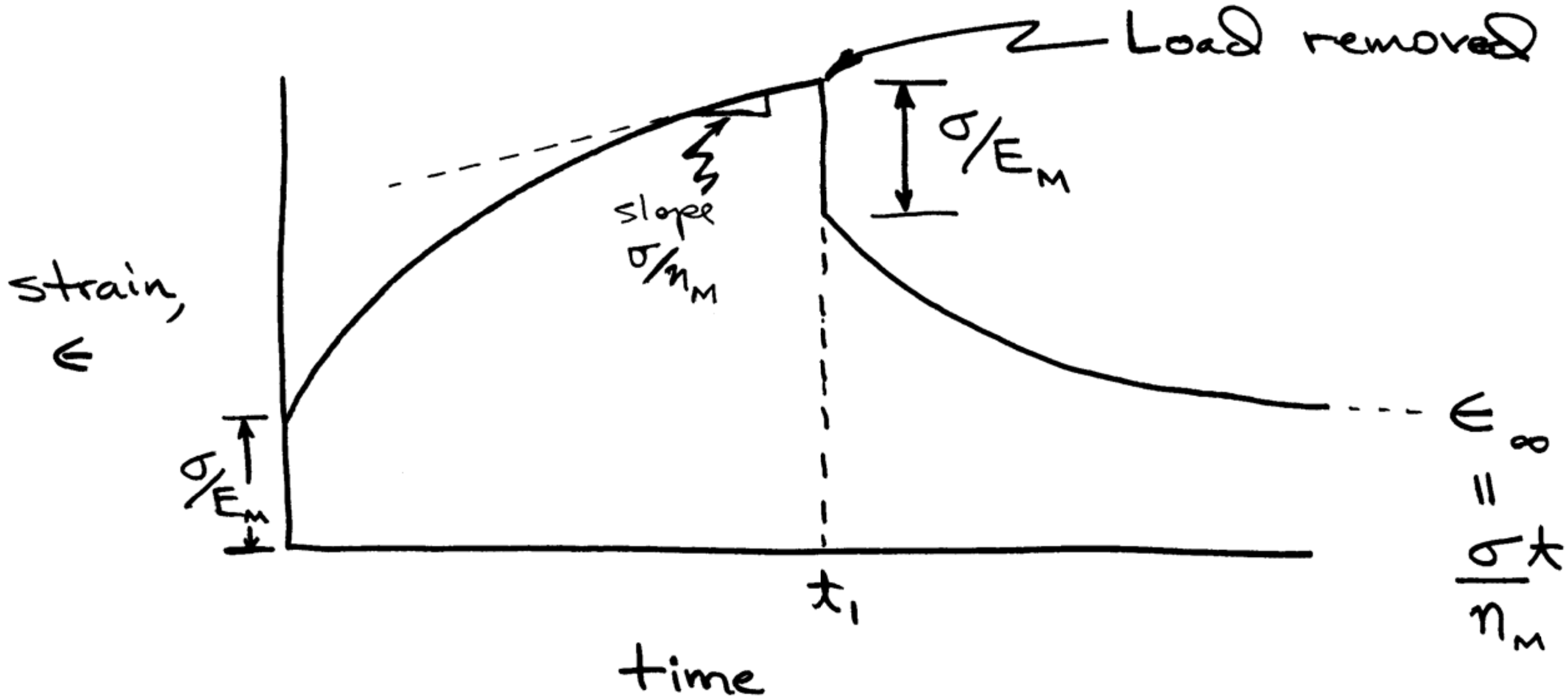
1. Instantaneous elastic deformation – bending and stretching of primary valence bonds.

2. Irrecoverable viscous flow – slippage of the polymer chain or chain segments past one another.

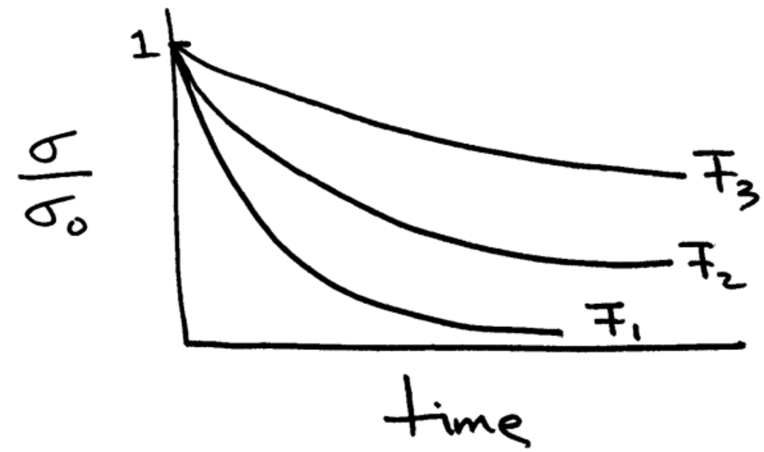
3. Retarded elastic deformation – the transformation of a given equilibrium conformation into a biased conformation in which elongated and oriented structures are favored.

for constant load:

$$\epsilon = \frac{\sigma}{E_M} + \frac{\sigma t}{\eta_M} + \frac{\sigma}{E_V}\left(1 - e^{-t/\tau_V}\right)$$



Material Response Time



for Maxwell model

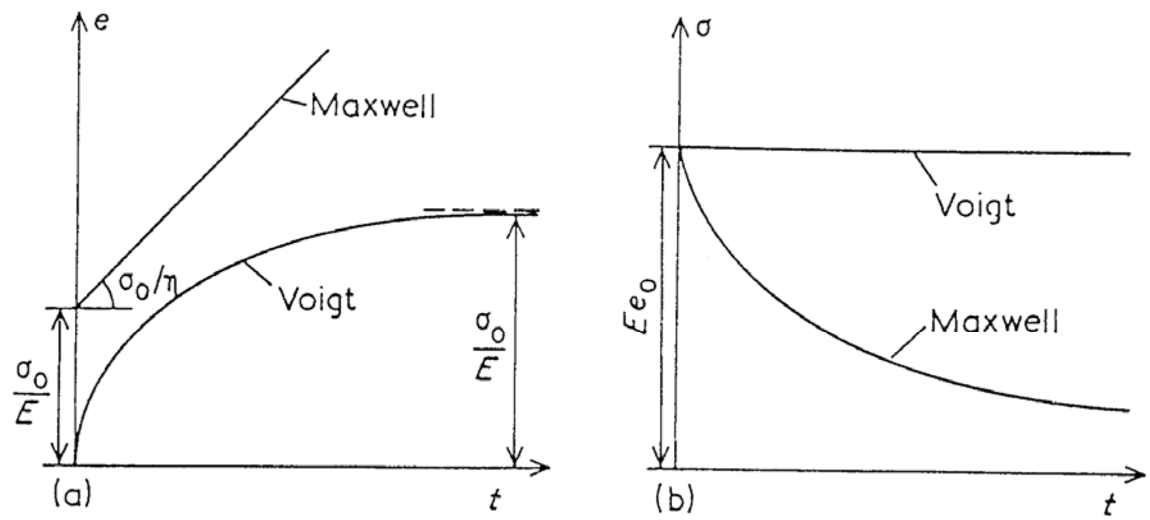
$\mathcal{T}_1 < \mathcal{T}_2 < \mathcal{T}_3$

as $\mathcal{T} \to 0$; rapid decay ; liquidlike behavior ; completely viscous

as $\mathcal{T} \to \infty$; maintenance of high stress for long time ; solidlike behavior ; completely elastic

$\mathcal{T} = f(\eta)$ $\eta = f(T)$ $\therefore$ $\mathcal{T}$ low @ high $T \to$ fluidlike

$\mathcal{T}$ high @ low $T \to$ solidlike



ig. 5.8 *The behaviour of the Maxwell and Voigt models during different types of loading.* 'reep *(constant stress σ_0)*, (b) *Relaxation (constant strain e_0) (after Williams).*