

of mean stress in controlling FCP response is similar to the smaller role played by mean stress in cyclic life tests as portrayed with Gerber and Goodman diagrams (Chapter 12).

Detailed procedures for conducting a fatigue crack propagation test have been established and are to be found in ASTM Standard E647-93.⁹ The interested reader is referred to this document and to a collection of papers dealing with various laboratory experiences pertaining to this standard.¹⁰ Furthermore, the reader *must* recognize that data generated in the low crack growth regime, according to conventional test methods described by this standard, may lead to nonconservative estimates of component life. This topic is considered at greater length in Section 13.4.

13.1.1 Fatigue Life Calculations

When conducting a failure analysis, it is often desirable to compute component life for comparison with the actually recorded service life. Alternatively, if one were designing a new part and wished to establish for it a safe operating service life, fatigue life calculations would be required. Such a computation can be performed by integrating Eq. 13-3 (where $\Delta K = Y\Delta\sigma\sqrt{a}$) with the starting and final flaw size as limits of the integration. When the geometrical correction factor Y does not change within the limits of integration (e.g., for the case of a circular flaw where $K = (2/\pi)\sigma\sqrt{\pi a}$ (Fig. 8.7*i*)), the cyclic life is given by

$$N_f = \frac{2}{(m-2)AY^m\Delta\sigma^m} \left[\frac{1}{a_0^{(m-2)/2}} - \frac{1}{a_f^{(m-2)/2}} \right] \quad \text{for } m \neq 2 \quad (13-6)$$

where N_f = number of cycles of failure
 a_0 = initial crack size
 a_f = final crack size at failure
 $\Delta\sigma$ = stress range
 A, m = material constants
 Y = geometrical correction factor

Usually, however, this integration cannot be performed directly, since Y varies with the crack length. Consequently, cyclic life may be estimated by numerical integration procedures by using different values of Y held constant over a number of small crack length increments. It is seen from Eq. 13-6 that when $a_0 \ll a_f$ (the usual circumstance) the computed fatigue life is not sensitive to the final crack length a_f but, instead, is strongly dependent on estimations of the starting crack size a_0 .

EXAMPLE 13.1

Compare the differences in fatigue lifetimes for three components that experienced crack extension from 2 to 10 mm, versus where the initial crack length was four times smaller ($a_0 = 0.5$ mm), or where the final crack length was four times larger ($a_f = 40$ mm). Assume that crack growth rates follow a Paris relation where $m = 4$.

We have three scenarios:

Case A: $a_0 = 2$ mm, $a_f = 10$ mm

Case B: $a_0 = 0.5$ mm, $a_f = 10$ mm

Case C: $a_0 = 2$ mm, $a_f = 40$ mm

We are only interested in examining the relative influence of crack length on fatigue lifetimes. Hence, from Eq. 13-6 where $m = 4$,

$$N_f \propto \left[\frac{1}{a_0} - \frac{1}{a_f} \right]$$

Therefore, for Case A (2–10 mm):

$$\left[\frac{1}{0.002} - \frac{1}{0.010} \right] = 500 - 100 = 400$$

For Case B (0.5–10 mm):

$$\left[\frac{1}{0.0005} - \frac{1}{0.010} \right] = 2000 - 100 = 1900$$

For Case C (2–40 mm):

$$\left[\frac{1}{0.002} - \frac{1}{0.040} \right] = 500 - 25 = 475$$

For the case where the initial crack length is smaller, fatigue lifetime increases by 375%.

$$\frac{1900 - 400}{400} \times 100 = 375\%$$

By comparison, when the final crack length is increased by the same relative proportion, fatigue lifetime increases by only 18.75%.

$$\frac{475 - 400}{400} \times 100 = 18.75\%$$

Clearly, fatigue lifetime is far more sensitive to initial rather than final crack size. In a practical sense, one finds that the major portion of fatigue lifetime in a service component is consumed prior to the point where the crack is discovered. Several additional examples¹¹ demonstrating this integration procedure are now considered.

In the first example, an extruded aluminum alloy is assumed to contain a semi-