

Express the following transfer function in matrix state variable format, i.e.

$$\dot{X} = AX + Bu$$

$$y = CX + Du$$

$$Z(s) = \left[\frac{10}{s^3 + 20s^2 + 30s + 40} \right] U(s)$$

Note, the output of interest is \ddot{z} .

$$\ddot{z} + 20\dot{z} + 30z + 40z = 10u \quad z(0) = 0 \quad \dot{z}(0) = 0$$

$$\begin{aligned}
 x_1 &= z & \dot{x}_1 &= x_2 \\
 x_2 &= \dot{z} & \dot{x}_2 &= x_3 \\
 x_3 &= \ddot{z} & \dot{x}_3 &= 10u - 20\dot{z} - 30z - 40z \\
 & & &= 10u - 20x_3 - 30x_2 - 40x_1
 \end{aligned}$$

$$\dot{X} = A X + B u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -40 & -30 & -20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u \quad \checkmark$$

$$y = Cx + Du$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u \quad \checkmark$$