CE 2313 / MAE 2312
Mechanics of Materials

Examination III

April 26, 2007

Name: ____

- There are 4 numbered problems on this exam. The relative weight of each problem is provided in parentheses immediately after the problem number.
- Time allowed for the exam is 80 minutes.
- Read the problems carefully and don't waste time doing work that is not requested.
- Problem statements may include information that you do not really need.
- Provided values have at least three significant digits even when not expressed that way.
- **Draw Free Body Diagrams (or other diagrams) when appropriate!!!!**
- Show each problem solution in detail. Test credit will be based primarily on your solution method rather than your numerical solution.
- If you are filling each page, you are probably doing too much. If you do need additional space for a solution ask for a blank sheet of paper. On that sheet, write the 3 digit test ID in the lower right corner and clearly identify the part of the test you are working on.

Formulas

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) = -w(x)$$

$$\frac{d}{dx}\left(EI\frac{d^2v}{dx^2}\right) = V(x)$$

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

$$\sigma = -\frac{M_z\, y}{I_z}$$

$$\tau = \frac{V Q}{I t}$$

$$Q = \bar{y}' A'$$

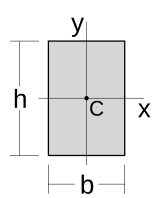
$$S = \frac{I}{c}$$

$$S_{\text{req'd}} = \frac{M}{\sigma_{\text{allow}}}$$

$$\tau_{\text{allow}} \geq \frac{V_{\max} Q}{I t}$$

$$\tau_{\text{allow}} \geq 1.5\frac{V_{\max}}{A}$$

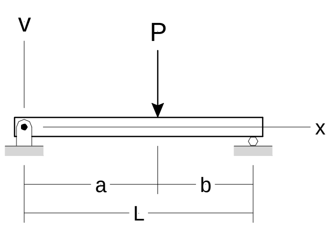
$$\tau_{\text{allow}} \geq \frac{V_{\max}}{A_{\text{web}}}$$



$$A = b h$$

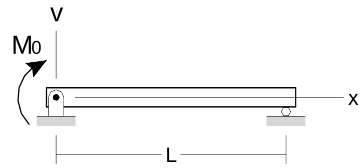
$$I_x = \frac{b h^3}{12}$$

$$I_y = \frac{h b^3}{12}$$

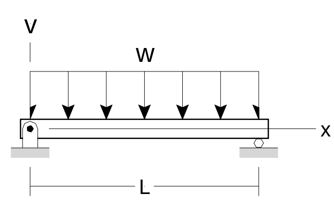


$0 \le x \le a$

$$v = \frac{-P b x}{6 EI L}\left(L^2 - b^2 - x^2\right)$$



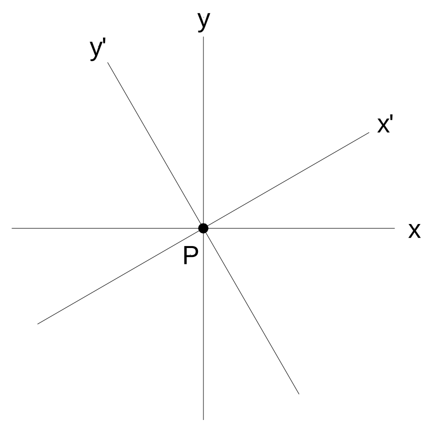
$$v = \frac{-M_0 x}{6 EI L}\left(x^2 - 3 L x + 2 L^2\right)$$



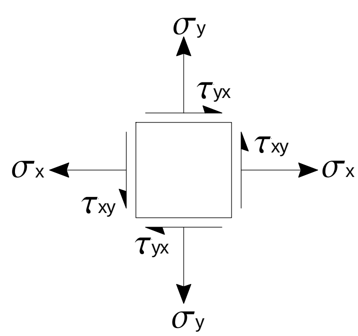
$$v = \frac{-w x}{24 EI}\left(x^3 - 2 L x^2 + L^3\right)$$

056

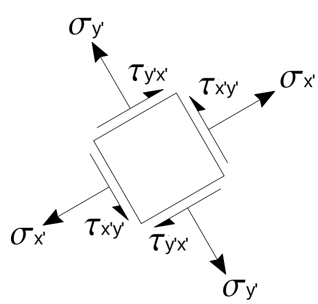
1) (20) A point P is in a state of plane stress. The diagram at the right shows the relationship between P and two different coordinate systems, x-y and x'-y'.



a) Consider a stress element at point P oriented as shown below. Assume that all stress components are positive (which implies non-zero). **Sketch the stresses on the element, using the orientation that corresponds to a positive stress. Give each stress a correct, symbolic label (the numeric values are not known).**

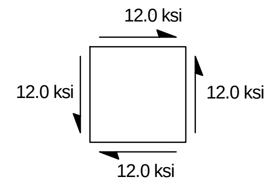


b) Consider a stress element at point P oriented as shown below. Assume that all stress components are positive (which implies non-zero). **Sketch the stresses on the element, using the orientation that corresponds to a positive stress. Give each stress a correct, symbolic label (the numeric values are not known).**



056

2) (20) The diagram shows the state of plane stress of a point.



a) Determine the principal stresses at the point and corresponding orientation of the stress element that shows the principal stresses.

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x + \sigma_y}{2}}$$

$$\tan 2\theta_p = \frac{12}{\frac{0+0}{2}}$$

$$\tan 2\theta_p = \frac{12}{0}$$

$$2\theta_p = 90^\circ$$

$$\boxed{\theta_p = 45^\circ}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{0+0}{2} + \sqrt{\left(\frac{0-0}{2}\right)^2 + 12^2}$$

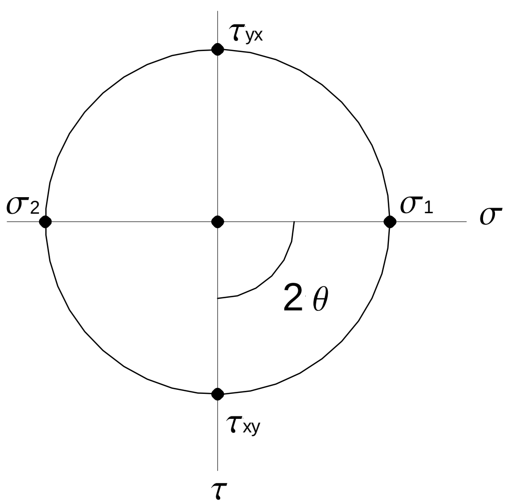
$$\boxed{\sigma_1 = 12.0 \text{ ksi}}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{0+0}{2} - \sqrt{\left(\frac{0-0}{2}\right)^2 + 12^2}$$

$$\boxed{\sigma_2 = -12.0 \text{ ksi}}$$

OR



$$2\theta_p = 90^\circ$$

$$\boxed{\theta_p = 45^\circ}$$

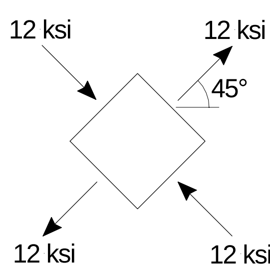
$$\sigma_1 = \tau_{xy}$$

$$\boxed{\sigma_1 = 12.0 \text{ ksi}}$$

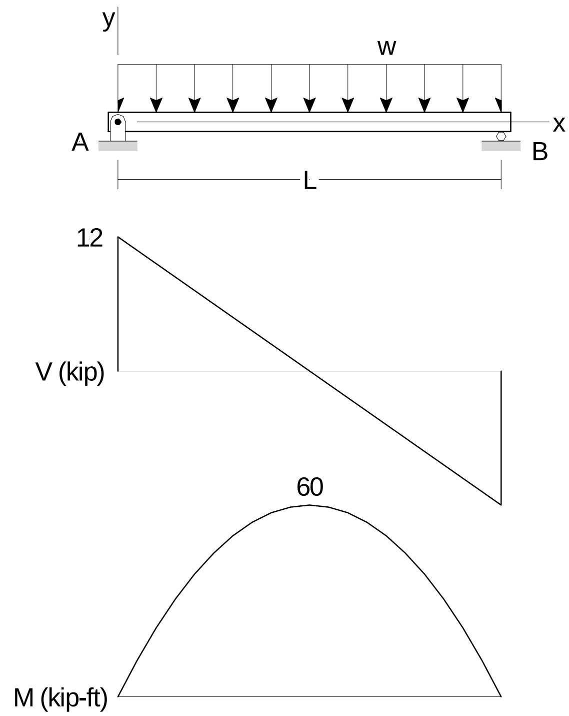
$$\sigma_2 = -\tau_{xy}$$

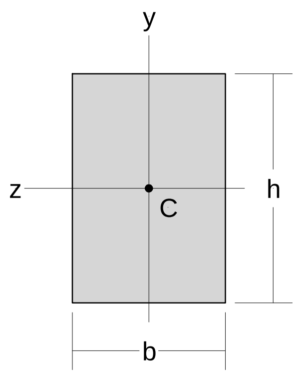
$$\boxed{\sigma_2 = -12.0 \text{ ksi}}$$

b) Sketch the stress element that shows the principal stresses. Show all the stresses acting on the stress element in this orientation. Show the stresses using their actual direction and label them with their magnitude.



3) (20) A beam is to be designed to support the load as shown. The shear and moment diagrams for the beam are provided. The allowable stresses in the beam material are $\sigma_{\text{allow}} = 1.4$ ksi and $\tau_{\text{allow}} = 0.5$ ksi. The beam will have a rectangular cross section as shown in the diagram and it is required that $h = 3b$.





Determine the required height, h, of the cross section of the beam.

$M = 60 \cdot 12$

$M = 720$ kip-in

$h = 3b$

$b = \dfrac{h}{3}$

$I = \dfrac{bh^3}{12}$

$I = \dfrac{h}{3}\dfrac{h^3}{12}$

$I = \dfrac{h^4}{36}$

$c = \dfrac{h}{2}$

$S = \dfrac{I}{c}$

$S = \dfrac{\frac{h^4}{36}}{\frac{h}{2}}$

$S = \dfrac{h^3}{18}$

Design for bending stress

$S_{\text{req'd}} = \dfrac{M}{\sigma_{\text{allow}}}$

$\dfrac{h^3}{18} = \dfrac{720}{1.4}$

$h = 20.997$ in

Check shear stress

$A = bh$

$A = \dfrac{h}{3}h$

$A = \dfrac{h^2}{3}$

$A = \dfrac{h^2}{3}$

$A = \dfrac{20.997^2}{3}$

$A = 146.96 \text{ in}^2$

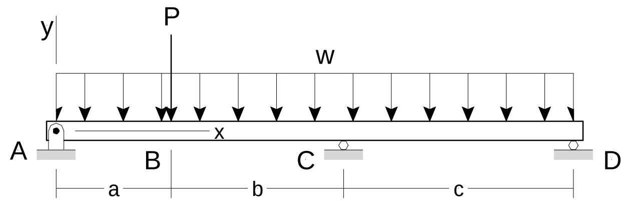
$\tau_{\text{allow}} \geq 1.5\dfrac{V_{\max}}{A}$

$0.5 \text{ ksi} \geq 1.5\dfrac{12}{146.96}$

$0.5 \text{ ksi} \geq 0.1225 \text{ ksi}$ OK

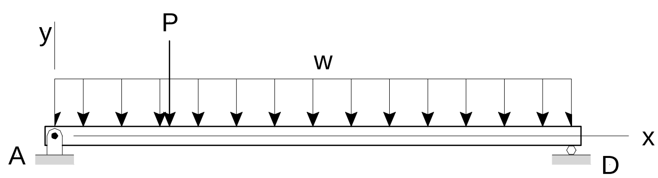
$\boxed{h = 21.0 \text{ in}}$

4) (40) The continuous, prismatic beam has moment of inertia $I = 300$ in^4 and modulus of elasticity $E = 29000$ ksi . It is supported by a pin at A and rollers at C and D. The downward, uniformly distributed load has magnitude $w = 1.8\ \dfrac{\text{kip}}{\text{ft}}$ and the downward, concentrated load at B has magnitude $P = 30$ kip . The required distances are $a = 10$ ft , $b = 15$ ft , and $c = 20$ ft .



Determine the vertical reactions at A, C, and D using the Method of Superposition. Follow the order of steps and utilize the corresponding space that is provided below.

a) Complete the diagram to show the primary structure you have chosen.



C_y is the redundant reaction.

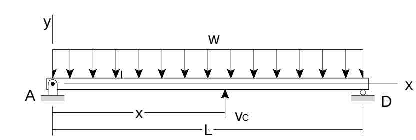
b) Determine the deflection(s) of the primary structure at the position(s) of the redundant force(s) due to the applied loads.

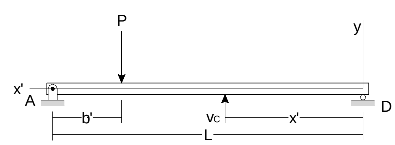
$$v_C = \frac{-w\,x}{24\,EI}\left(x^3 - 2\,L\,x^2 + L^3\right) + \frac{-P\,b'\,x'}{6\,EI\,L}\left(L^2 - b'^2 - x'^2\right)$$

$$v_C = \frac{-1.8\cdot 25}{24\,EI}\left(25^3 - 2\cdot 45\cdot 25^2 + 45^3\right) + \frac{-30\cdot 10\cdot 20}{6\,EI\,45}\left(45^2 - 10^2 - 20^2\right)$$

$$v_C = -\frac{94687.5}{EI} - \frac{33888.9}{EI}$$

$$v_C = -\frac{128576.4}{EI}$$



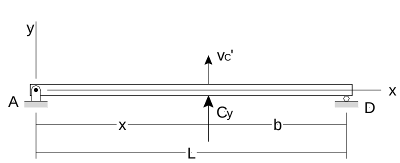


c) Determine the deflection of the primary structure at the position of each redundant force due to each redundant force. (If the redundants are A_y and B_y , calculate the deflection at A due to A_y , the deflection at A due to B_y , the deflection at B due to A_y , and the deflection at B due to B_y . This hint is not meant to suggest that there are two redundants nor that A_y or B_y is a suitable choice for a redundant.)

$$v_C\,' = \frac{-P\,b\,x}{6\,EI\,L}\left(L^2 - b^2 - x^2\right)$$

$$v_C\,' = \frac{C_y\,20\cdot 25}{6\,EI\,45}\left(45^2 - 20^2 - 25^2\right)$$

$$v_C\,' = \frac{1851.85}{EI}C_y$$



d) Using the results of the first three steps, give the superposition equation(s) and determine the values of the redundant reactions.

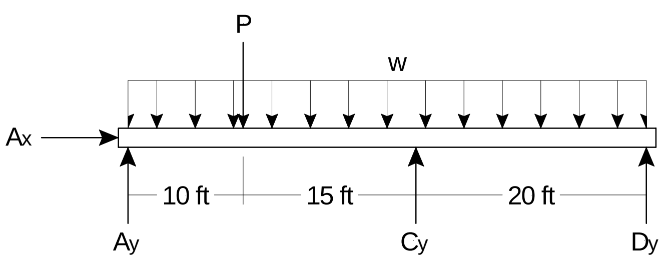
$$v_C + v_C' = 0$$

$$-\frac{128576.4}{EI} + \frac{1851.85}{EI} C_y = 0$$

$$C_y = 69.431$$

$$\boxed{C_y = 69.4 \text{ kip}}$$

e) Determine the remaining reactions.



$$\sum F_x = 0$$

$$\boxed{A_x = 0}$$

$$\sum M_A = 0$$

$$\frac{L}{2}\left[L(-w)\right] + x_B(-P) + x_C C_y + x_D D_y = 0$$

$$\frac{45}{2}\left[45(-1.8)\right] + 10(-30) + 25 \cdot 69.431 + 45 D_y = 0$$

$$D_y = 8.5939$$

$$\boxed{D_y = 8.59 \text{ kip}}$$

$$\sum F_y = 0$$

$$A_y + L(-w) - P + C_y + D_y = 0$$

$$A_y + 45(-1.8) - 30 + 69.431 + 8.5939 = 0$$

$$A_y = 32.975$$

$$\boxed{A_y = 33.0 \text{ kip}}$$