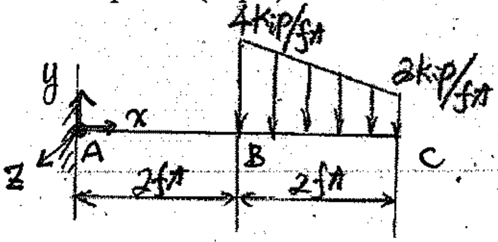
MAE2312 Solid Mechanics/CE 2313 Strength of Materials 11/06/06

Student Name Answer Key

**Exam #2**

Policy: Close notes; close book; no cell phone use during the exam; one formula sheet is allowed; calculator is allowed; work on distributed exam sheets only; addition paper can be requested.

**Problem 1:** Draw the shear and bending diagram for the beam. You must derive the shear and bending functions for each section of the beam using free-body diagrams to get full points (20pts)



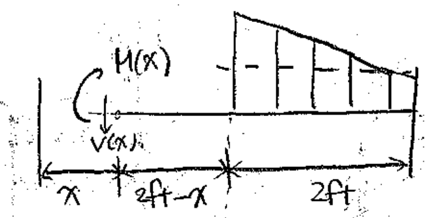

Between A&B $0 < x \le 2$

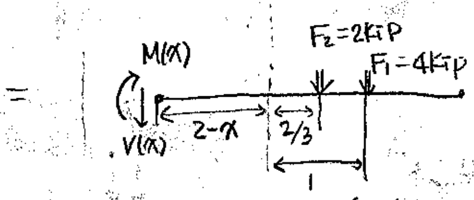
$\Sigma F_y = V(x) + F_2 + F_1 = 0$

$\Rightarrow V(x) = -(F_2 + F_1) = -6\text{kip}$

$\Sigma M = 0 = M(x) + 2(2 - x + \frac{2}{3}) + 4(2 - x + 1) = 0$

$\Rightarrow M(x) = 6x + 17\cdot\frac{1}{3}$




Between B& C $2 < x \le 4$

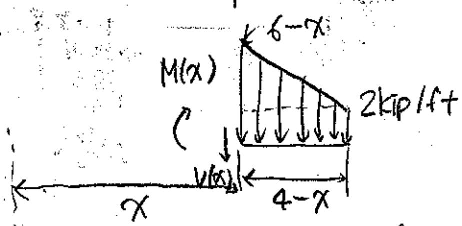
$\Sigma F_y = V(x) + F_2 + F_1 = 0$

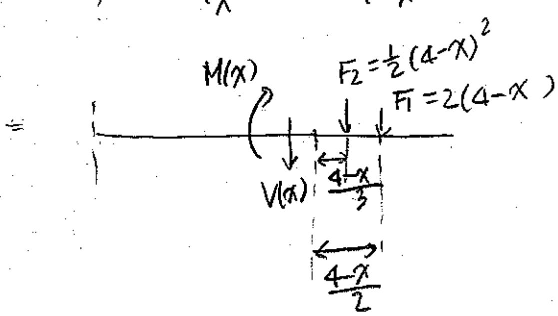
$V(x) = -F_1 - F_2 = -(\frac{1}{2}x^2 - 6x + 16)$

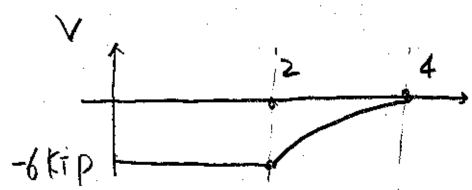
$\Sigma M = M(x) + F_2\cdot\frac{4-x}{3} + F_1\frac{4-x}{2} = 0$

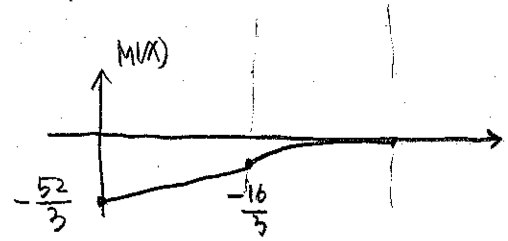
$M(x) = -\frac{1}{2}\cdot\frac{(4-x)^3}{3} - 2(4-x)\cdot\frac{(4-x)}{2}$

$M(x) = -\frac{(4-x)^3}{6} - (4-x)^2$



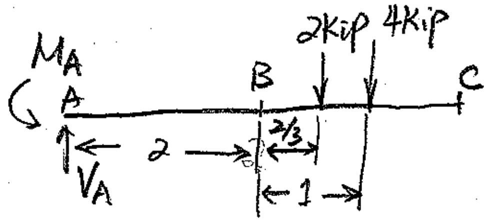



Method 2:

Step 1. calculate reaction force & moment @ A.

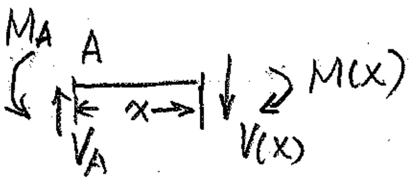



$\Sigma F_y = V_A - 2 - 4 = 0 \Rightarrow V_A = 6\,\text{Kip}$

$\Sigma M_A = M_A - 2\cdot(2+\tfrac{2}{3}) - 4\cdot(2+1) = 0$

$\Rightarrow M_A = 17\tfrac{1}{3}\ \text{Kip}\cdot\text{ft}$

Step 2. section between A & B, $0<x<2$



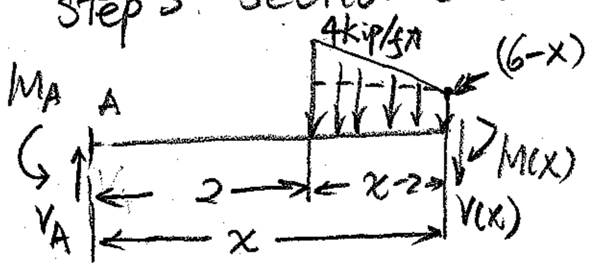

$\Sigma F_y = V_A - V(x) = 0$

$\Rightarrow V(x) = V_A = 6\,\text{Kip}$ const. between A & B

$\Sigma M_A = M_A - M(x) - V(x)\cdot x = 0$

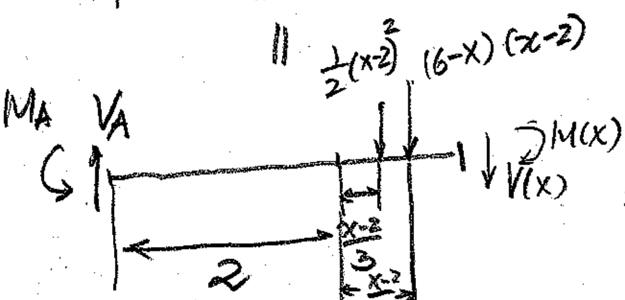
$M(x) = M_A - V(x)\cdot x = 17\tfrac{1}{3} - 6x \qquad M = 5\tfrac{1}{3}\ @\ x=2$

step 3. section between B & C, $2<x<4$




$\Sigma F_y = V_A - \tfrac{1}{2}(x-2)^2 - (6-x)(x-2) - V(x) = 0$

$V(x) = \tfrac{1}{2}x^2 - 6x + 16 \qquad V(x) = 0\ @\ x=4$



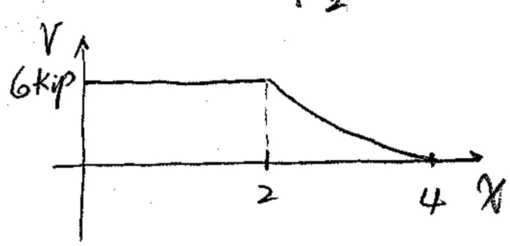

$\Sigma M_A = M_A - \tfrac{1}{2}(x-2)^2\cdot(2+\tfrac{x-2}{3}) - (6-x)(x-2)(2+\tfrac{x-2}{2})$

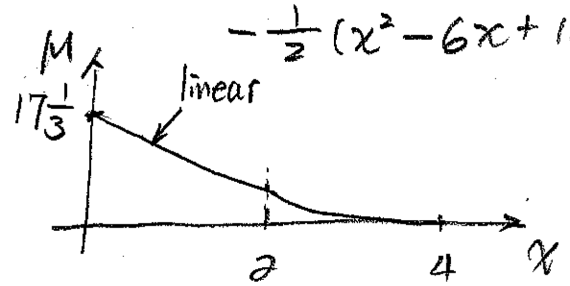
$- V(x)\cdot x - M(x) = 0$

$\Rightarrow M(x) = 17\tfrac{1}{3} - \tfrac{1}{6}(x-2)^2(x+4) + \tfrac{1}{2}(x-6)(x^2-4)$

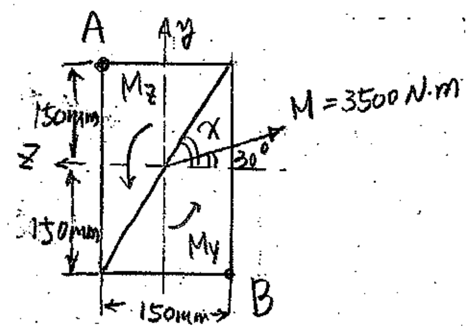
$- \tfrac{1}{2}(x^2 - 6x + 16)x$

$M(x) = 0\ @\ x=4$



**Problem 2:** The beam has a rectangular cross-section. If it is subjected to a moment of 3500 $N \bullet m$ directed as shown, determine the maximum bending stress in the beam and the orientation of the neutral axis. (25pts)




$M_z = -M\cos 30^\circ = -3031.09$ N·m

$M_y = M \cdot \sin 30^\circ = 1750$ N·m

Point A & Point B have maximum bending stress.

$I_y = \frac{1}{12} \times 0.3 \times 0.15^3 = 84.375 \times 10^{-6}\ m^4$

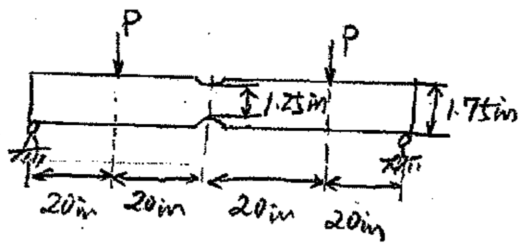
$I_z = \frac{1}{12} \times 0.15 \times 0.3^3 = 0.3375 \times 10^{-3}\ m^4$

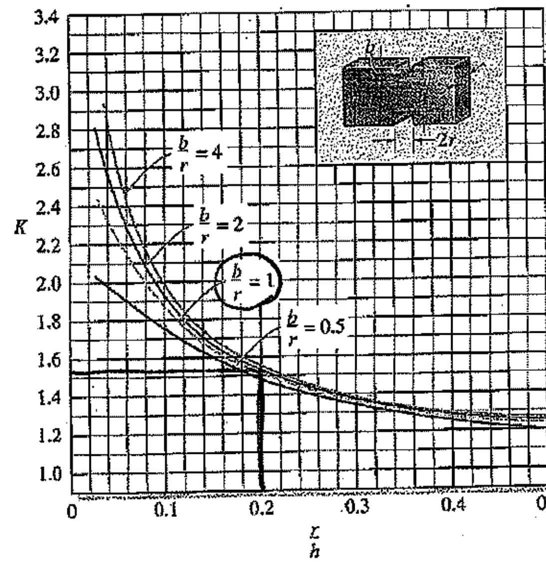
$$\sigma_A = \frac{M_y \cdot Z_{max}}{I_y} + \frac{M_z \cdot Y_{max}}{I_z} = \frac{1750 \times 0.075}{84.375 \times 10^{-6}} + \frac{3031.09 \times 0.15}{0.3375 \times 10^{-3}} = 2.90\ MPa$$

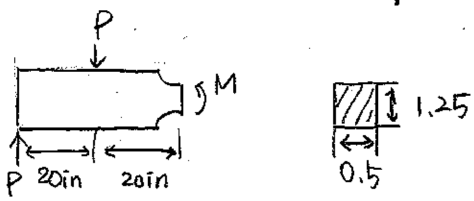
$$\sigma_B = -\frac{M_y \cdot Z_{max}}{I_y} - \frac{M_z \cdot Y_{max}}{I_z} = -2.90\ MPa$$

$$\tan\alpha = \frac{I_z}{I_y} \cdot \tan\theta = \frac{0.3375 \times 10^{-3}}{84.375 \times 10^{-6}} \times \tan(30^\circ) = -66.6^\circ$$

**Problem 3:** The simply supported notched bar is subjected to the two loads, each having a magnitude of P=100lb. Determine the maximum bending stress developed in the bar. The thickness of the beam is 0.5in (15pts)




$M = 20 \times P$

$b = \dfrac{1.75 \times 1.25}{2} = 0.25\text{ in}$

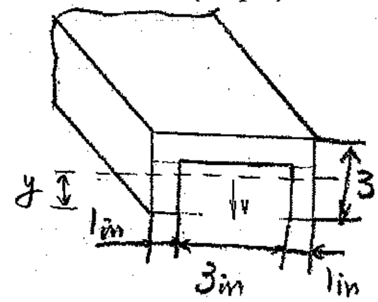
$r = b = 0.25\text{ in}$

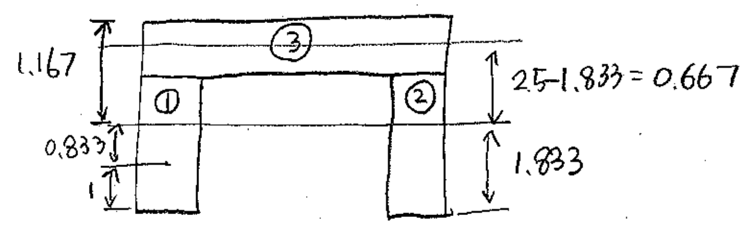
$\dfrac{b}{r} = 1. \quad \dfrac{r}{h} = \dfrac{0.25}{1.25} = 0.2 \quad \xrightarrow{\text{graph}} \quad K = 1.55$

$I = \frac{1}{12} \times 0.5 \times 0.125^3 = 0.0813$

$\sigma_{max} = K \cdot \dfrac{MY}{I} = \dfrac{20 \times 100 \times 0.625}{0.0813} \times 1.55 = \underline{\underline{23808\text{ psi}}}$

**Problem 4:** If the applied shear force V=18Kip. Determine the maximum shear stress in the member (15 pts)



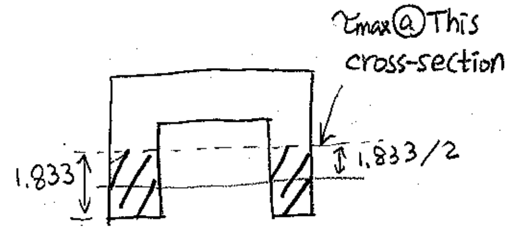

$$\bar{y} = \frac{2\times1\times2\times1 + 5\times1\times2.5}{2\times1\times2 + 5\times1} = 1.833$$

$$I = \frac{1}{12}\times5\times1^3 + 5\times1\times(2.5-1.833)^2 + \left[\frac{1}{12}\times1\times2^3 + 1\times2\times\underbrace{0.833^2}_{(1.833-1)^2}\right]\times2 = 6.75\ in^4$$

Method 1 : consider bottom blocks

$$Q_{max} = 2\times\frac{1.8333}{2}\times1.8333\times1 = 3.3611\ in$$

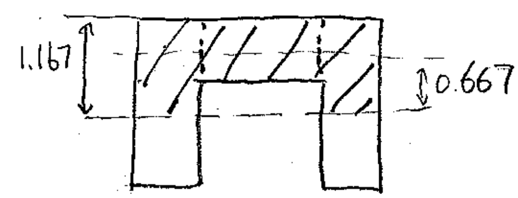
$$\tau_{max} = \frac{VQ_{max}}{I\cdot A} = \frac{18\times3.3611}{6.75\times2\times1} = 4.48\ Ksi.$$



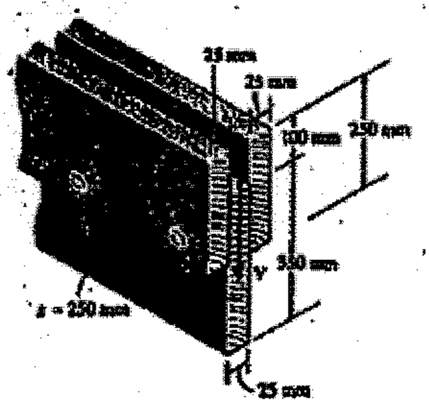

Method 2 : consider top portions

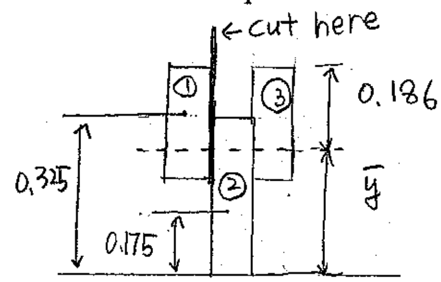
$$\begin{aligned} Q_{max} &= 2\times(1.167)\times1\times0.583 \\ &\quad +3\times1\times0.667 \\ &= 3.36\ in \end{aligned}$$

$$\tau_{max} = \frac{18\times3.361}{6.75\times2} = 4.48\ Ksi$$



**Problem 5:** A beam is constructed from three boards bolted together as shown. Determine the shear force developed in each bolt if the bolts are spaced s=250mm apart and the applied shear is V=35KN. (15 pts)



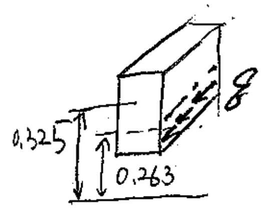

$$\bar{y} = \frac{0.175 \times 0.025 \times 0.35 + 2 \times 0.325 \times 0.025 \times 0.25}{0.025 \times 0.35 + 0.025 \times 0.25 \times 2}$$

$$\bar{y} = 0.263$$

$$I = \frac{1}{12} \times 0.025 \times 0.35^3 + 0.025 \times 0.35 \times (0.263 - 0.175)^2$$
$$+ 2 \times \frac{1}{12} \times 0.025 \times 0.25^3 + 0.025 + 0.35 \times (0.325 - 0.263)^2 = 0.270236 \times 10^{-3}\ m^4$$

Method 1: Consider block ①, shear flow @ one side

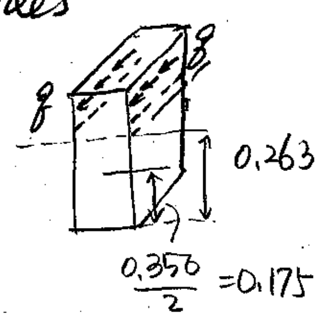
$$Q = \bar{y}A = (0.325 - 0.263) \times 0.025 \times 0.25 = 3.875 \times 10^{-4}\ (m^3)$$




$$q = \frac{VQ}{I} = \frac{35 \times 3.875 \times 10^{-4}}{0.27 \times 10^{-3}} = 50.23\ KN/m$$

$$F = q \times S = 50.23 \times 0.25 = \underline{\underline{12.56\ KN}}$$

Method 2: Consider block②, shear flow @ both sides



**Bonus problem**: draw the variation and direction of shear flow for the following thin-wall structure and (10pts)