MAE2312/CE 2313 Strength of Materials 10/16/06

Student Name____

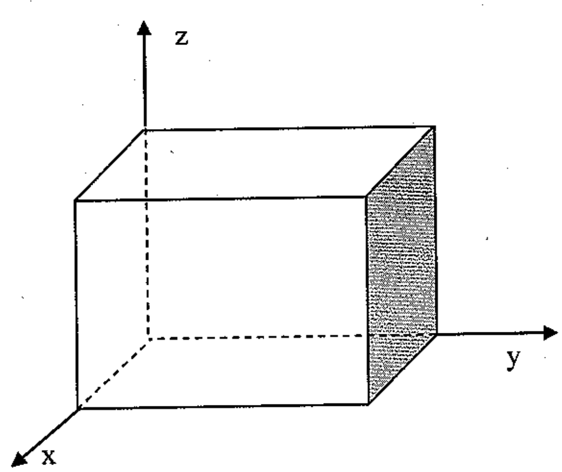
Exam #1

Policy: Close notes; close book; no cell phone use during the exam; one formula sheet is allowed; calculator is allowed; work on distributed exam sheets only; addition paper can be requested.

Problem 1: Draw the stress-strain curve for a typical ductile material. Identify Young's modulus, proportional limit, yield stress, ultimate stress, fracture stress, elastic region, yielding region, strain hardening region, necking region, and plastic region in the graph. (10pts)

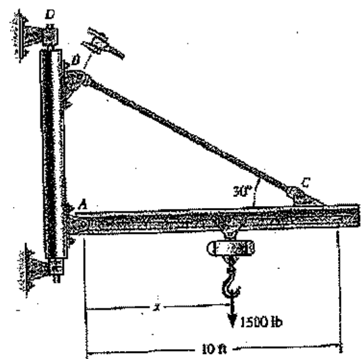
see Textbook Fig 3-4 on Page 88

Problem 2: A cubic volume element of a component is shown below. Draw the stress components on the surfaces whose norm $\vec{n}$ is aligned with positive x, y, z axis. (5pts)



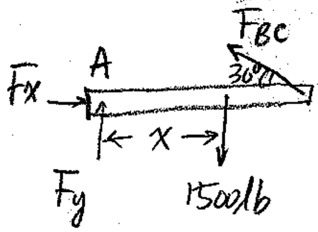
see textbook Fig 1-12 on Page 23

Problem 3: The jib crane is pinned at A and supports a chain hoist that can travel along the bottom flange of the beam, $1\,ft \le x \le 12\,ft$. The weight applied at the hoist is 1500lb.



$L = 10ft$

3.a) Calculate the largest internal force experienced by rod BC (5pt)



$$\sum M_A = 1500x - F_{BC}\cdot \sin 30^0 \cdot L$$

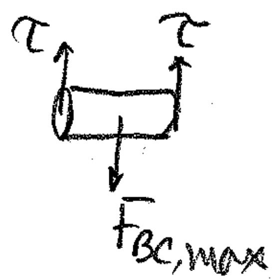
$$\Rightarrow F_{BC} = \frac{1500x}{L\sin 30^0}$$

$$F_{BC,max} = \frac{1500 \times 12}{10 \times \sin 30^0} = 3600(lb)$$

$$\sigma_{max} = \frac{F_{BC}}{A} = \frac{3600}{\pi/4 \cdot 0.75^2} = 8.15\,ksi$$

3.b) Determine the maximum average normal stress experienced by the ¾ inch diameter tie rod BC (5pt)

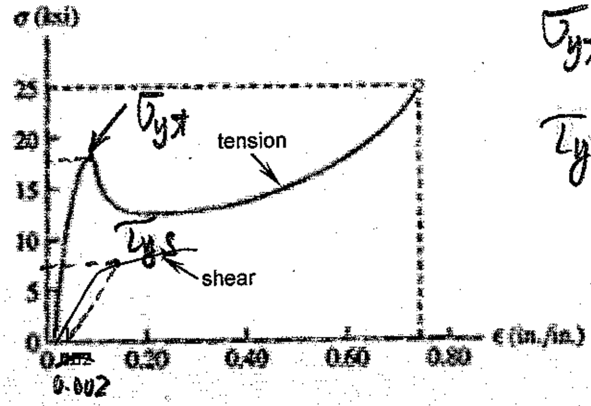
3.c) Determine the maximum average shear stress experienced by the 5/8 in diameter pin at B. (5pts)



$$2\tau A = F_{BC,max}$$

$$\tau = \frac{F_{BC,max}}{2A} = \frac{3600}{2 \times \frac{\pi}{4} \times \left(\frac{5}{8}\right)^2} = 5.87\,ksi$$

3.d) The normal stress-strain diagram for rod BC and the shear stress-strain diagram for the pin are shown below. Identify the yield stresses for both tension and shear on the graph. Calculate the allowable normal stress for rod BC and the allowable shear stress for the pin at support B if yielding is not allowed, assuming the factor of safety n=2 for normal stress and n=1.5 for shear stress. (5pts)



$\sigma_{yt} = 18\ ksi$

$\tau_{ys} = 8\ ksi$

$\sigma_{allow,t} = \frac{18}{2} = 9\ ksi$

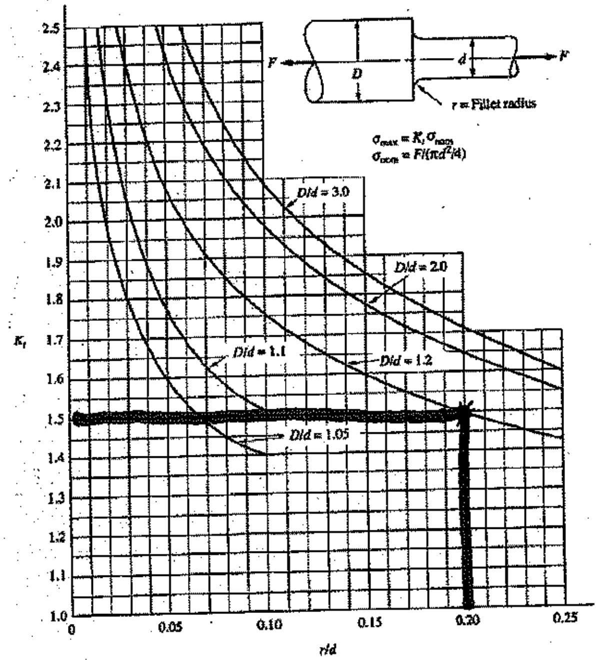
$\tau_{allow,s} = \frac{8}{1.5} = 5.3\ ksi$

3.e) will rod BC or the pin fail? (5pt)

$\sigma_{max} < \sigma_{allow,t}$ safe

$\tau_{max,s} > \tau_{allow,s} \rightarrow$ not safe

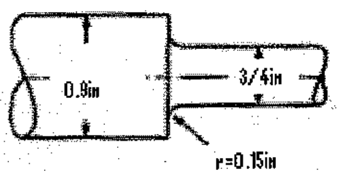
3.f) Determine the stress concentration factor (SCF) at the geometry discontinuity of rod BC for the dimensions given below. (5pt)



$$r/d = \frac{0.15}{3/4} = 0.2$$

$$D/d = \frac{0.9}{3/4} = 1.2$$

$$k = 1.5$$

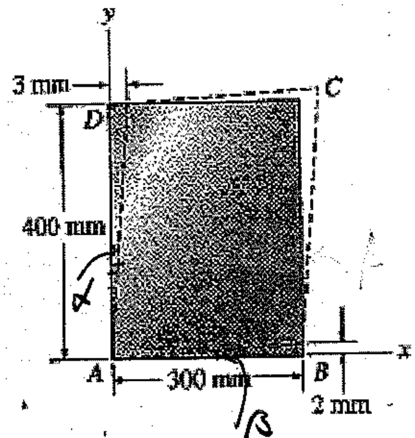


3. g) If the fatigue limit of rod BC is 15ksi and SCF is considered, what is the safety factor n for rod BC (5pt)

$\sigma_{fail} = 15 ksi$ $\qquad$ $\sigma_{max} = S.F \cdot \sigma_{nom} = 1.5 \times 8.15$

$$n = \frac{\sigma_{fail}}{\sigma_{max}} = \frac{15}{8.15 \times 1.5} = 1.2$$

Problem 4: The piece of rubber is originally rectangular and subjected to the deformation shown by the dashed lines. Determine the average normal strain along the side AD, and average shear strain γ_{xy} at the corners A. (10pts)



$$L_{AD} = 400\,mm$$

$$L'_{AD} = \sqrt{400^2+3^2} = 400.011\,mm$$

$$\varepsilon = \frac{L'_{AD}-L_{AD}}{L_{AD}} = \frac{400.011-400}{400} = 2.75e4$$

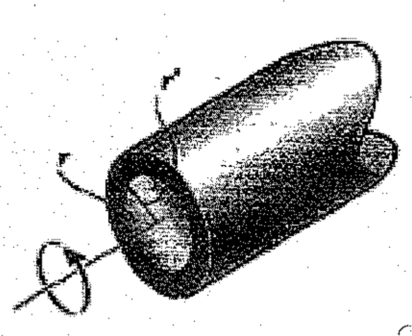
$$\alpha = \tan^{-1}\left(\frac{3}{400}\right)$$

$$\beta = \tan^{-1}\left(\frac{2}{300}\right)$$

$$\gamma_{xy} = \alpha+\beta = \tan^{-1}\left(\frac{3}{400}\right)+\tan^{-1}\left(\frac{2}{300}\right)$$

$$= 0.0142\ (rad)$$

Problem 5: The solid shaft of radius r is subjected to a torque T. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque (T/4). (15pts)

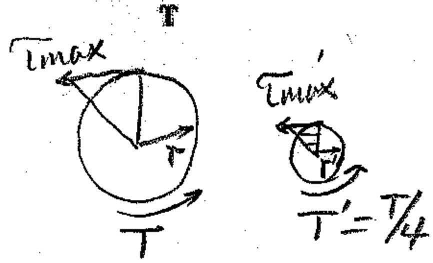


Method #1:

$$\tau_{max} = \frac{Tc}{J} = \frac{T\cdot r}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$\tau = \frac{r'}{r}\cdot\tau_{max} = \frac{r'}{r}\,\frac{2T}{\pi r^3} = \frac{r'}{r^4}\,\frac{2T}{\pi}$$

$$\tau'_{max} = \frac{2T'}{\pi r'^3} = \frac{2\cdot T/4}{\pi r'^3} = \frac{r'}{r^4}\cdot\frac{2T}{\pi}$$

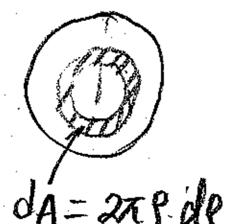


$$\frac{r'^4}{r^4} = 1/4 \Rightarrow r' = \sqrt[4]{1/4}\,r = \frac{r}{\sqrt{2}} = 0.707r$$

$T' = T/4$

Method #2.

$$\tau = \frac{\rho}{c}\tau_{max} = \frac{\rho}{r}\cdot\left(\frac{2T}{\pi r^3}\right) = \frac{2T}{\pi r^4}\rho$$

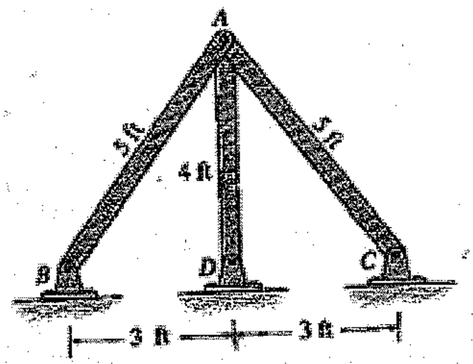


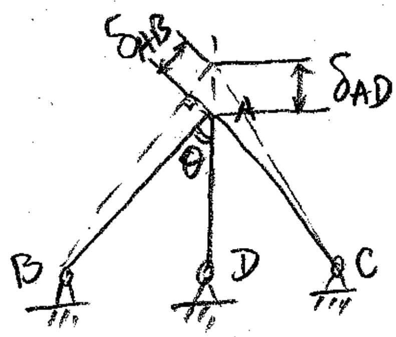
$$dT = \rho\tau\, dA = \rho\left(\frac{2T}{\pi r^4}\rho\right)(2\pi\rho\, d\rho) = \frac{4T}{r^4}\rho^3 d\rho$$

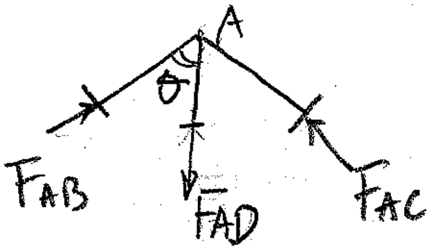
$$\int_0^{T/4} dT = \frac{4T}{r^4}\int_0^{r'}\rho^3 d\rho \Rightarrow \frac{T}{4} = \frac{4T}{r^4}\left.\frac{\rho^4}{4}\right|_0^{r'}$$

$$\Rightarrow \frac{1}{4} = \left(\frac{r'}{r}\right)^4 \Rightarrow r' = \frac{1}{\sqrt{2}}r = 0.707r$$

Problem 6: The three bars are made of A-36 steel (CTE α=6.60e-6 in/°F, E=29e3 lb/in^2) and form a pin-connected truss. If the truss is constructed when T_1=50°F, determine the internal forces of bar AB and bar AD when T_2=150°F. Each bar has a cross-sectional area of 2in^2. (Hint: the structure is symmetric, i.e. A moves vertically) (20pts)







$$\delta_{AB} = \delta_{AD} \cdot \cos\theta = \frac{4}{5}\delta_{AD} \quad (1)$$

$$\delta_{AB} = \delta_{AB,T} - \delta_{AB,F_{AB}}$$
$$= \alpha T \cdot L_{AB} - \frac{F_{AB} \cdot L_{AD}}{EA}$$
$$= 6.60e\text{-}6 \times (150-50) \cdot 5 \times 12 - \frac{F_{AB} \cdot 5 \times 12}{29E3 \times 2} \quad (2)$$

$$\delta_{AD} = \delta_{AD,T} + \delta_{AD,F_{AD}}$$
$$= \alpha T L_{AD} + \frac{F_{AD} \cdot L_{AD}}{EA}$$
$$= 6.60e\text{-}6 \times (150-50) \times 4 \times 12 + \frac{F_{AD} \cdot 4 \times 12}{29E3 \times 2} \quad (3)$$

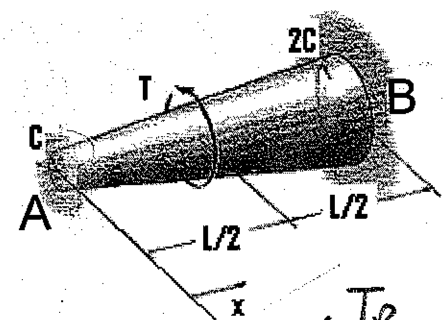
Substitute (2) & (3) into (1)

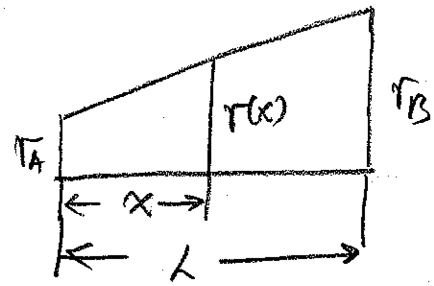
$$4F_{AD} + 6.25F_{AB} = 86.13 \quad --- (4)$$

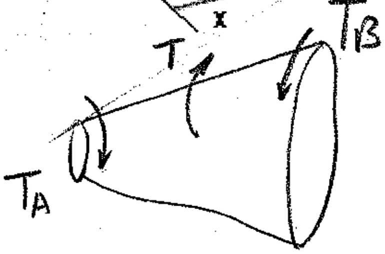
$$\Sigma F_{yA} = F_{AD} - 2 \times F_{AB} \cdot \cos\theta = F_{AD} - \frac{8}{5}F_{AB} = 0 \quad -- (5)$$

Solve for (4) & (5)
$$\begin{cases} F_{AB} = 6.81\text{ kip} \\ F_{AD} = 10.894\text{ kip} \end{cases}$$

Problem 7: The tapered shaft is confined by the fixed supports at A & B. The radius of the cross-section at A is c and the radius of the cross-section at B is 2c. If a torque T is applied at its mid-point, determine the reactions at the supports. (15pts)



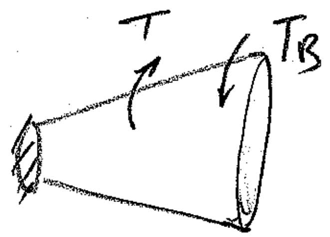




$$T_A + T_B = T \quad \text{---} \quad (1)$$

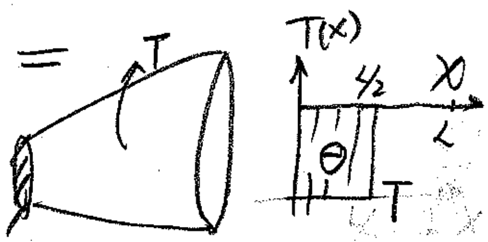
$$r(x) = r_A + \frac{r_B - r_A}{L} x = \frac{c}{L}(L+x)$$

$$J(x) = \frac{\pi}{2} r(x)^4 = \frac{\pi}{2} \frac{c^4}{L^4} (L+x)^4$$



$$\phi_T = \int_0^{L/2} \frac{T(x)}{G J(x)} dx$$

=

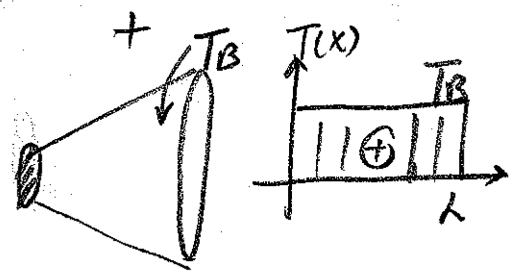


$$= -\int_0^{L/2} \frac{T}{G \frac{\pi}{2} \frac{c^4}{L^4} (L+x)^4} dx$$

$$= -\frac{T}{\frac{\pi}{2} G \frac{c^4}{L^4}} \cdot \left(-\frac{1}{3}\right)(L+x)^{-3} \Big|_0^{L/2}$$

$$= -\frac{38}{81} \frac{TL}{\pi c^4 G}$$

+

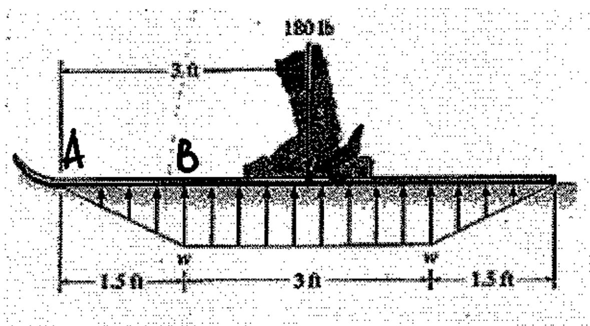


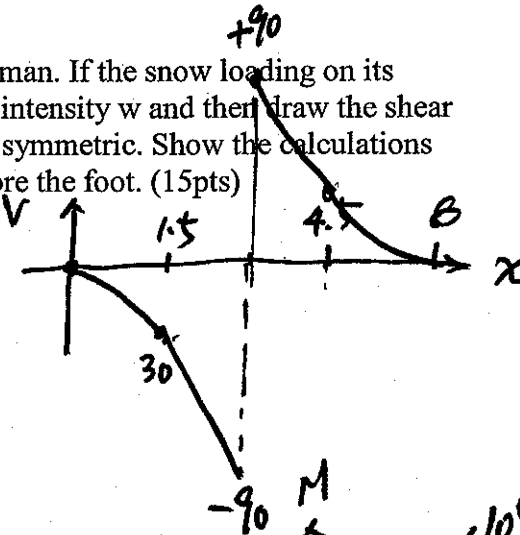
$$\phi_{T_B} = \int_0^L \frac{T_B}{\frac{\pi}{2} \frac{c^4}{L^4} (L+x)^4 G} dx = \frac{7 T_B L}{12 \pi c^4 G}$$

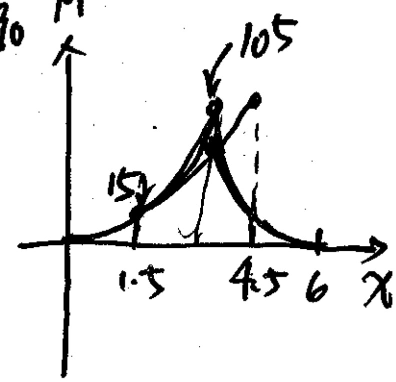
$$\phi_T + \phi_{T_B} = -\frac{38}{81} \frac{TL}{\pi c^4 G} + \frac{7 T_B L}{12 \pi c^4 G} = 0 \quad \text{---} \quad (2)$$

$$\Rightarrow \begin{cases} T_B = \frac{152}{189} T \\ T_A = \frac{37}{189} T \quad \text{from eq. (1)} \end{cases}$$

Problem 8: The ski supports the 180-lb weight of the man. If the snow loading on its bottom surface is trapezoidal as shown, determine the intensity w and then draw the shear and moment diagram for the ski. Notice that the ski is symmetric. Show the calculations of internal shear and moment for the two sections before the foot. (15pts)



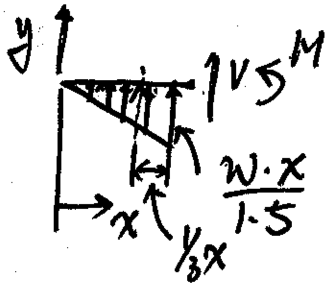




$$\Sigma F_y = 2 \cdot \frac{1}{2} w \cdot 1.5 + 3 \cdot w - 180 = 0$$

$$\Rightarrow w = \frac{180}{1.5+3} = 40 \text{ lb/ft}$$

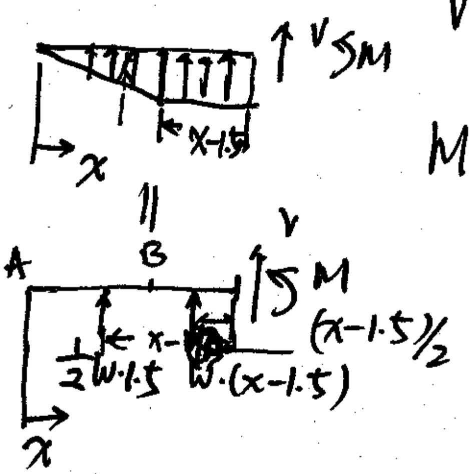
Section AB



$$V = -\frac{1}{2} \frac{w}{1.5} \cdot x \cdot x = -\frac{w}{3} \cdot x^2 = -13.33 x^2$$

$$M = +V \cdot \frac{1}{3} x = 13.33/3 \cdot x^2 \cdot x = 4.44 x^3$$

Section BC:



$$V = -\left[\frac{1}{2} \cdot w \cdot 1.5 + w \cdot (x-1.5)\right] = -wx + 0.75w$$

$$= -40x + 30$$

$$M = \frac{1}{2} w \, 1.5 \cdot (x - 1) + w(x-1.5) \cdot \frac{x-1.5}{2}$$

$$= 0.75 w (x - 1) + 0.5 w (x-1.5)^2$$

$$= 20x^2 - 30x + 15$$