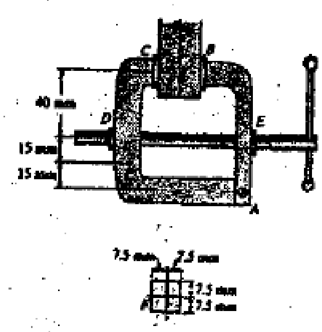
MAE2312 Solid Mechanics/CE 2313 Strength of Materials 11/29/06

Student Name____

Exam #3

Policy: Close notes; close book; no cell phone use during the exam; one formula sheet is allowed; calculator is allowed; work on distributed exam sheets only; addition paper can be requested.

Problem 1: The clamp is made from members AB and AC, which are pin connected at A. If the compressive force at C & B is 180N, determine the state of stress at point F, and indicate the results on a volume element. The screw DE is subjected to a tensile force along its axis. (25pts)



Support Reactions :

$\circlearrowleft + \Sigma M_A = 0;\quad 180(0.07) - T_{DE}(0.03) = 0$

$T_{DE} = 420\ \text{N}$

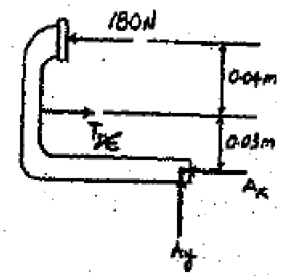
Internal Forces and Moment :

$\xrightarrow{+} \Sigma F_x = 0;\quad 420 - 180 - V = 0 \qquad V = 240\ \text{N}$

$+\uparrow \Sigma F_y = 0;\quad N = 0$

$+\Sigma M_O = 0;\quad 180(0.055) - 420(0.015) - M = 0$

$M = 3.60\ \text{N}\cdot\text{m}$

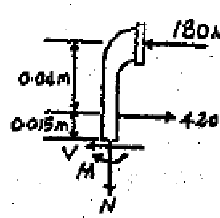


Section Properties :

$A = 0.015(0.015) = 0.225(10^{-3})\ \text{m}^2$

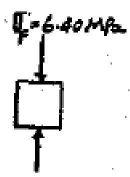
$I = \frac{1}{12}(0.015)(0.015^3) = 4.21875(10^{-9})\ \text{m}^4$

$Q_F = 0$



Normal Stress : Since $N = 0$, the normal stress is caused by bending stress only.

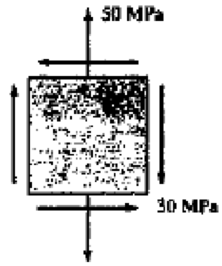
$$\sigma_F = \frac{Mc}{I} = \frac{3.60(0.0075)}{4.21875(10^{-9})} = 6.40\ \text{MPa (C)} \qquad \text{Ans}$$



Shear Stress : Applying shear formula, we have

$$\tau_F = \frac{VQ_F}{It} = 0 \qquad \text{Ans}$$

Problem 2: Determine (a) the principle stress and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case (20pts)



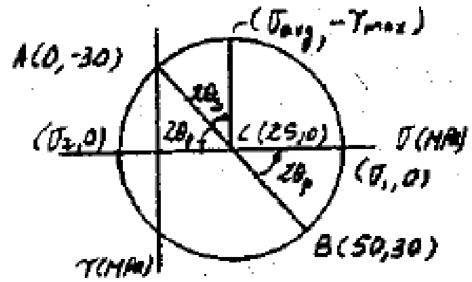
$A(0,-30) \qquad B(50,30) \qquad C(25,0)$

$R = CA = CB = \sqrt{25^2 + 30^2} = 39.05$

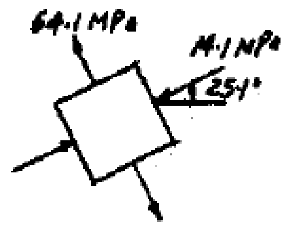
a)

$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa}$ **Ans**

$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa}$ **Ans**



$\tan 2\theta_p = \frac{30}{25} \qquad 2\theta_p = 50.19° \qquad \theta_p = 25.1°$



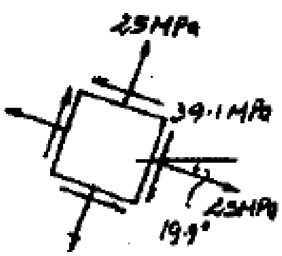
b)

$\tau_{\substack{max \\ in\text{-}plane}} = R = 39.1 \text{ MPa}$ **Ans**

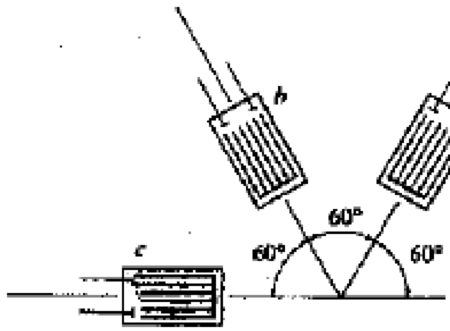
$\sigma_{avg} = 25 \text{ MPa}$ **Ans**

$2\theta_s = 90 - 2\theta_p$

$\theta_s = -19.9°$



Problem 3: The 60^o strain rosette is mounted on the surface of an aluminum plate. The following readings are obtained for each gauge: $\varepsilon_a=950(10^{-6})$, $\varepsilon_b=380(10^{-6})$, $\varepsilon_c= -220(10^{-6})$. Determine the strain components ε_x, ε_y, & γ_{xy}. (15pts)



$\varepsilon_a = 250(10^{-6})$ $\quad\varepsilon_b = -400(10^{-6})$ $\quad\varepsilon_c = 280(10^{-6})$

$\theta_a = 60^\circ$ $\quad\theta_b = 120^\circ$ $\quad\theta_c = 180^\circ$

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

$$280(10^{-6}) = \varepsilon_x \cos^2 180^\circ + \varepsilon_y \sin^2 180^\circ + \gamma_{xy} \sin 180^\circ \cos 180^\circ$$

$$\varepsilon_x = 280(10^{-6})$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

$$250(10^{-6}) = \varepsilon_x \cos^2 60^\circ + \varepsilon_y \sin^2 60^\circ + \gamma_{xy} \sin 60^\circ \cos 60^\circ$$

$$250(10^{-6}) = 0.25\varepsilon_x + 0.75\varepsilon_y + 0.433\gamma_{xy} \qquad (1)$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

$$-400(10^{-6}) = \varepsilon_x \cos^2 120^\circ + \varepsilon_y \sin^2 120^\circ + \gamma_{xy} \sin 120^\circ \cos 120^\circ$$

$$-400(10^{-6}) = 0.25\varepsilon_x + 0.75\varepsilon_y - 0.433\gamma_{xy} \qquad (2)$$

Subtract Eq. (2) from Eq. (1)

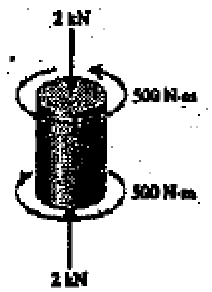
$$650(10^{-6}) = 0.866\gamma_{xy}$$

$$\gamma_{xy} = 750.56(10^{-6})$$

$$\varepsilon_y = -193.33(10^{-6})$$

Note: ε_a, ε_b, ε_c in the solution are different from the problem, but the approach should be exactly the same.

Problem 4: A short concrete cylinder having a diameter of 50mm is subjected to a torque of 500N.m and an axial compression force of 2KN. Determine if it fails according to the maximum-normal-stress theory. The ultimate stress of the concrete is σ_{ult}=28MPa. (20pts)



$$A = \frac{\pi}{4}(0.05)^2 = 1.9635(10^{-3})\ m^2$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.61359(10^{-6})\ m^4$$

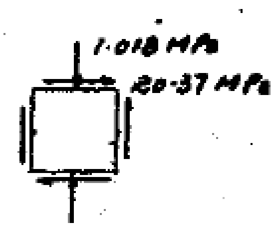
$$\sigma = \frac{P}{A} = \frac{2(10^3)}{1.9635(10^{-3})} = 1.019\ MPa$$

$$\tau = \frac{Tc}{J} = \frac{500(0.025)}{0.61359(10^{-6})} = 20.372\ MPa$$

$$\sigma_x = 0 \qquad \sigma_y = -1.019\ MPa \qquad \tau_{xy} = 20.372\ MPa$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{0 - 1.018}{2} \pm \sqrt{\left(\frac{0-(-1.019)}{2}\right)^2 + 20.372^2}$$



$$\sigma_1 = 19.87\ MPa \qquad \sigma_2 = -20.89\ MPa$$

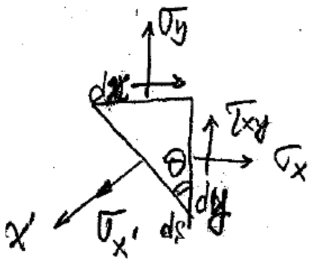
Failure criteria :

$|\sigma_1| < \sigma_{ult} = 28\ MPa$ OK

$|\sigma_2| < \sigma_{ult} = 28\ MPa$ OK

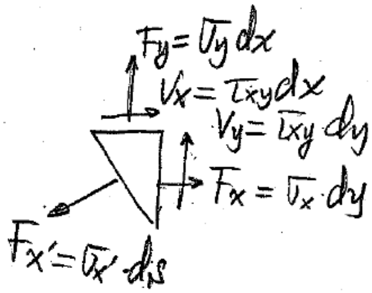
No. Ans

Problem 5: express $\sigma_{x'}$ in terms of σ_x, σ_y, & τ_{xy}. Show derivation steps. (10 pts)



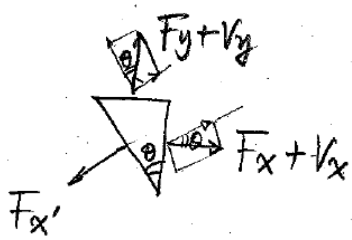
$$dx = ds \cdot \sin\theta, \quad dy = ds\cos\theta$$

$$\sum F_{x'} = F_{x'} - (F_x + V_x)\cos\theta - (F_y + V_y)\cdot\sin\theta = 0$$



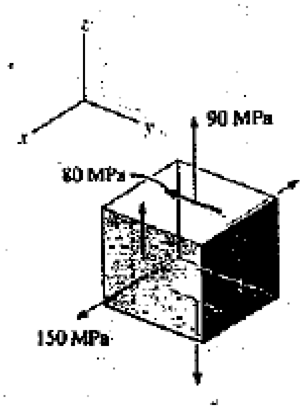
$$\sigma_{x'} ds - (\sigma_x dy + \tau_{xy} dx)\cos\theta - (\sigma_y dx + \tau_{xy} dy)\sin\theta = 0$$

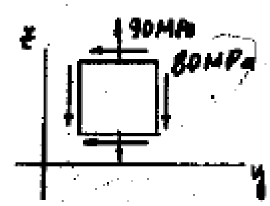
$$\sigma_{x'} ds - (\sigma_x ds\cos\theta + \tau_{xy} ds\sin\theta)\cos\theta - (\sigma_y \cdot ds\sin\theta + \tau_{xy} \cdot ds\cos\theta)\sin\theta = 0$$



$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin 2\theta$$

Bonus problem: The stress at a point is shown on the element. Determine the principle stresses and the absolute maximum shear stress. (10pts)





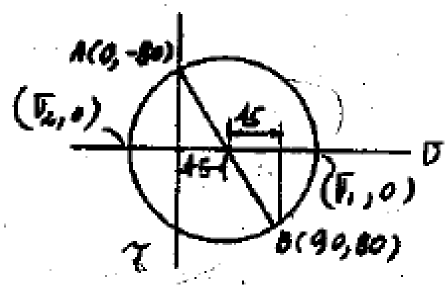
For y - z plane :

$A(0,-80)$ $B(90,80)$ $C(45,0)$

$R = \sqrt{45^2 + 80^2} = 91.79$

$\sigma_1 = 45 + 91.79 = 136.79$ MPa

$\sigma_2 = 45 - 91.79 = -46.79$ MPa



Thus,

$\sigma_1 = 150$ MPa **Ans**

$\sigma_2 = 137$ MPa **Ans**

$\sigma_3 = -46.8$ MPa **Ans**

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{150 - (-46.8)}{2} = 98.4 \text{ MPa}$$ **Ans**