HW #2 SOLUTION ME 5339

Solu^n 2.1

Given:-

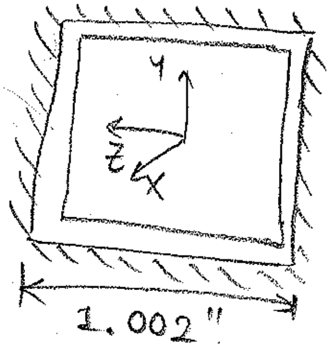
Square Aluminium bar 1" each side 10" long.

$E = 10 \times 10^6$ psi

$\nu = 0.33$

$\alpha = 13.1 \times 10^{-6}$ in/in/°F.

a)



$P_x = -50{,}000$ lb.

Now, due to compressive load stresses in axial direction

$$\sigma_x = \frac{P}{A} = -\frac{50{,}000}{1 \times 1} = -50{,}000 \text{ psi}$$

First of all consider there is no constraints on bar and only axial stress σ_x acting, while $\sigma_y = \sigma_z = 0$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{-50{,}000}{10 \times 10^6} = -5 \times 10^{-3} \text{ in/in}$$
$$= -0.005 \text{ in/in}$$

$$\varepsilon_y = -\nu \frac{\sigma_x}{E} = -0.33 \times \frac{-50{,}000}{10 \times 10^6} = 0.00165 \text{ in/in}$$

$$\varepsilon_z = -\nu \frac{\sigma_x}{E} = 0.00165 \text{ in/in}$$

The deformed size of the square bar $= 1 + 1 \times 0.00165$
$= 1.00165$ in

which is < 1.002

Hence $\sigma_y = \sigma_z = 0$ & $\sigma_x = -50{,}000$ psi.

$\varepsilon_y = \varepsilon_z = 0.00165$ & $\varepsilon_x = -0.005$ in/in

(B) Now, in case A, temp of 250°F added to bar

temp. strain $= \alpha \Delta T$

$= 13.1 \times 10^{-6} \times 250$

$= 3.275 \times 10^{-3}$ in/in

$\therefore$ Total Strain in this case $\varepsilon = \varepsilon^M + \varepsilon^T$

$= 0.00165 + 0.003275$

$= 0.004925$

Hence size of bar $= (1 + 1 \times 0.004925) = 1.004925$ in which is > 1.002 in, the hole size in which bar has kept. So there will be stresses σ_y & σ_z

Also, stresses in y & z direction will be same.

$\sigma_y = \sigma_z$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} + \alpha \Delta T$$

$$\varepsilon_x = -0.005 - \frac{0.33}{10 \times 10^6} \times 2\sigma_y + 0.003275 \quad \text{--- (1)}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_z}{E} + \alpha \Delta T$$

$$0.002 = \frac{\sigma_y}{10 \times 10^6}(1 - 0.33) - 0.33(-0.005) + 0.003275$$

$\sigma_y = \sigma_z = -43656$ PSI $= -43.66$ KSI

Substituting in eqn 1

$\therefore \varepsilon_x = 1.156 \times 10^{-3}$ in/in $= 0.001156$ in/in

with temperature
Stresses $\rightarrow \sigma_x = -50{,}000$ PSI , $\sigma_y = \sigma_z = -43656.71$ PSI
Strains $\rightarrow \varepsilon_x = 0.001156$ in/in
$\varepsilon_y = \varepsilon_z = 0.002$ in/in

(c) Strain energy for case (a)

$$U = \frac{1}{2}\sigma_x\varepsilon_x + \cancel{\frac{1}{2}\sigma_y\varepsilon_y} + \cancel{\frac{1}{2}\sigma_z\varepsilon_z} + \cancel{\frac{1}{2}\tau_{xy}\varepsilon_{xy}} + \cancel{\frac{1}{2}\tau_{yz}\varepsilon_{yz}} + \cancel{\frac{1}{2}\tau_{xz}\varepsilon_{xz}}$$

$$= \frac{1}{2}(-50{,}000)(-0.005)$$

$$= 125 \ \text{lb-in/in}^3$$

(d) Strain energy for case (b)

Only mechanical strain contributes to the strain energy

$$U = \frac{1}{2}\left(\sigma_x\varepsilon_x^M + \sigma_y\varepsilon_y^M + \sigma_z\varepsilon_z^M + \cancel{\tau_{xy}\varepsilon_{xy}^M} + \cancel{\tau_{yz}\varepsilon_{yz}^M} + \cancel{\tau_{xz}\varepsilon_{xz}^M}\right)$$

Now, $\varepsilon_x^M = \varepsilon_x - \alpha\Delta T$

$= -2.119\times10^{-3}$ in/in

$\varepsilon_y^M = \varepsilon_z^M = \varepsilon_y - \alpha\Delta T = -1.275\times10^{-3}$ in/in

$$\therefore U = \frac{1}{2}\left[(-50{,}000 \times -2.119\times10^{-3}) + 2(-43656.71 \times -1.275\times10^{-3})\right]$$

$$= 108.63 \ \text{lb-in/in}^3$$

or you can use formulae

$$U = \frac{1}{2E}(\sigma_x^2+\sigma_y^2+\sigma_z^2) - \frac{\nu}{E}(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x) + \cancel{\frac{1}{2G}(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

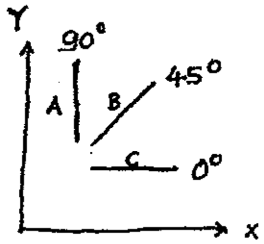
$$= \frac{1}{2\times10\times10^6}\left((-50{,}000)^2 + 2(-43656.71)^2\right)$$

$$- \frac{0.33}{10\times10^6}\left[(-50{,}000\times-43656.71)\times2 + (-43656.71)^2\right]$$

$$= 108.63 \ \text{lb-in/in}^3$$

Problem 2

Given :



$\epsilon_a = 1000\ \mu\epsilon \qquad \theta_a = 90°$

$\epsilon_b = 4000\ \mu\epsilon \qquad \theta_b = 45°$

$\epsilon_c = 2000\ \mu\epsilon \qquad \theta_c = 0°$

$\alpha = 11.7 \times 10^{-6}$ in/in/°C $\qquad E = 200\,GPa \qquad \nu = 0.25$

$\Delta T = 100°C$

Solution :

$$\epsilon_x' = \epsilon_x^M \cdot m^2 + \epsilon_y^M \cdot n^2 + \Gamma_{xy} \cdot mn + \alpha\Delta T \ldots\ m = \cos\theta\ ;\ n = \sin\theta$$

$$\therefore\ \epsilon_A = \epsilon_x^M \cdot m_A^2 + \epsilon_y^M \cdot n_A^2 + \Gamma_{xy} \cdot m_A n_A + \alpha.\Delta T$$

$$\Rightarrow 1000 = \epsilon_x^M.(0) + \epsilon_y^M \cdot (1) + \Gamma_{xy} \cdot (0) + (11.7 \times 10^{-6} \times 100)$$

$$\epsilon_y^M = -170\ \mu\epsilon$$

$$\epsilon_c = \epsilon_x^M . m_c^2 + \epsilon_y^M n_c^2 + \Gamma_{xy} . m_c n_c + \alpha . \Delta T$$

$$\Rightarrow 2000 = \epsilon_x^M . (1) + \epsilon_y^M (0) + \Gamma_{xy} \cdot (0) + (11.7 \times 10^{-6} \times 100)$$

$$\epsilon_x^M = 830\ \mu\epsilon$$

$$\epsilon_B' = \epsilon_x^M \cdot m_B^2 + \epsilon_y^M . n_B^2 + \Gamma_{xy} \cdot m_B n_B + \alpha\Delta T$$

$$\Rightarrow 4000 = 830(1/2) + (-170)(1/2) + \Gamma_{xy}(1/2) + (11.7 \times 10^{-6} \times 100)$$

$$\Gamma_{xy} = 5000\ \mu\epsilon$$

CASE I : Plane Strain Condition

$$\epsilon_z = \Gamma_{yz} = \Gamma_{xz} = 0 \Rightarrow \tau_{xz} = \tau_{yz} = 0 \text{ but } \sigma_z \neq 0$$

$$\epsilon_z = \epsilon_z^M + \alpha . \Delta T = 0$$

$$\Rightarrow \epsilon_z^M = -\alpha . \Delta T = -1170\ \mu\epsilon$$

State of plane strain

$$\begin{pmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \varepsilon_z^M \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

$$\therefore \begin{pmatrix} 830 \\ -170 \\ -1170 \end{pmatrix} \times 10^{-6} = \frac{1}{200 \times 10^9} \begin{bmatrix} 1 & -0.25 & -0.25 \\ -0.25 & 1 & -0.25 \\ -0.25 & -0.25 & 1 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix}$$

by solving above equations, we get

$\sigma_x = 92$ MPa

$\sigma_y = -68$ MPa

$\sigma_z = -228$ MPa

Note:- In this case $\varepsilon_z = 0$, but because of temperature strain ε_z^M exist.

$\tau_{xy} = G \cdot \gamma_{xy}$

$$G = \frac{E}{2(1+\nu)} = \frac{200 \times 10^9}{2(1+0.25)} = 80 \text{ GPa}$$

$\therefore \tau_{xy} = 80 \times 5000 \times 10^{-6}$

$= 400$ MPa

$$\therefore [\sigma] = \begin{bmatrix} 92 & 400 & 0 \\ 400 & -68 & 0 \\ 0 & 0 & -228 \end{bmatrix} \text{MPa}$$

B) Strain Energy :

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{92-68-228}{3} = -68 \text{ MPa}$$

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)\right]^{1/2}$$

$$= \frac{1}{3}\left[(92+68)^2+(-68+228)^2+(-228-92)^2+6(400)^2\right]^{1/2}$$

$$= 351.75 \text{ MPa}$$

$$U_{ov} = \frac{3(1-2\nu)}{2E}\cdot\sigma_m^2$$

$$= \frac{3(1-0.5)}{2\times 200\times 10^9}(-68\times 10^6)^2 = 17.34 \text{ kN-m/m}^3$$

$$U_{od} = \frac{3}{4G}\cdot\tau_{oct}^2$$

$$= \frac{3}{4\times 80\times 10^9}\cdot(351.75)^2 = 1160 \text{ kN-m/m}^3$$

CASE II : Plane Stress Condition

$\sigma_z = \tau_{yz} = \tau_{xz} = 0 \Rightarrow \Gamma_{yz} = \Gamma_{xz} = 0$ but $\epsilon_z \neq 0$

$$\therefore \epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E} = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

A) State of the Stress:

$$\therefore \quad \sigma_x = \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y)$$

$$= \frac{200\times10^9}{(1-0.25^2)}\cdot(830\times10^{-6} + 0.25\times-170\times10^{-6})$$

$$= 168 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x)$$

$$= \frac{200\times10^9}{(1-0.25^2)}\cdot(-170\times10^{-6} + 0.25\times830\times10^{-6})$$

$$= 8 \text{ MPa}$$

$$\tau_{xy} = G.\Gamma_{xy} = 400 \text{ MPa}$$

$$\therefore \quad [\sigma] = \begin{bmatrix} 168 & 400 & 0 \\ 400 & 8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

B) Strain Energy:

$$\sigma_m = \frac{\sigma_x+\sigma_y+\sigma_z}{3} = \frac{168+8+0}{3} = 58.67 \text{ MPa}$$

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)\right]^{1/2}$$

$$= \frac{1}{3}\left[(168-8)^2+(8-0)^2+(0-168)^2+6(400^2+0+0)\right]^{1/2}$$

$$= 335.64 \text{ MPa}$$

$$\mathcal{U}_{ov} = \frac{3(1-2\nu)}{2E} \cdot \sigma_m^2$$

$$= \frac{3(1-0.5)}{2 \times 200 \times 10^9} \cdot (58.67)^2 = 12.9 \text{ kN-m/m}^3$$

$$\mathcal{U}_{od} = \frac{3}{4G} \cdot \tau_{oct}^2$$

$$= \frac{3}{4 \times 80 \times 10^9} \cdot (335.64 \times 10^6)^2 = 1052 \text{ kN-m/m}^3$$

Problem 23

$E = 70 \times 10^3$ MPa

$\nu = 0.33$

$\alpha = 23.6 \times 10^{-6}$ m/m/°C

$$\sigma = \begin{bmatrix} 200 & 20 & 10 \\ & -50 & 0 \\ \text{sym} & & 40 \end{bmatrix} \text{MPa}$$

a) Corresponding strain components

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{70 \times 10^3}\left[200 - 0.33 \cdot (-50) - 0.33(40)\right]$$

$$= 0.0029 \text{ m/m} = 2904\ \mu\varepsilon$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x - \nu\sigma_y)$$

$$= \frac{1}{70 \times 10^3}\left[-50 - 0.33 \times 200 - 0.33 \times 40\right]$$

$$= -0.00185 \text{ m/m} = -1845\ \mu\varepsilon$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y)$$

$$= \frac{1}{70 \times 10^3}\left[40 - 0.33 \times 200 - 0.33(-50)\right]$$

$$= 0.0001357 \text{ m/m} = -135\ \mu\varepsilon$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{20}{26.31 \times 10^3} = 7.6 \times 10^{-4} \text{ m/m}$$

$$= 760\ \mu\varepsilon$$

$$G = \frac{E}{2(1+\nu)} = \frac{70 \times 10^3}{2(1+0.33)} = 26.31 \times 10^3 \text{ MPa}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = 0$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G} = \frac{10}{26.31 \times 10^3} = 3.8 \times 10^{-4} \text{ m/m}$$

$$= 380\ \mu\varepsilon$$

Strain Invariants

$$J_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= 2904 - 1845 - 135$$

$$= 924\ \mu\varepsilon = 924 \times 10^{-6}$$

$$J_2 = \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x - \frac{1}{4}\left(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2\right)$$

$$= (2904)(-1845) + (-1845)(-135) + (-135)(2904) - \frac{1}{4}\left[(760)^2 + 0 + (380)^2\right]$$

$$= -5681345 = -5.68 \times 10^{-6}$$

$$J_3 = \begin{vmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \text{sym} & & \varepsilon_z \end{vmatrix}$$

$$= \begin{vmatrix} 2904 & 380 & 190 \\ & -1845 & 0 \\ \text{sym} & & -135 \end{vmatrix}$$

$$= 809412300 = 8.09 \times 10^{-10}$$

Principal Strain

$$\begin{vmatrix} 2904 - \varepsilon_p & 380 & 190 \\ & -1845 - \varepsilon_p & 0 \\ \text{sym} & & -135 - \varepsilon_p \end{vmatrix} = 0$$

$$\varepsilon_{p1} = -1.8753 \times 10^{3}\ \mu\varepsilon$$

$$\varepsilon_{p2} = -0.1465 \times 10^{3}\ \mu\varepsilon$$

$$\varepsilon_{p3} = 2.9459 \times 10^{3}\ \mu\varepsilon$$

Corresponding Direction Cosines

When $\varepsilon_{P1} = -1.8753 \times 10^{3}\ \mu\varepsilon = -1875\ \mu\varepsilon$

$$\begin{bmatrix} 2904+1875 & 380 & 190 \\ & -1845+1875 & 0 \\ & & -135+1875 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$$\begin{bmatrix} 4879 & 380 & 190 \\ 380 & 30 & 0 \\ 190 & 0 & 1740 \end{bmatrix} \begin{bmatrix} l_1 \\ m_1 \\ n_1 \end{bmatrix} = 0$$

Also, $l_1^2 + m_1^2 + n_1^2 = 1$

$4879\, l_1 + 380\, m_1 + 190\, n_1 = 0$

$380\, l_1 + 30\, m_1 = 0 \longrightarrow m_1 = -\frac{380}{30} l_1$

$190\, l_1 + 1740\, n_1 = 0 \longrightarrow n_1 = -\frac{190}{1740} l_1$

$$\therefore\ l_1^2 + \left(-\frac{380}{30} l_1\right)^2 + \left(-\frac{190}{1740} l_1\right)^2 = 1$$

$$l_1^2 + 160.44\, l_1^2 + 0.01192\, l_1^2 = 1$$

$$161.45\, l_1^2 = 1$$

$l_1 = 0.0787$

$m_1 = -0.9968$

$n_1 = -0.00859$

Similarly

When $\varepsilon_{P2} = -0.1465 \times 10^{3}\ \mu\varepsilon$

$l_2 = 0.0605$

$m_2 = 0.0135$

$n_2 = -0.9981$

When $\varepsilon_{P3} = 2.9459 \times 10^{3}\ \mu\varepsilon$

$l_3 = -0.9950$

$m_3 = -0.0789$

$n_3 = -0.0614$

b) If solid is placed at a temperature environment $\Delta T = 100°C$

$$\varepsilon^{th} = \alpha \Delta T$$
$$= 23.6 \times 10^{-6} \times 100$$
$$= 23.6 \times 10^{-4} \text{ m/m}$$
$$= 2360 \ \mu\varepsilon$$

Strain components

$$\varepsilon_x = 2904 + 2360 = 5264 \ \mu\varepsilon$$
$$\varepsilon_y = -1845 + 2360 = 515 \ \mu\varepsilon$$
$$\varepsilon_z = -135 + 2360 = 2225 \ \mu\varepsilon$$

There is no effect on shear stress due to temperature change

Principal strains

$$\begin{vmatrix} 5264 - \varepsilon_p & 380 & 190 \\ 380 & 515 - \varepsilon_p & 0 \\ 190 & 0 & 2225 - \varepsilon_p \end{vmatrix} = 0$$

$$\varepsilon_{p1} = 484.7 \ \mu\varepsilon$$
$$\varepsilon_{p2} = 2213.5 \ \mu\varepsilon$$
$$\varepsilon_{p3} = 5305.9 \ \mu\varepsilon$$

Maximum Shear Strain

$$\varepsilon_{max} = \left| \frac{\varepsilon_{p3} - \varepsilon_{p1}}{2} \right| = 2410.6 \ \mu\varepsilon$$