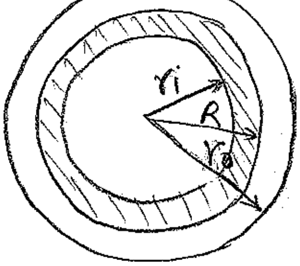
Solu^n 1



$E = 30 \times 10^6$ psi

$\vartheta = 0.3$

$r_i = 5$ in

$R = 10$ in

$r_o = 11$ in

a) What diametric interference will give a peak $\sigma_\theta = 50$ ksi in outer cylinder.

For outer cylinder maxi tangential stress will occur at inner surface R, and $p_o = 0$

$$\therefore K_1 = \frac{P_i r_i^2}{r_o^2 - r_i^2} \qquad K_2 = r_o^2 \cdot K_1$$

$$\therefore \sigma_\theta^o = P \frac{r_o^2 + R^2}{r_o^2 - R^2}$$ where P = Interference pressure

$$50 = P \frac{(11)^2 + (10)^2}{(11)^2 - (10)^2} \qquad \therefore P = 4.75 \text{ Ksi}$$

Now, as both materials of outer & inner cylinder same

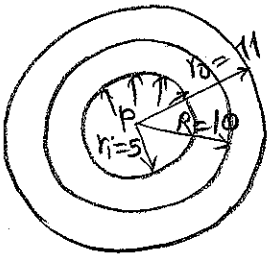
$$P = \frac{E\delta}{R} \frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)}$$

$$4.75 \times 10^3 = \frac{30 \times 10^6 \times \delta}{10} \frac{(11^2 - 10^2)(10^2 - 5^2)}{2 \times 10^2 \times (11^2 - 5^2)}$$

$$\delta = 1.93 \times 10^{-2} \text{ in}$$
$$= 0.0193 \text{ in}$$

$\therefore$ Diametric Interference $= 2\delta = 2 \times 0.0193 = 0.0386$ inch

(b) Internal pressure → $p = (?)$ which will create



$$\sigma_\theta \Big|_{r=r_i} = 70 \text{ Ksi}$$

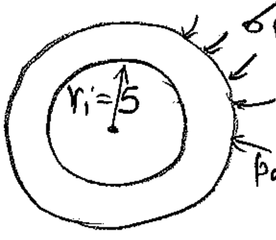
$r_i \downarrow 5$

Stresses due to

↓

Due to Hydraulic Pressure + Due to interference

Now, due to interference tangential stresses

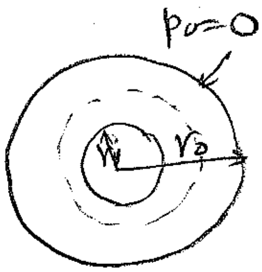


$$\sigma_{\theta_1}\Big|_{r=r_i=5} = \frac{-2 p_o R^2}{R^2 - r_i^2}$$

$$= -\frac{2 \times 4.75 \times 10^2}{10^2 - 5^2}$$

$$= -12.67 \text{ KSI}$$

Now, due to hydraulic pressure at inner cylinder



$$\sigma_{\theta_2}\Big|_{r=r_i=5} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}$$

$$= p_i \frac{11^2 + 5^2}{11^2 - 5^2}$$

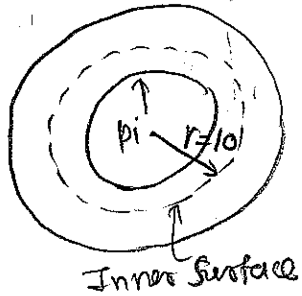
$$= 1.52\, p_i$$

$$\sigma_\theta = \sigma_{\theta_1} + \sigma_{\theta_2}$$

$$70 = -12.67 + 1.52\, p_i$$

$$\therefore \underline{p_i = 54.39 \text{ KSI}}$$

c) When $p_i = 54.39$ KSI total stress σ_r & σ_θ at inner surface of outer cylinder



Stresses at inner surface of outer cylinder considering only inner pr. = 54.39 KSI

So, for this case $r_i = 5$, $r_o = 11$, $r = 10$, $p_i = 54.39$ KSI, $p_o = 0$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

$$= \frac{5^2 \times 54.39}{11^2 - 5^2}\left(1 - \frac{11^2}{10^2}\right)$$

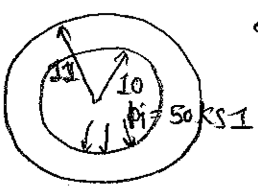
$$= -2.97 \text{ KSI}$$

$$\sigma_\theta = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$= \frac{5^2 \times 54.39}{11^2 - 5^2}\left(1 + \frac{11^2}{10^2}\right)$$

$$= 31.30 \text{ KSI}$$

Now, due to interference pressure $p = 4.75$ KSI in outer cylinder



$$\sigma_r = -p \qquad\qquad \sigma_\theta = 50 \text{ KSI}$$

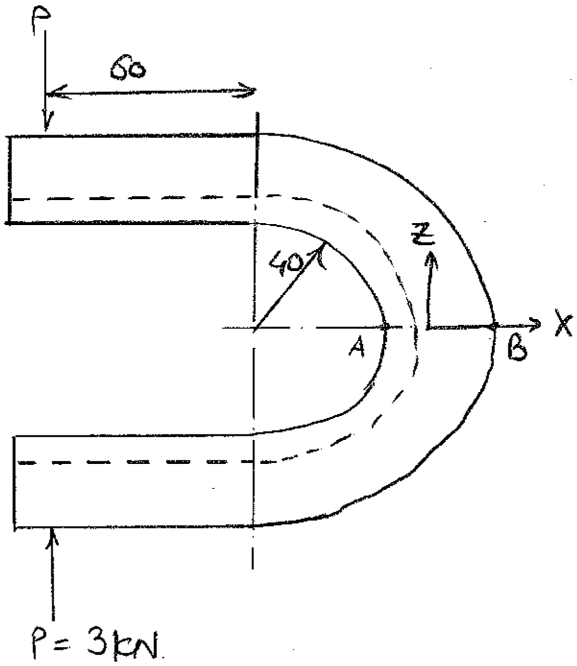
$$= -4.75 \text{ KSI}$$

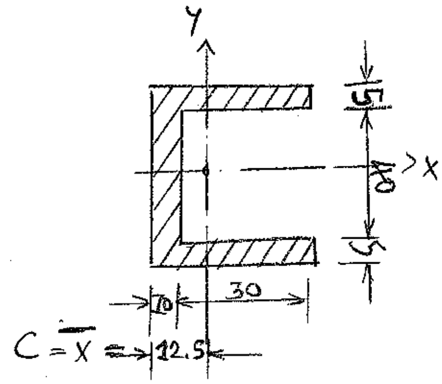
$\therefore$ Total Stresses

$$\sigma_\theta = 50 + 31.30 = 81.30 \text{ KSI}$$

$$\sigma_r = -4.75 - 2.97 = -7.72 \text{ KSI}$$

Soluⁿ #2





Section A-B

Calculate centroid $\bar{x}$

$$\bar{x} = \frac{\int x\,dA}{dA} = \frac{2\left(5\times40\times\frac{40}{2}\right) + \left(40\times10\times\frac{10}{2}\right)}{2(5\times40) + (40\times10)}$$

$$= 12.5 \text{ mm}$$

From table 5.3 of book page 228, for section AB

$$Z = -1 + \frac{R}{A}\left[t\cdot\ln(R+C_1) + (b-t)\ln(R-C_2) - b\ln(R-C)\right]$$

From the notations used in book

$R = 40 + 12.5 = 52.5$ mm $t = 10$ mm

$A = 800 \text{ mm}^2$ $C_1 = 40 - 12.5 = 27.5$ mm

$b = 40 + 2\times5 = 50$ mm $C_2 = 12.5 - 10 = 2.5$ mm

$C = \bar{x} = 12.5$ mm

$$\therefore Z = -1 + \frac{52.5}{800}\left[10\cdot\ln(52.5+27.5) + (50-10)\ln(52.5-2.5) - 50\ln(52.5-12.5)\right]$$

$$= 0.0406$$

∴ Tangential Stress at pt A.

$$\sigma\theta_A = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{C_A}{Z\,(R + C_A)}\right]$$

Where $P = -3$ kN

$M = 3\,(60 + 40 + 12.5) = 337.5$ N-m

$$\therefore \sigma\theta_A = -\frac{3000}{800\times10^{-6}} + \frac{337.5}{800\times10^{-6}\times52.5\times10^{-3}}\left[1 + \frac{-12.5}{0.0406\,(52.5 - 12.5)}\right]$$

$= -57.57\times10^{6}$ Pa

$= -57.57$ MPa

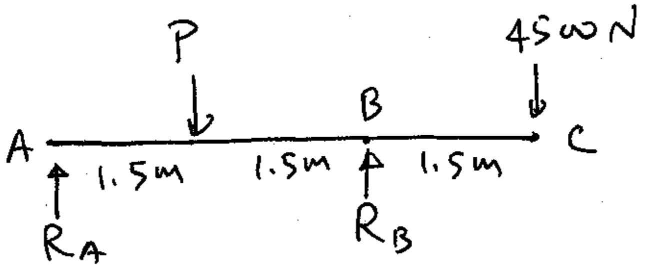
Similarly for pt B, $C_B = 40 - 12.5 = 27.5$

$$\therefore \sigma\theta_B = -\frac{3000}{800\times10^{-6}} + \frac{337.5}{800\times10^{-6}\times52.5\times10^{-3}}\left[1 + \frac{27.5}{0.0406\,(52.5 + 27.5)}\right]$$

$= 72.32\times10^{6}$ Pa

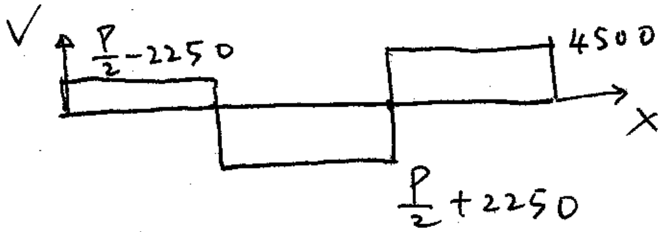
$= 72.32$ MPa

Problem 5.3

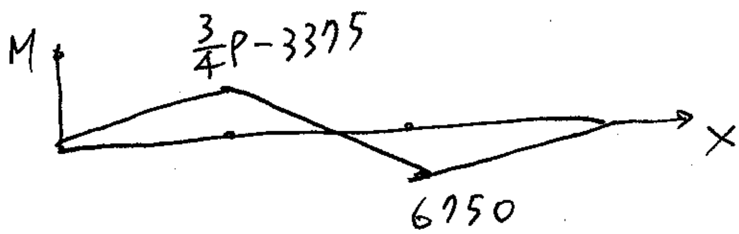


$R_A = \frac{P}{2} - 2250$ N

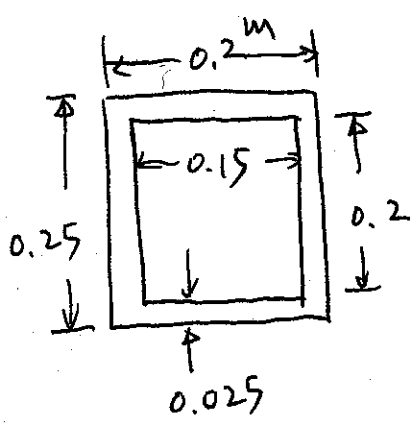
$R_B = \frac{P}{2} + 6750$ N



shear diagram



moment diagram



$I = \frac{1}{12}(0.2)(0.25)^3 - \frac{1}{12}0.15\,(0.2)^3$

$= 1.604 \times 10^{-4}\ m^4$

$\sigma_{max} = \frac{Mc}{I}$ $\quad c = 0.125$, $\sigma_{max} = 7$ MPa

$M = \frac{3}{4}P - 3375$

$P_{allow} = 16.478$ kN (based on σ_{max})

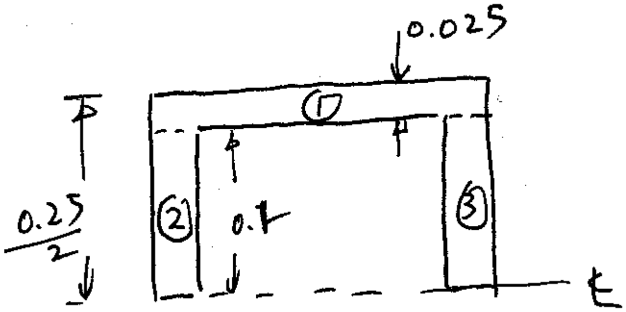
$$\tau_{max} = \frac{VQ}{Ib}$$

occurs at the central line

$$V = \frac{P}{2}\; 6750$$

$$b = 0.025 \times 2 \quad (2\ \text{arms})$$

$$I = 1.604 \times 10^{-4}$$



$$Q = Q^1 + Q^2 + Q^3$$

$$Q^1 = A^* y^* = (0.2 \times 0.025) \times \left(\frac{0.25}{2} - \frac{0.025}{2}\right)$$

$$= (0.2)(0.025)(0.1125) = 5.625 \times 10^{-4}$$

$$Q^2 = Q^3 = \underbrace{\left(\frac{0.25}{2} - 0.025\right)(0.025)}_{A^*} \underbrace{\frac{1}{2}\left(\frac{0.25}{2} - 0.025\right)}_{y^*} = (0.\cancel{1})(0.025)(0.\cancel{05}) =$$

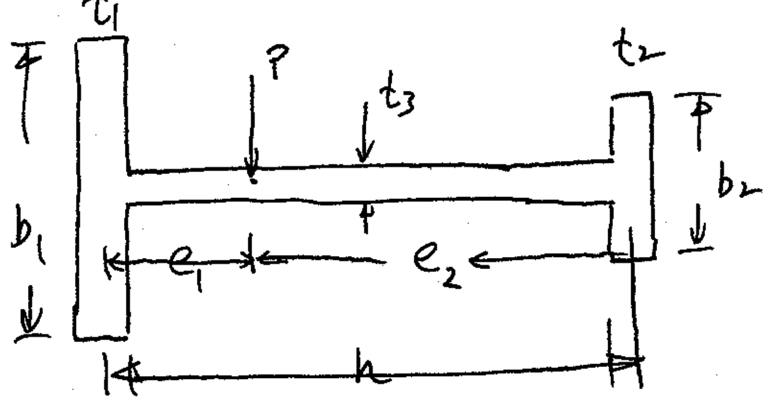
$$= (0.1)(0.025)\frac{1}{2}(0.1) = 1.25 \times 10^{-4}$$

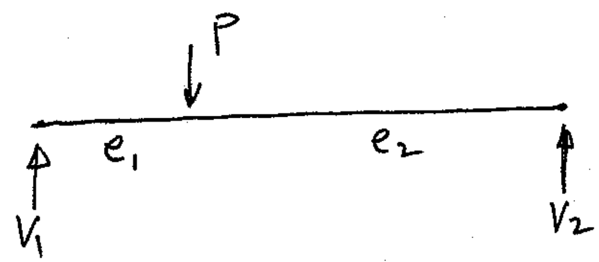
$$Q = (5.625 + 1.25 \times 2)\, 10^{-4} = 8.125 \times 10^{-4}$$

$$\tau_{max} = 0.7 \times 10^6 = \frac{\left(\frac{P}{2} + 6750\right) 8.125 \times 10^{-4}}{1.604 \times 10^{-4} \times 0.05}$$

$P_{all} = 9.327$ kN (based on shear)

Problem 5.4



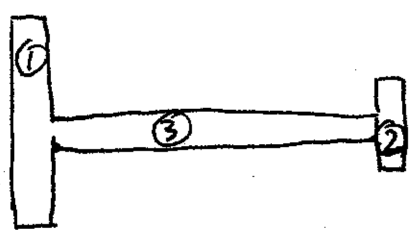


$$V_1 + V_2 = P$$

$$V_2 = \frac{P e_1}{e_1 + e_2}$$

$$V_1 = \frac{P e_2}{e_1 + e_2}$$

Note: actual V_1 & V_2 forces ~~are~~ points down.

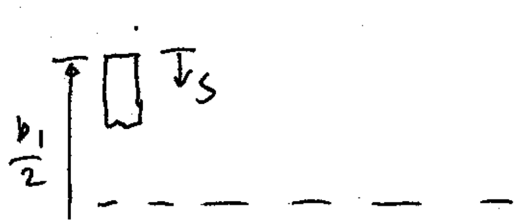


$$I_1 = \frac{1}{12} t_1 b_1^3 \qquad I_2 = \frac{1}{12} t_2 b_2^3$$

$$I_3 = \frac{1}{12}\left(h - \frac{t_1}{2} - \frac{t_2}{2}\right) t_3^3$$

$$I = I_1 + I_2 + I_3$$

subelement 1

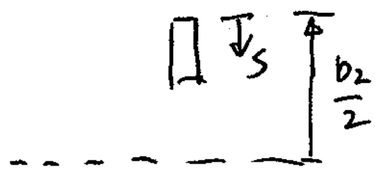


$$Q_1 = A^* y^* = s t_1 \left(\frac{b_1}{2} - \frac{s}{2}\right)$$

$$= \frac{1}{2}\left[-s^2 t_1 + s t_1 b_1\right]$$

$$q_1 = \frac{V_1}{I}\left(-\frac{s^2 t_1}{2} + s t_1 b_1\right)$$

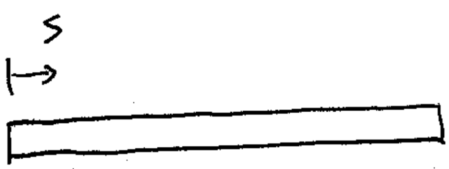
Subelement 2



$$q_2 = \frac{V_2}{I}\left(-\frac{s t_2}{2} + s t_2 b_2\right)$$

Subelement 3

$$Q = A^* y^* = s \cdot t_3 \cdot 0$$



$$q_3 = 0$$