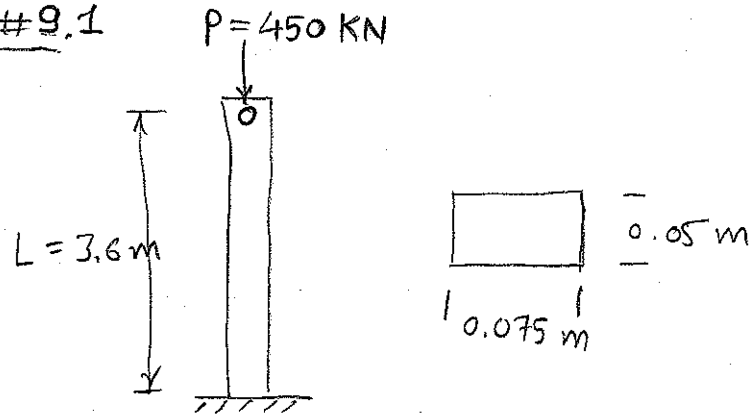
HW SOLUTION # 9

Solu^n # 9.1



$\sigma_{yp} = 280$ MPa

$E = 210$ GPa

a) $L_{eff} = 0.7 L$ (As one end fixed one end hinge-connection)

$= 0.7 \times 3.6$

$= 2.52$ m

$$I_{least} = \frac{1}{12} bh^3 = \frac{1}{12} \times 0.075 \times (0.05)^3$$

$$= 7.8125 \times 10^{-7} \ m^4$$

$$I = A \cdot r^2$$

$$\therefore r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7.8125 \times 10^{-7}}{0.075 \times 0.05}} = 0.0144$$

$$\left(\frac{L_{eff}}{r}\right) = \frac{2.52}{0.0144} = 174.59$$

$$\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{\pi^2 E}{0.5 \times \sigma_y}} = \sqrt{\frac{\pi^2 \times 210 \times 10^9}{0.5 \times 280 \times 10^6}} = 121.67$$

As, $\left(\frac{L_{eff}}{r}\right) > \left(\frac{L_{eff}}{r}\right)_c$ so long column.

As per Euler column theory, critical load

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{\pi^2 \times 210 \times 10^9 \times 7.8125 \times 10^{-7}}{(2.52)^2}$$

$$= 254.98 \ KN$$

As Factor of safety F.S. = 1.5

$\therefore$ Critical Load

$$P_{cr} = \frac{P_{cr}}{F.S.} = \frac{254.98}{1.5} = 169.98 \text{ kN}$$

$$\boxed{\therefore P_{cr} = 170 \text{ kN}}$$

Critical Euler stress

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{169.98}{0.05 \times 0.075} = \underline{\underline{45.32 \text{ MPa}}}$$

b) If length is reduced to 1.2 m

1st of all check slenderness ratio

$$L_{eff} = 0.7 \times 1.2 = 0.84 \text{ m}$$

$$\left(\frac{L_{eff}}{r}\right) = \frac{0.84}{0.0144} = 58.33 < \left(\frac{L_{eff}}{r}\right)_c = 121.67$$

So this is short column, and as per Jhonson short column theory

$$\sigma_{cr} = a - b\left(\frac{L_{eff}}{r}\right)^2$$

$$a = \sigma_y = 280 \times 10^6$$

$$b = \frac{\sigma_y}{2} \cdot \frac{1}{\left(\frac{L_{eff}}{r}\right)_c^2} = \frac{280 \times 10^6}{2} \times \frac{1}{(121.67)^2}$$

$$= 9457.17$$

$$\therefore \sigma_{cr} = 280 \times 10^6 - 9457.17\,(58.33)^2$$

$$= 247.82 \text{ MPa}$$

$$\therefore P_{cr} = A \cdot \sigma_{cr} = (0.05 \times 0.075)\, 247.82$$

$$\boxed{P_{cr} = 929. \text{ KN}}$$

Problem 9.2

Given: $E = 30$ Msi $\sigma_y = 36$ ksi $L_1 = 20'$ $K = 1$

$d_i = 3$ in $d_o = 4$ in $L_2 = 10'$

Solution

a) Slenderness ratio

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(4^4 - 3^4) = 8.59 \text{ in}^4$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(4^2 - 3^2) = 5.49 \text{ in}^2$$

$$\therefore r = \sqrt{\frac{I}{A}} = \sqrt{\frac{8.59}{5.49}} = 1.25 \text{ in}$$

$$\therefore \text{Slenderness ratio} = \frac{L_e}{r} = \frac{20 \times 12}{1.25} = 192$$

b) The critical buckling load:

To calculate critical buckling load, it is essential to determine if the column is long column or short column.

$$\left(\frac{L_e}{r}\right)_c = \sqrt{\frac{2E\pi^2}{\sigma_y}} = \sqrt{\frac{2 \times 30 \times 10^6 \times \pi^2}{36 \times 10^3}} = 128.25$$

$\because \frac{L_e}{r} > \left(\frac{L_e}{r}\right)_c \Rightarrow$ long column.

$$\therefore P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 30\times10^6 \times 8.59}{(20\times12)^2} = 44.15 \text{ kips}$$

c) Axial stress at buckling load :

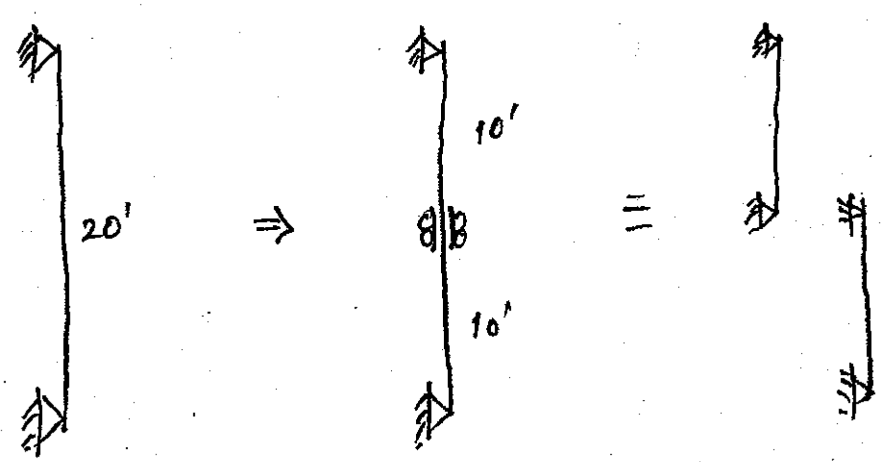
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{44.15}{5.49} = 8.04 \text{ ksi}$$

d) Buckling load w/ roller support at center

The roller support at center transforms a 20′ simply supported column into <u>TWO</u> 10′ simply supported columns.

The buckling load in such cases would be the least of buckling loads for newly transformed columns.

Here, both columns have same end conditions



$\therefore$ L_e for new column = 10'

$\therefore \quad \dfrac{L_e}{r} = \dfrac{10 \times 12}{1.25} = 96$

$\because \quad \dfrac{L_e}{r} < \left(\dfrac{L_e}{r}\right)_c \Rightarrow$ Short column.

$\therefore \quad \dfrac{P_{cr}}{A} = a - b\left[\dfrac{L_e}{r}\right]^2$

where

$a = \sigma_y = 36$ ksi

$b = \dfrac{\sigma_y^2}{4\pi^2 E} = \dfrac{(36 \times 10^3)^2}{4\pi^2 \times 30 \times 10^6} = 1.094 \times 10^{-3}$ ksi

$\therefore \quad P_{cr} = A\left[a - b\left(\dfrac{L_e}{r}\right)^2\right]$

$= 5.49\left[36 - 1.094 \times 10^{-3} \times 96^2\right]$

$= 142.28$ kips

Solu^n # 9.3 $P = 60$ KN , $\sigma_y = 252$ MPa , $E = 210$ GPa
$= 252\ N/mm^2$ $= 210\times10^3\ N/mm^2$

a) Length is 750 mm

As both ends have pinned connection $l_{eff} = 750$ mm

Assuming the column in long column, and as per Euler calum critical load

$$P_{cr} = \pi^2 \frac{EI}{L_{eff}^2}$$

$$60\times10^3 = \pi^2 \frac{210\times10^3 \times I}{(750)^2}$$

$$\therefore I = 16283.76\ mm^4$$

As, $I = \frac{\pi}{64} d^4 = 16283.76$

$$\therefore d = 23.99\ mm \approx 24\ mm$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}(24)^4}{\frac{\pi}{4}(24)^2}} = 6\ mm$$ (Radious of gyration)

$\therefore$ Slenderness Ratio $\left(\frac{L_{eff}}{r}\right) = \frac{750}{6} = 125$

Critical slenderness Ratio $\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{\pi^2 E}{0.5\sigma_y}}$

$$= \sqrt{\frac{\pi^2 \times 210\times10^3}{0.5\times 252}}$$

$$= 128.25$$

$\left(\frac{L_{eff}}{r}\right) < \left(\frac{L_{eff}}{r}\right)_c$ column is short column, As per Jhonson short column

$$\sigma_{cr} = a - b\left(\frac{L_{eff}}{r}\right)^2$$

$$a = \sigma_y = 252 \ N/mm^2$$

$$b = \frac{\sigma_y}{2} \times \frac{1}{\left(\frac{L_{eff}}{r}\right)_c^2} = \frac{\sigma_y}{2} \times \frac{1}{\frac{\pi^2 E}{0.5\sigma_y}} = \frac{\sigma_y^2}{4\pi^2 E}$$

$$= \frac{(252)^2}{4 \times \pi^2 \times 210 \times 10^3} = 7.66 \times 10^{-3}$$

$$\therefore \frac{P_{cr}}{\frac{\pi}{4}(d)^2} = 252 - 7.66 \times 10^{-3}\left(\frac{750}{r}\right)^2$$

Where r = Radius of gyration $= \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2}} = \sqrt{\frac{d^2}{16}}$

$$= \frac{d}{4}$$

$$\therefore \frac{60 \times 10^3}{\frac{\pi}{4}(d)^2} = 252 - 7.66 \times 10^{-3}\left(\frac{750}{d/4}\right)^2$$

$$d = 24.01 \ mm$$

$$\boxed{\therefore d = 25 \ mm}$$

As you can compare the diameter, by long column (23.99 mm) & by short column (24.01 mm), because we are in transition region, very close to critical slenderness ratio.

b) If length is 300 mm,

this column will also be short column as same material properties as above case

∵ By Johnson short column

$$\frac{P_{cr}}{A} = a - b\left(\frac{L_{eff}}{r}\right)^2$$

Substituting values of a, b, P_{cr} from above case a.

$$\frac{60 \times 10^3}{\frac{\pi}{4}(d)^2} = 252 - 7.66 \times 10^{-3}\left(\frac{300}{d/4}\right)^2$$

$d = 19.34$ mm ≈ 20 mm

$\boxed{d = 20\text{mm}}$