Solution 10.1

Given: $E = 30 \times 10^3$ ksi $\sigma_f' = 150$ ksi

$\varepsilon_f' = 1.0$ $b = -0.105$ $c = -0.640$

a) Transition fatigue life

$$2N_t = \left[\frac{E \cdot \varepsilon_f'}{\sigma_f'}\right]^{\frac{1}{b-c}}$$

$$= \left[\frac{30 \times 10^6 \times 1.0}{150}\right]^{\frac{1}{-0.105-(-0.640)}}$$

$$= 1.999 \times 10^4 \approx 2 \times 10^4 \text{ cycles.}$$

Strain amplitude at transition life

$$\left.\frac{\Delta\varepsilon}{2}\right|_{2N_f = 2N_t} = \varepsilon_f' \cdot (2N_t)^c$$

$$= 1 \cdot (2 \times 10^4)^{-0.640}$$

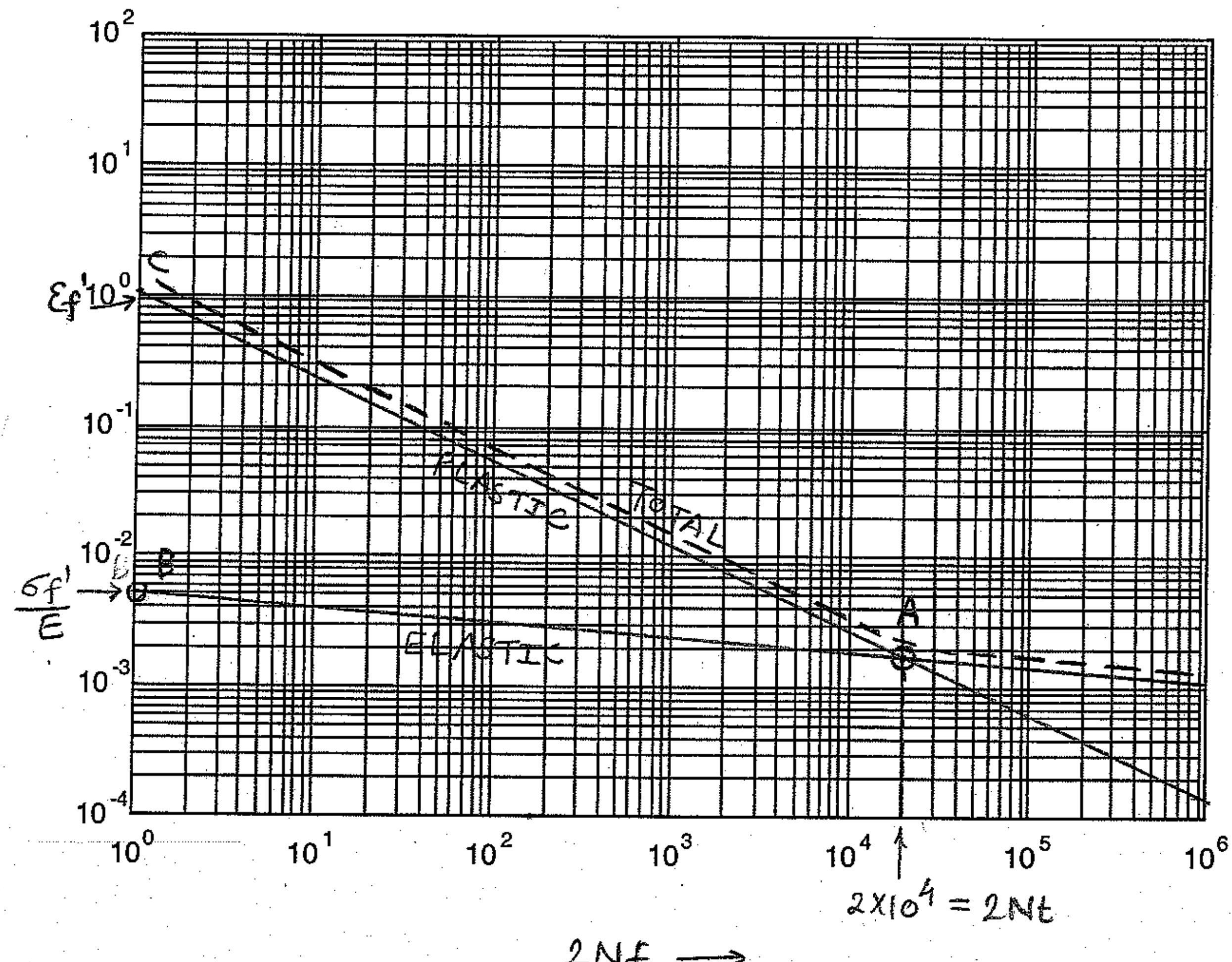
$$= 0.0018 \approx 1.8 \times 10^{-3}$$

To plot graph:

i) Point A $\left(2N_t, \left.\frac{\Delta\varepsilon}{2}\right|_{2N_t}\right) = (2 \times 10^4, 1.8 \times 10^{-3})$

ii) Point B $\left(0, \frac{\sigma_f'}{E}\right) = (0, 5 \times 10^{-3})$

iii) Point C $(0, \varepsilon_f') = (0, 1)$



10^2
10^1
ε_f' 10^0
C
10^{-1}
PLASTIC
TOTAL
10^{-2}
B
$\frac{\sigma_f'}{E}$
A
ELASTIC
10^{-3}
10^{-4}
10^0
10^1
10^2
10^3
10^4
10^5
10^6
$2\times10^4 = 2N_t$
$2N_f$

b) $2N_f = 2\times10^6$ rev.

$$\varepsilon_{Total} = \varepsilon_{Elastic} + \varepsilon_{plastic}$$

$$\Rightarrow \quad \varepsilon_E = \frac{\sigma_f'}{E}\cdot(2N_f)^b$$

$$= \frac{150}{30\times10^3}\cdot(2\times10^6)^{-0.105}$$

$$= 1.1\times10^{-3}$$

$$\varepsilon_p = \varepsilon_f'\cdot(2N_f)^c = 1\,(2\times10^6)^{-0.640}$$

$$= 9.2756\times10^{-5}$$

$$\varepsilon_T = \varepsilon_E + \varepsilon_p = 0.0012$$

c) $2N_f = 500$ rev.

$$\varepsilon_E = \frac{\sigma_f'}{E}(2N_f)^b = \frac{150}{30\times10^3}\cdot(500)^{-0.105}$$

$$= 0.0026$$

$$\varepsilon_p = \varepsilon_f'(2N_f)^c = 1\,(500)^{-0.640} = 0.0187$$

$$\varepsilon_T = \varepsilon_E + \varepsilon_p = 0.0213$$

d) Transition life as per part (a)

Solution 10.2

$S_{ult} = 100$ ksi $\quad S_e = 25$ ksi

$S_y = 80$ ksi $\quad \sigma_f = 130$ ksi

$\sigma_a = 16/d^2 \quad \sigma_m = 30/d^2$ in ksi.

[1] If d = 2 inches

$\sigma_a = 16/2^2 = 4$ ksi $\quad \sigma_m = 30/2^2 = 7.5$ ksi.

a) Goodman model

$$\frac{\sigma_a}{S_n} + \frac{\sigma_m}{S_{ult}} = 1$$

Where $S_n \rightarrow$ endurance stress for a given stresses

$$\therefore \frac{4}{S_n} + \frac{7.5}{100} = 1$$

$$S_n = 4.324 \text{ ksi.}$$

$$F.S. = \frac{S_e}{S_n} = \frac{25}{4.324} = 5.78$$

b) Gerber Model

$$\frac{\sigma_a}{S_n} + \left(\frac{\sigma_m}{S_{ult}}\right)^2 = 1 \quad \therefore \frac{4}{S_n} + \left(\frac{7.5}{100}\right)^2 = 1$$

$$\therefore S_n = 4.02 \text{ ksi.}$$

$$F.S. = \frac{S_e}{S_n} = \frac{25}{4.02} = 6.21$$

c) Morrow model

$$\frac{\sigma_a}{S_n} + \frac{\sigma_m}{\sigma_f} = 1 \quad \therefore \frac{4}{S_n} + \frac{7.5}{130} = 1$$

$$\therefore S_n = 4.24$$

$$F.S. = \frac{S_e}{S_n} = \frac{25}{4.24} = 5.89$$

[2] If $F.S. = 3$

d) Goodman Model

$$F.S. = \frac{S_e}{S_n} \quad \therefore 3 = \frac{25}{S_n} \qquad S_n = 8.33\ ksi$$

$$\therefore \frac{\sigma_a}{S_n} + \frac{\sigma_m}{S_{ult}} = 1$$

$$\frac{16}{d^2 \times 8.33} + \frac{30}{d^2 \times 100} = 1$$

$$d = 1.4\ in$$

e) Gerber Model

$$\frac{\sigma_a}{S_n} + \left(\frac{\sigma_m}{S_{ult}}\right)^2 = 1$$

$$\frac{16}{d^2 \times 8.33} + \left(\frac{30}{d^2 \times 100}\right)^2 = 1$$

$$\therefore d = 1.4\ in$$

f) Morrow model

$$\frac{\sigma_a}{S_n} + \frac{\sigma_m}{\sigma_f} = 1$$

$$\frac{16}{d^2 \times 8.33} + \frac{30}{d^2 \times 130} = 1$$

$$d = 1.47\ in$$

Problem 10.3

Given: $S_e = 12$ MPa $S_{ult} = 385$ MPa $N = 70{,}000$

Solution:

$$S_{1000} = 0.9 \times S_{ult}$$
$$= 0.9 \times 385 = 346.5 \text{ MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{S_{1000}}{S_e}\right)$$
$$= -\frac{1}{3}\log\left(\frac{346.5}{12}\right) = -0.486$$

$$c = \log\frac{(S_{1000})^2}{S_e}$$
$$= \log\frac{(346.5)^2}{12} = 4$$

$$a = 10^c = 10^4$$

$$S_n = a \cdot N^b \Rightarrow S_{70{,}000} = 10^4 \cdot (70{,}000)^{-0.486}$$
$$= 44.2 \text{ MPa}$$

Solution 10.4

$d = 32\ mm\ \phi$

AISI 1035 steel, machined finish & heat-treated

$S_{ult} = 710\ MPa$

$S_e' = 0.5\ S_{ult} = 0.5 \times 710 = 355\ MPa$

$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot K_e \cdot S_e'$

$\Rightarrow$ Where surface factor

$K_a = a\, S_u^{\,b}$

from table in notes, for machined component

$a = 4.51\ MPa \qquad b = -0.265$

$\therefore K_a = 4.51 \times (710)^{-0.265}$

$= 0.7918$

$\Rightarrow$ Size factor $K_b = 1$

(Assuming tensile loading)

$\Rightarrow$ $K_c = K_d = K_e = 1$

$\therefore S_e = K_a \cdot K_b\ S_e'$

$= 0.7918 \times 1 \times 355$

$= 281.07\ MPa$