Problem 11.1

Given: $2b = 12$ in $\quad E = 30 \times 10^6$ psi $\quad \nu = 0.3 \quad \sigma_f = 380$ ksi

Solution

a) The ultimate strength of plate with 8" crack size.

Fracture toughness is SIF at ultimate strength and is constant.

$$\Rightarrow \beta_1 \cdot \sigma_1 \cdot \sqrt{\pi a_1} = \beta_2 \cdot \sigma_2 \cdot \sqrt{\pi a_2}$$

$$\therefore \sigma_2 = \sigma_1 \cdot \frac{\beta_1}{\beta_2} \cdot \sqrt{\frac{a_1}{a_2}}$$

$$= \sigma_1 \cdot \left[\sqrt{\frac{2b}{\pi a_1} \cdot \tan\left(\frac{\pi a_1}{2b}\right)} \Big/ \sqrt{\frac{2b}{\pi a_2} \cdot \tan\left(\frac{\pi a_1}{2b}\right)} \right] \cdot \sqrt{\frac{a_1}{a_2}}$$

$$= 30 \left[\sqrt{\frac{12}{\pi \cdot 2} \tan\left(\frac{\pi \cdot 2}{12}\right)} \Big/ \sqrt{\frac{12}{\pi \cdot 4} \tan\left(\frac{\pi \cdot 4}{12}\right)} \right] \cdot \sqrt{\frac{2}{4}}$$

$$= \boxed{17.32} \text{ ksi}$$

b) Strain energy release rate (4 in, 30 ksi)

$$K_I = \beta \cdot \sigma \cdot \sqrt{\pi a}$$

$$\therefore \; K_I = \sqrt{\frac{2b}{\pi a} \cdot \tan\left(\frac{\pi a}{2b}\right)} \cdot \tau \cdot \sqrt{\pi a}$$

$$= \sqrt{\frac{12}{\pi . 2} \tan\left(\frac{\pi . 2}{12}\right)} \cdot 30 \cdot \sqrt{\pi . 2}$$

$$= 78.95 \text{ ksi} \sqrt{\text{in}}$$

Strain energy release rate,

$$G = \frac{(K_I)^2}{E}$$

$$= \frac{(78.95 \times 10^3)^2}{30 \times 10^6} = 207.8 \text{ psi}$$

c) Maximum crack opening displacement:

Max. displacement at $r = a = 2''$, $\theta = 180°$

$$u_y = \frac{K_I}{8\mu} \sqrt{\frac{2r}{\pi}} \cdot \left[(2k+1) \cdot \sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right]$$

$$\mu = \frac{E}{2(1+\nu)} = \frac{30 \times 10^6}{2(1+0.3)} = 11.54 \times 10^6 \text{ psi}$$

$$k = \frac{3-\nu}{1+\nu} = \frac{3-0.3}{1+0.3} = 2.08$$

Substituting, we get

$$u_y = 5.9 \times 10^{-3} \text{ in} \sim 0.006 \text{ in}$$

$$\therefore \; COD|_{max} = 2u_y = 0.012 \text{ in}$$

Problem 11.2

a) $$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]-\frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left[2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]+\frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

at "A", $r=1$ $\theta=\pi/2$

$$4000=\frac{K_I}{\sqrt{2\pi}}\cos\frac{\pi}{4}\left(1-\sin\frac{\pi}{2}\sin\frac{3\pi}{4}\right)-\frac{K_{II}}{\sqrt{2\pi}}\sin\frac{\pi}{4}\left(2+\cos\frac{\pi}{4}\cos\frac{3\pi}{4}\right)$$

$$=\frac{K_I-3K_{II}}{4\sqrt{\pi}} \qquad K_I-3K_{II}=16000\sqrt{\pi} \quad (1)$$

$$16000=\frac{K_I}{\sqrt{2\pi}}\frac{1}{\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\right)+\frac{K_{II}}{\sqrt{2\pi}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)$$

$$3K_I-K_{II}=64000\sqrt{\pi} \quad (2)$$

$$K_I = 39.0\,\text{ksi}\sqrt{\text{in}} = 22\sqrt{\pi}, \qquad K_{II}=3.55\,\text{ksi}\sqrt{\text{in}} = 2\sqrt{\pi} \quad \#$$

crack opening displacement at B, $r=1$. $\theta=\pi$

$$u_y=\frac{K_I}{8\mu}\sqrt{\frac{2r}{\pi}}\left[(2K+1)\sin\frac{\theta}{2}-\sin\frac{3\theta}{2}\right]+\frac{K_{II}}{8\mu}\sqrt{\frac{2r}{\pi}}\left[-(2K-3)\cos\frac{\theta}{2}-\cos\frac{3\theta}{2}\right]$$

$$K=\frac{3-\nu}{1+\nu}=2.08 \quad \text{plane stress}$$

$$G=\frac{30\times10^6}{2(1+0.3)}=11.54\times10^6\,\text{psi}$$

$$K_I=\sigma\sqrt{\pi a}=11\times10^3\sqrt{4\pi}=22\times10^3\sqrt{\pi}$$

Problem 10.2 (cont'd)

$$U_y\Big|_{r=1,\ \theta=\pi} = \frac{22\sqrt{\pi}\times10^3}{8\times11.54\times10^6}\sqrt{\frac{2}{\pi}}\left[\left(2(2.08)+1\right)-(-1)\right]+\cancel{K_{II}}\cos\frac{\theta}{2}\ldots$$

$$= 2.08\times10^{-3}\ in$$

crack opening displacement, $2U_y = 4.16\times10^{-3}$ in

b)

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2 \quad \text{for plane stress}$$

$$= \frac{1}{2\pi}\left(\frac{11\times10^3\sqrt{2\pi}}{47\times10^3}\right)^2 \qquad K_I = 11\times10^3\sqrt{2\pi}$$

$$= \frac{1}{2\pi}\cdot\left(\frac{11}{47}\right)^2\cdot 2\pi = 0.055\ in$$

c)

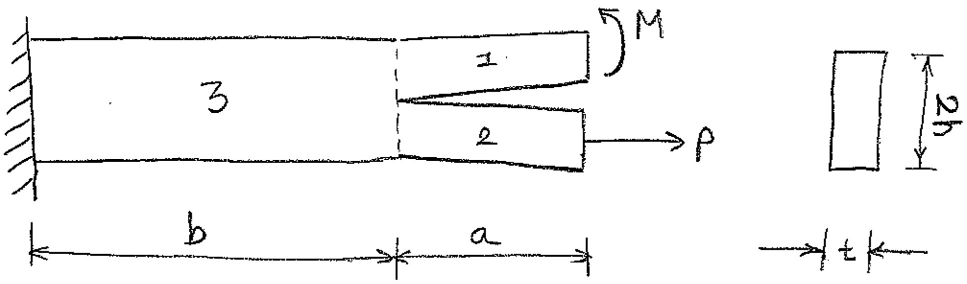
$$a = \frac{1}{\pi}\left(\frac{K_c}{\sigma_{max}}\right)^2 = \frac{1}{\pi}\cdot\left(\frac{110\times10^3}{55\times10^3}\right)^2 = 1.27\ in.$$

Maximum permissible crack length

$$= 2a = 2\times1.27 = 2.54\ in$$

Solution 11.3

a)



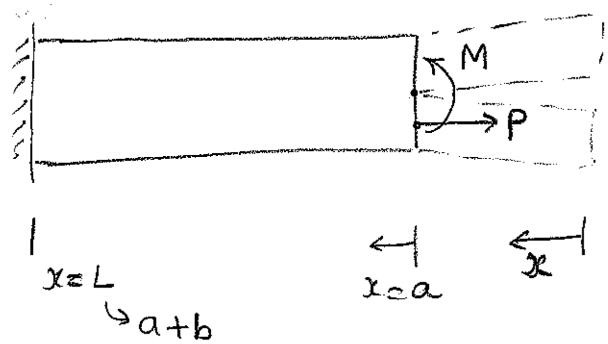
For beam 1 $$U_1 = \int_0^a \frac{M^2}{2EI_1}$$

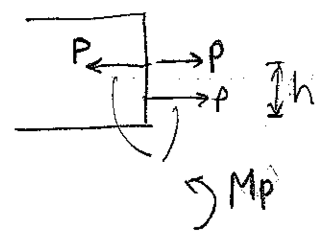
$$= \frac{M^2 a}{2E\left[\frac{1}{12}\cdot t\cdot (h)^3\right]}$$

$$= \frac{6M^2 a}{E t h^3}$$

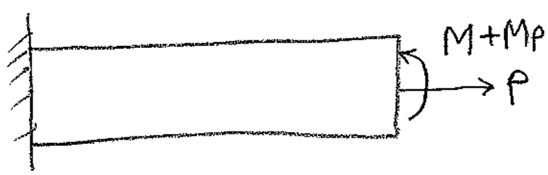
For beam 2 $$U_2 = \int_0^a \frac{P^2}{2EA_2}\,dx = \frac{P^2 a}{2E(t\cdot h)} = \frac{P^2 a}{2Eth}$$

For beam 3





$$\therefore M_P = \frac{Ph}{2}$$



$$U_3 = \int_a^L \left[\frac{(M+M_P)^2}{2E\left[\frac{1}{12}t(2h)^3\right]} + \frac{P^2}{2E(t\cdot 2h)}\right] dx$$

$$= \left[\frac{3(M+M_P)^2}{4Eth^3} + \frac{P^2}{4Eth}\right](L-a)$$

∴ Total Strain energy

$$U = U_1 + U_2 + U_3$$

$$= \frac{6M^2a}{Eth^3} + \frac{P^2a}{2Eth} + \frac{3\left(M+\frac{Ph}{2}\right)^2}{4Eth^3}(L-a) + \frac{P^2(L-a)}{4Eth}$$

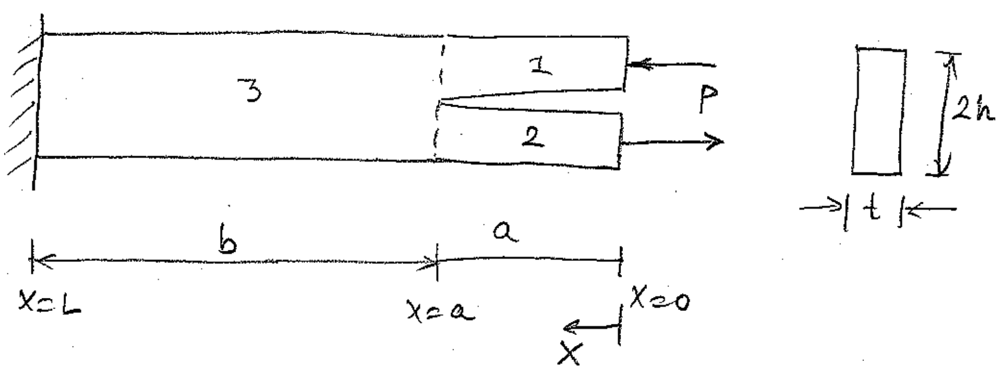
Strain energy release rate

$$G = \frac{dU_T}{dA} = \frac{dU_T}{t\,da}$$

$$= \frac{1}{t}\left[\frac{6M^2}{Eth^3} + \frac{P^2}{2Eth} + \frac{3\left(M+\frac{Ph}{2}\right)^2}{4Eth^2}(-1) + \frac{P^2(-1)}{4Eth}\right]$$

$$= \frac{1}{t}\left[\frac{6M^2}{Eth^3} + \frac{P^2}{2Eth} - \frac{3\left(M+\frac{Ph}{2}\right)^2}{4Eth^3}\right] = \frac{P^2}{16Eht^2} + \frac{21M^2}{4Et^2h^2} - \frac{3MP}{4Et^2h}$$

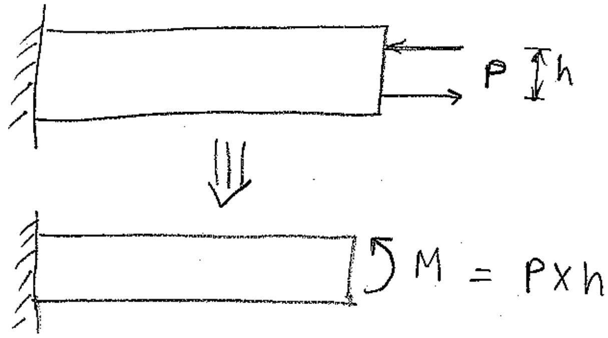
b)



For beam 1 $\quad U_1 = \int_0^a \frac{(-P)^2}{2EA_1}\,dx = \frac{P^2a}{2Eth}$

For beam 2 $\quad U_2 = \int_0^a \frac{P^2}{2EA_2}\,dx = \frac{P^2a}{2Eth}$

For beam 3



$$U_3 = \int_a^L \frac{M^2}{2EI}\,dx = \frac{M^2 (L-a)}{2E\left[\frac{1}{12}\,t\cdot(2h)^3\right]}$$

$$= \frac{3\,M^2(L-a)}{4Eth^3}$$

Total Strain energy $U_T = U_1 + U_2 + U_3$

$$= \frac{P^2 a}{Eth} + \frac{3M^2(L-a)}{4Eth^3}$$

Strain energy release rate

$$G = \frac{dU_T}{dA} = \frac{dU_T}{t\cdot da}$$

$$= \frac{1}{t}\left[\frac{P^2}{Eth} + \frac{3M^2(-1)}{4Eth^3}\right]$$

$$= \frac{1}{t}\left[\frac{P^2}{Eth} - \frac{3M^2}{4Eth^3}\right]$$

$$= \frac{1}{t}\left[\frac{P^2}{Eth} - \frac{3(Ph)^2}{4Eth^3}\right]$$

$$= \frac{1}{t}\left[\frac{P^2}{Eth} - \frac{3P^2}{4Eth}\right]$$

$$= \frac{P^2}{4Et^2h}$$