SOLUTION HW#12

Solution 12.1 :-

Infinite plate of unit thickness.

$2a_0 = 0.5''$

$K_{IC} = 80 \text{ KSI} \sqrt{in}$

$E = 10 \times 10^6$ psi. $\quad \nu = 0.3$

$\sigma_{max} = 60$ Ksi

$R = 0.1$

$R = \dfrac{\sigma_{min}}{\sigma_{max}} \quad \therefore \sigma_{min} = 6$

$$\frac{da}{dN} = 1.6 \times 10^{-12} (\Delta K)^4$$

a)

$$K_{IC} = \sigma_{max} \sqrt{\pi a_f}$$

$$80 = 60 \sqrt{\pi a_f}$$

$$\therefore a_f = 0.5658 \text{ in}$$

From fatigue crack growth

$$N_f = \frac{1}{C \pi^2 (\Delta\sigma)^4} \left[\frac{1}{a_0} - \frac{1}{a_f} \right]$$

$$= \frac{1}{1.6 \times 10^{-12} \times \pi^2 \times (60-6)^4} \left[\frac{1}{0.25} - \frac{1}{0.5658} \right]$$

$$= 16627 \text{ cycles.} \qquad N_f = 16,630 \text{ cycles}$$

b) Maximum crack opening displacement at time of failure Considering loading as mode I

$$u_y = \frac{K_I}{8\mu} \sqrt{\frac{2r}{\pi}} \left[(2k-1) \sin\frac{\theta}{2} - \sin\frac{3\theta}{2} \right]$$

Considering plane strain

$$\mu = \frac{E}{2(1+\nu)} = \frac{10 \times 10^6}{2(1+0.3)} = 3.846 \times 10^6$$

$$k = 3 - 4\nu = 3 - 4(0.3) = 1.8$$

$$\therefore u_y = \frac{80,000}{8 \times 3.846 \times 10^6} \sqrt{\frac{2 \times 0.5658}{\pi}} \left[(2 \times 1.8 - 1) \sin 90° - \sin 270° \right]$$

$$= 0.00874 \text{ in}$$

$\therefore$ Maximum Crack Opening displacement

$$= 2u_y = 0.0174 \text{ in}$$

c) Residual strength of plate after 4×10^3 fatigue life

$$N_1 = \frac{1}{C \times \pi^2 \times (\Delta\sigma)^4} \left[\frac{1}{a_0} - \frac{1}{a_1} \right]$$

$$4 \times 10^3 = \frac{1}{1.6 \times 10^{-12} \times \pi^2 \times (54)^4} \left[\frac{1}{0.25} - \frac{1}{a_1} \right]$$

$$a_1 = 0.2887 \text{ in.} \approx 0.289 \text{ in}$$

$\therefore$ Residual Strength $\sigma_c = \frac{K_{IC}}{\sqrt{\pi \cdot a_c}}$

$$= \frac{80000}{\sqrt{\pi \cdot 0.2887}} = 83991 \text{ psi.}$$

$$= 84.0 \text{ ksi}$$

d) By cracksize $2a_0 = 1$ in

i) $$N_f = \frac{1}{(1.6 \times 10^{-12}) \, \pi^2 \times (54)^4} \left[\frac{1}{0.5} - \frac{1}{0.5658} \right]$$

$$= 1732 \text{ cycles}$$

ii) Maximum crack opening displacement remain unchanged

iii) $N_1 = 4 \times 10^3$ cycles is greater than $N_f = 1732$ cycles so it will not be able to this loading cycle.

Solution 12.2 :

$S_{ult} = 150$ ksi

$S_e = 50$ ksi

$S_{1000} = 100$ ksi

$\sigma_f = 175$ ksi

a) For Load 1

$\sigma_{max} = 60$ ksi $\quad \sigma_{min} = 0 \quad n_1 = 50{,}000$ cycles

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 30 \text{ ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 30 \text{ ksi}$$

By using Goodman relationship

$$\frac{\sigma_a}{S_n} + \frac{\sigma_m}{S_{ult}} = 1$$

$$\frac{30}{S_n} + \frac{30}{150} = 1 \qquad S_n = 37.5 \text{ ksi}$$

Also, $S_n = a(N_1)^b$

$$a = \frac{S_{1000}^2}{S_e} = \frac{(100)^2}{50} = 200$$

$$b = -\frac{1}{3}\log\left(\frac{S_{1000}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{100}{50}\right) = -0.1003$$

$$\therefore \quad S_n = 37.5 \text{ ksi} = 200(N_1)^{-0.1003}$$

$\Rightarrow$ Cycles to fail by 1 $\rightarrow N_1 = 17710 \times 10^3$ cycles

For Load 2

$\sigma_{max} = 90$ ksi $\quad \sigma_{min} = 30$ ksi $\quad n_2 = 30{,}000$ cycles

$\sigma_a = 30$ ksi $\quad \sigma_m = 60$ ksi.

By goodman relationship

$$\frac{30}{S_n} + \frac{60}{150} = 1$$

$S_n = 50$ ksi.

$$S_n = a N_2^{\,b}$$

$$50 = 200 (N_2)^{-0.1003}$$

$$N_2 = 1006 \times 10^3 \text{ cycles}$$

Cycle to fail for segment 2.

b) Fatigue Damage at the end of 1st block of loading

$$D_1 = \frac{n_1}{N_1} + \frac{n_2}{N_2}$$

$$= \frac{50 \times 10^3}{17710 \times 10^3} + \frac{30 \times 10^3}{1006 \times 10^3}$$

$$= 0.0326$$

c) Fatigue life if block loading is repeated

Total fatigue life $N_f = \frac{1}{0.0326} = 30.675$ blocks.

That means 30 full blocks, and fraction of either block I or I, to find that

damage due to 0.675 block = 0.675 × 0.0326
= 0.022

Now, damage due to load 1 = $\frac{n_1}{N_1}$ = 2.823×10^{-3} ≈ 0.00282 } in a block
load 2 = $\frac{n_2}{N_2}$ = 0.0298

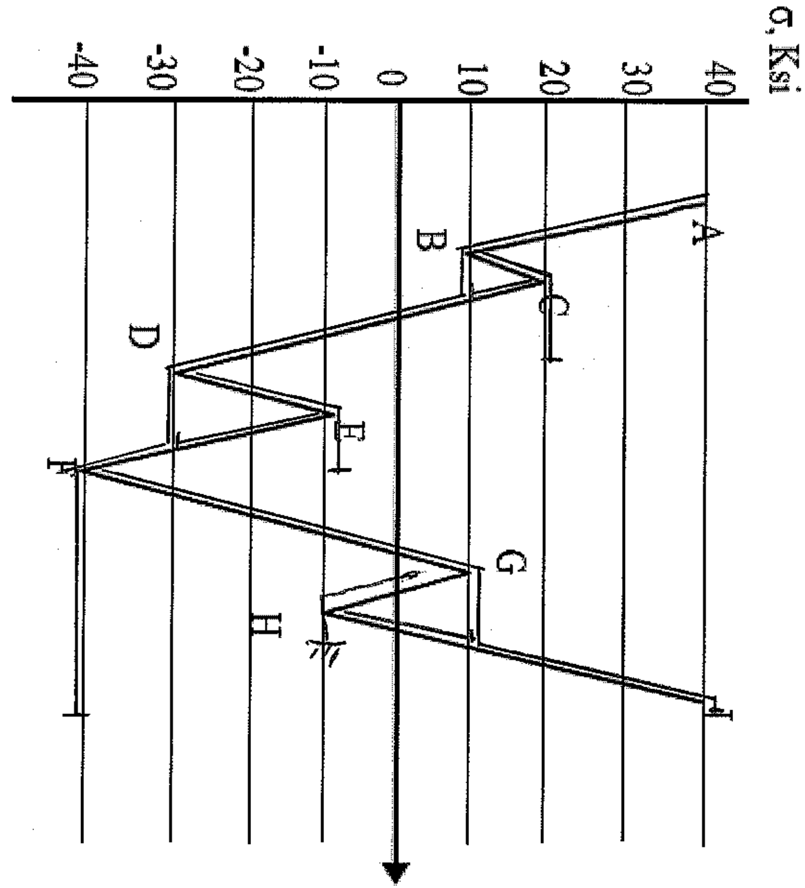
Damage in 31st block

Total		load 1		load 2
0.022	=	0.00282	+	$\frac{\text{No. of cycles}}{1006 \times 10^3}$

No. of cycles = 19291

∴ Fatigue life = 30 blocks + (50,000) cycles load 1 + (19291) cycles load 2
= 30 (50,000 + 30,000) + 50,000 + 19291
= 2469291 cycles.

Solution 12.3:- a) Rainflow Method



Cycle	Range
ABDF - FGI	80
BC - CB	10
DE - ED	20
GH - HG	20

b) Peak Couting Method

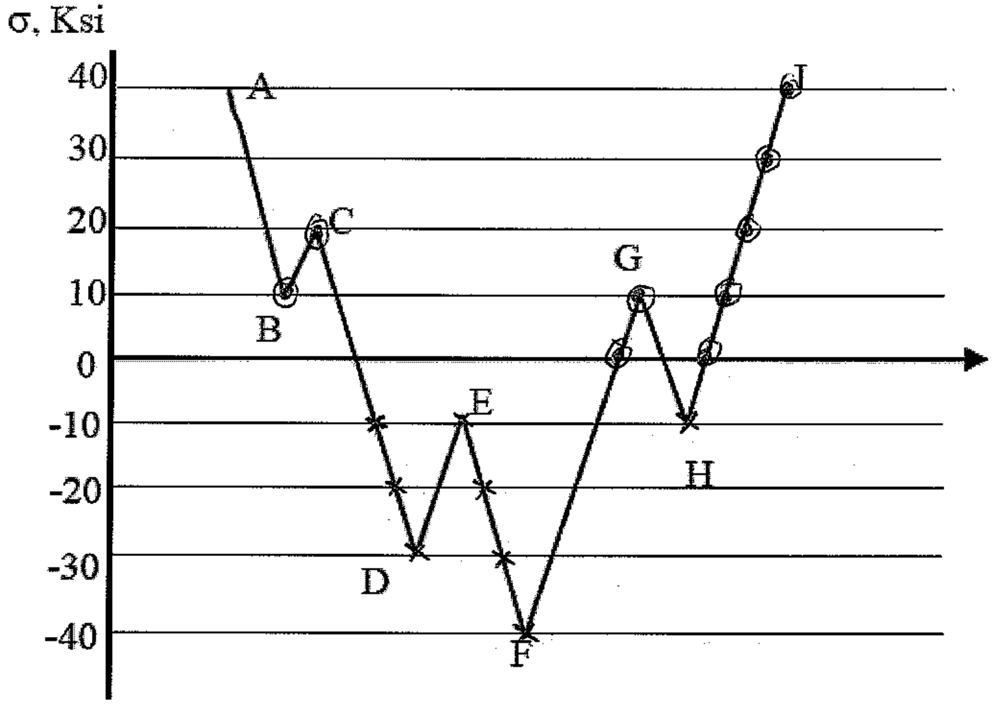
	Level Range
AF	80
CD	50
GE	20

c) Simple Range Couting

	Strain Level	Cycle Counts
AB	30	0.5
BC	10	0.5
CD	50	0.5
DE	20	0.5
EF	30	0.5
FG	50	0.5
GH	20	0.5
HI	50	0.5

Range (Levels)	Cycle Counts
50	1.5
30	1.0
20	1.0
10	0.5

d) Level Crossing Method



Level	Counts	Revised Count	Revised Count
40	1	0	–
30	1	0	–
20	2	1	0
10	3	2	1
0	2	1	0
−10	3	2	1
−20	2	1	0
−30	2	1	0
−40	1	0	–

Pairing level	Counts	Range Level
40 & −40	1	80
20 & −30	1	50
10 & −10	1	20