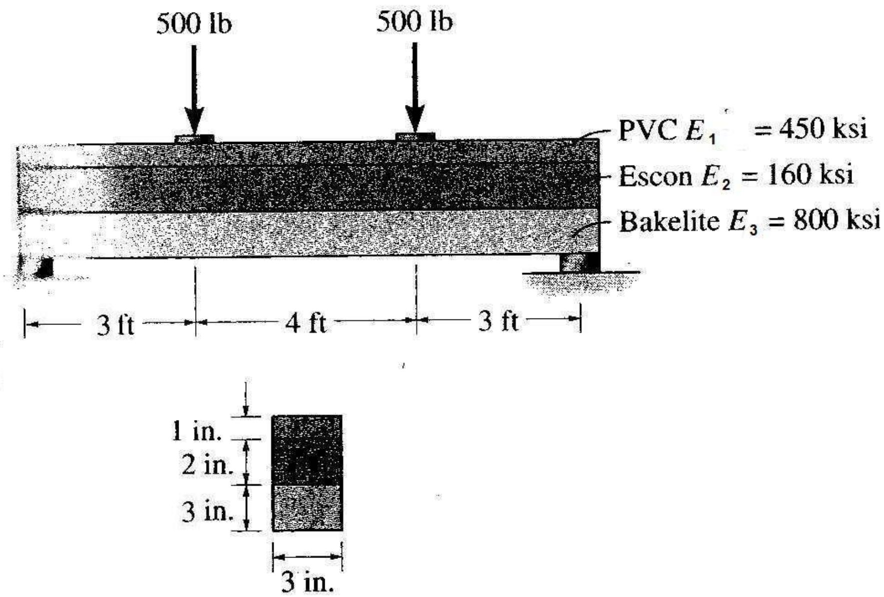
MAE5339

SUPPLEMENTS 5-1

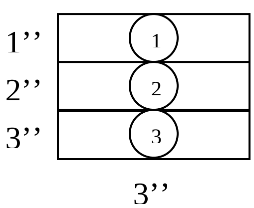
Problem 5-1

The beam is made from three types of plastic that are identified and have the modulus of elasticity shown in the figure. Determine the maximum bending stress in the PVC.



Solution:

Find an equivalent bending rigidity for a beam with multiple materials.



$$E_1(PVC) = 450ksi \qquad A_1 = 3in^2$$

$$E_2(Escon) = 160ksi \qquad A_2 = 6in^2$$

$$E_3(Bakeline) = 800ksi \qquad A_3 = 9in^2$$

Take Bakeline as a reference material.

$$A_1^* = \frac{E_1}{E_o} A_1 = \frac{450ksi}{800ksi} \cdot 3in^2 = 1.6875in^2$$

$$A_2^* = \frac{E_2}{E_o} A_2 = \frac{160ksi}{800ksi} \cdot 6in^2 = 1.2in^2$$

$$A_3^* = A_3 = 9in^2$$

$$\sum_i A_i^* = 1.6875in^2 + 1.2in^2 + 9in^2 = 11.8875in^2$$

$$\bar{y}^* = \frac{\sum A_i^* y_i}{\sum A_i^*} = \frac{\left(1.6875in^2\right)\left(3in + 2in + \frac{1}{2}in\right) + \left(1.2in^2\right)\left(3in + \frac{2}{2}in\right) + \left(9in^2\right)\left(\frac{3}{2}in\right)}{11.8875in^2} = 2.320in$$

$$\bar{z}^* = 1.5in$$

$$I_{z1}^* = \frac{E_1}{E_o} I_1 = \frac{E_1}{E_o}\left(I_{o1} + A_1 d_1^2\right) = \frac{450ksi}{800ksi}\left(\frac{1}{12}(3in)(1in)^3 + (1in)(3in)(5.5in - 2.32in)^2\right) = 17.21in^4$$

$$I_{z2}^* = \frac{E_2}{E_o} I_2 = \frac{E_2}{E_o}\left(I_{o2} + A_2 d_2^2\right) = \frac{160ksi}{800ksi}\left(\frac{1}{12}(3in)(2in)^3 + (2in)(3in)(5.5in - 4in)^2\right) = 3.1in^4$$

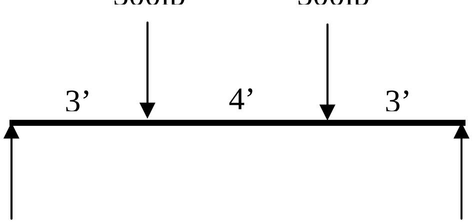
$$I_{z2}^* = \frac{E_3}{E_o} I_3 = \frac{E_3}{E_o}\left(I_{o3} + A_3 d_3^2\right) = \frac{800ksi}{800ksi}\left(\frac{1}{12}(3in)(3in)^3 + (3in)(3in)(5.5in - 1.5in)^2\right) = 150.75in^4$$

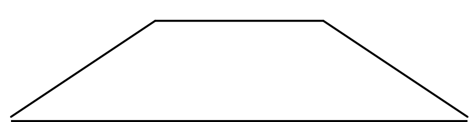
Finally,

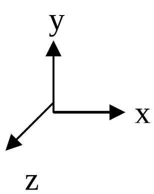
$$I^* = \sum I_{zi}^* = 17.21in^4 + 3.1in^4 + 150.75in^4 = 171.06in^4$$

Moment diagram,

$$M_z = +15{,}000 lb \cdot in$$







$$\sigma_x = \left(k_y^* M_z + \cancelto{0}{k_{yz}^* M_y}\right)y - \left(\cancelto{0}{k_z^* M_y} + \cancelto{0}{k_{yz}^* M_z}\right)z$$

$$k_y^* = \frac{I_y^*}{\left(I_{yz}^*\right)^2 - I_y^* I_z^*} = -\frac{1}{I_z^*}$$

Therefore,

$$\sigma_x = -\frac{M_z}{I_z^*} y$$

where, $y = -2.32in$. As a result,

$$\sigma_x = \frac{\left(-15{,}000 lb \cdot in\right)}{171.06 in^4}\left(-2.32 in\right) = 203 psi$$