ME 5339

Supplement Notes

Mid-Term Review

Example 1

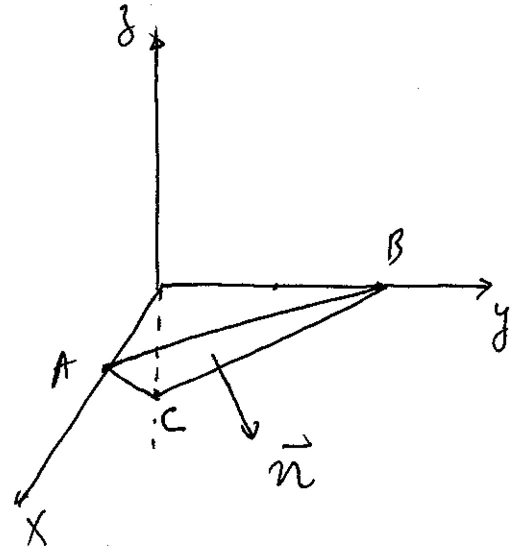
Given

$$[\sigma] = \begin{bmatrix} -10 & 20 & 30 \\ 20 & 10 & -20 \\ 30 & -20 & 40 \end{bmatrix} \text{MPa}$$

i) Evaluate the normal and shear stresses on a surface at the point where the surface is given by the three points $(1,0,0), (0,2,0), (0,0,-1)$

ii) Determine the directional cosine for the shear stress found in part i)

Soln:



$$[P] = [\sigma][n]$$

$$\sigma_n = \vec{P} \cdot \vec{n} \qquad \vec{P} = \sigma_n \vec{n} + \tau_n \vec{s}$$

$$\|P\|^2 = \|\sigma_n\|^2 + \|\tau_n\|^2$$

$$\vec{n}\|A\| = \frac{1}{2}(\vec{AB} \times \vec{AC})$$

$$\vec{AB} = (0,2,0) - (1,0,0)$$
$$= -i + 2j$$

$$\vec{AC} = (0,0,-1) - (1,0,0) = -i - k$$

$$\vec{n}\|\Delta\| = \frac{1}{2}\begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ -1 & 0 & -1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} i - \frac{1}{2} j \begin{vmatrix} -1 & 0 \\ -1 & -1 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix} k$$
$$= -i - \frac{1}{2}j + k$$

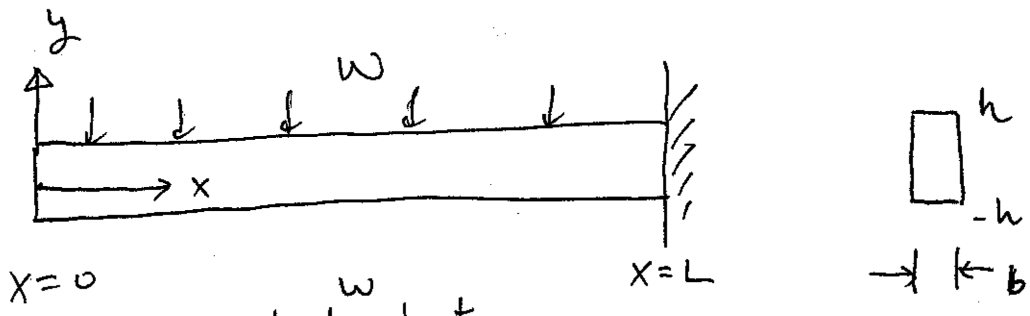
$$\text{unit vector} = \frac{-i - \frac{1}{2}j + k}{\sqrt{(-1)^2 + (-\frac{1}{2})^2 + 1^2}} = \frac{-i - \frac{1}{2}j + k}{\frac{3}{2}}$$

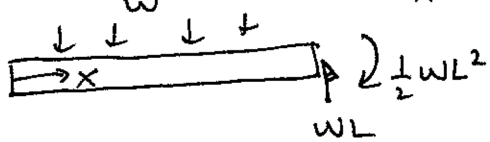
$$\vec{n} = \left(-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$$

$$[P] = \begin{pmatrix} -10 & 20 & 30 \\ 20 & 10 & -20 \\ 30 & -20 & 40 \end{pmatrix} \begin{pmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{pmatrix} =$$

$$\sigma_n = 5.56 \text{ MPa}$$

Example 2





B.C.

i) $\sigma_x = 0$, $\tau_{xy} = 0$ at $x = 0$

$$\int_{-h}^{h} \sigma_x \Big|_{x=0} b\,dy = 0 \quad \text{or} \quad \int_{-h}^{h} \sigma_x \Big|_{x=0} y\,b\,dy = 0 \ (\text{moment})$$

ii)

$$\int_{-h}^{h} \tau_{xy} \Big|_{x=L} b\,dy = wL \qquad x = L$$

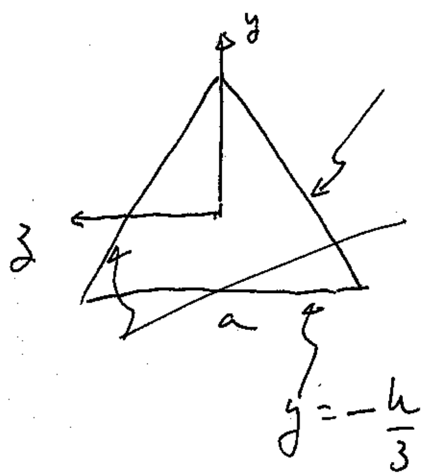
$$\int_{-h}^{h} \sigma_x \Big|_{x=L} b\,dy = 0 \qquad \int_{-h}^{h} \sigma_x \Big|_{x=L} y\,b\,dy = \frac{1}{2} w L^2$$

iii) $\sigma_y = -\frac{w}{b}$, $\tau_{xy} = 0$ at $y = h$

$\sigma_y = 0$ $\quad \tau_{xy} = 0$ at $y = -h$

Ex. 3

A bar with equilateral cross-section as shown is subjected to a T. Find shear stress



$$y = \frac{2h}{3a}(3z+a)$$

$$y = -\frac{2h}{3a}(3z-a)$$

$$h = \frac{\sqrt{3}}{2}a$$

$$\phi = k\left[y - \frac{2h}{3a}(3z+a)\right]\left[y + \frac{2h}{3a}(3z-a)\right]\left[y+\frac{h}{3}\right]$$

$$T = 2\iint \Phi \, dy\, dz$$

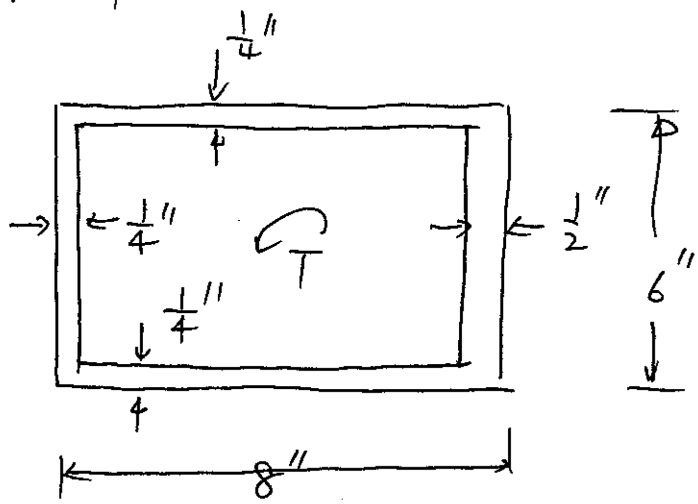
$$= 2\int_{-\frac{h}{3}}^{\frac{2h}{3}} \int_{\frac{a}{6h}(3y-2h)}^{-\frac{a}{6h}(3y-2h)} \phi \, dz\, dy$$

$$k = \frac{80T}{3a^5}$$

$$\tau_{xy} = \frac{\partial\phi}{\partial z}$$

$$\tau_{xy} = -\frac{\partial\phi}{\partial y}$$

Ex. 4



T = 50 kip-in

Find shear stress

J

$$T = 2\bar{A}q = 2\bar{A}\,\tau \cdot t \qquad q = \frac{T}{2\bar{A}} = \frac{50\times10^3}{2\,(43.8)}$$

$$\bar{A}_m = \left(8 - \tfrac{1}{8} - \tfrac{1}{4}\right)\left(6 - \tfrac{1}{8} - \tfrac{1}{8}\right) = 43.8\ in^2$$

$$\tau_{\frac{1}{2}''} = q \cdot \frac{1}{t_{\frac{1}{2}}} = 1140\ psi$$

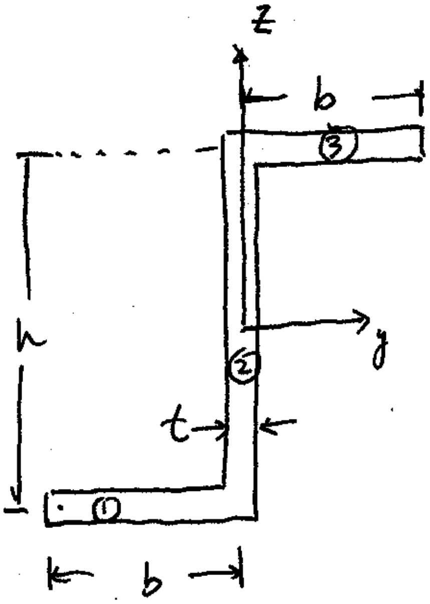
$$\tau_{\frac{1}{4}} = q \cdot \frac{1}{t_{\frac{1}{4}}} = 2280\ psi$$

$$J = \frac{4\bar{A}_m^2}{\oint \frac{ds}{t}} \qquad \oint \frac{ds}{t} = \frac{\left(8 - \frac{1}{8} - \frac{1}{4}\right)\cdot 2}{\frac{1}{4}} + \frac{\left(6 - \frac{1}{8} - \frac{1}{8}\right)}{\frac{1}{4}} + \frac{\left(6 - \frac{1}{8} - \frac{1}{8}\right)}{\frac{1}{2}} = 95.5$$

$$= \frac{4\times(43.8)^2}{95.5}$$

Ex.

A thin-walled Z-section shown below is subjected to a bending moment, M_y. Find the distribution of bending stress



$$I_y = (I_0 + Ad^2)_1 = \frac{bt^3}{12} + (bt)\left(\frac{h}{2}\right)^2$$

$$+ (I_0)_2 \qquad \frac{th^3}{12} +$$

$$+ (I_0 + Ad^2)_2 \qquad (bt)\left(\frac{h}{2}\right)^2$$

$$I_z = \left[\frac{tb^3}{12} + (2bt)\left(\frac{b}{2}\right)^2\right] + \frac{ht^3}{12} + \left[\frac{tb^3}{12} + (2bt)\left(\frac{b}{2}\right)^2\right]$$

$$I_{yz} = I_{yz}^{0} + Ad_y d_z$$

$$= (I_{yz}^{0} + Ad_y d_z)_1 + (0) + (I_{yz}^{0} + Ad_y d_z)_3$$

$$= bt\left(\frac{-b}{2}\right)\left(\frac{-h}{2}\right) + bt\left(+\frac{b}{2}\right)\left(+\frac{h}{2}\right) = \frac{b^2 t h}{2}$$