COURSE #: MAE 3315
PROFESSOR: DR. CHAN
FILE'S NAME: SOLUTION #1

HOMEWORK #1

Problem #1:

Part A) Equal bending stiffness:

bending stiffness is proportional to Eh^3

$$\text{bending stiffness} \propto Eh^3$$

$$h \propto \left(\sqrt[3]{\frac{\text{bending stiffness}}{\text{E}}}\right)$$

$$h \propto \left(\sqrt[3]{\text{bending stiffness}}\right)\left(\sqrt[3]{\frac{1}{E}}\right)$$

but as we know the bending stiffness is equal for all the beams; therefore, $\boxed{h \propto \sqrt[3]{\frac{1}{E}}}$

On the other hand, the weight of each beam can be related to its own density and volume.

$$weight = \rho \cdot Volume$$
$$weight = \rho \cdot (L \cdot b \cdot h)$$

but we assumed that all the beams have the same length (L) and width (b); therefore, the weight is proportional to the density (ρ) times the height (h).

$$\boxed{weight \propto \rho \cdot h}$$

Aluminum Beam:

$$h_{al} \propto \sqrt[3]{\frac{1}{E_{al}}} = \sqrt[3]{\frac{1}{71x10^9\,Pa}} = \boxed{2.4150x10^{-4}\,m/\sqrt{N}}$$

$$weight_{al} \propto \rho_{al} \cdot h_{al} = (2.78g/cm^3)(2.4150x10^{-3}cm) = \boxed{6.7136x10^{-3}\,g/cm^2}$$

Titanium Beam:

$$h_{Ti} \propto \sqrt[3]{\frac{1}{E_{Ti}}} = \sqrt[3]{\frac{1}{110x10^9\,Pa}} = \boxed{2.0871x10^{-4}\,m/\sqrt{N}}$$

$$weight_{Ti} \propto \rho_{Ti} \cdot h_{Ti} = (4.46g/cm^3)(2.0871x10^{-3}cm) = \boxed{9.3083x10^{-3}\,g/cm^2}$$

Steel Beam:

$$h_{st} \propto \sqrt[3]{\frac{1}{E_{st}}} = \sqrt[3]{\frac{1}{200x10^9\, Pa}} = \boxed{1.7100x10^{-4}\, m/\sqrt{N}}$$

$$weight_{st} \propto \rho_{st} \cdot h_{st} = (7.8 g/cm^3)(1.7100x10^{-3}\, cm) = \boxed{1.3338x10^{-2}\, g/cm^2}$$

Composite Beam:

$$h_c \propto \sqrt[3]{\frac{1}{E_c}} = \sqrt[3]{\frac{1}{140x10^9\, Pa}} = \boxed{1.9259x10^{-4}\, m/\sqrt{N}}$$

$$weight_c \propto \rho_c \cdot h_c = (1.55 g/cm^3)(1.9259x10^{-3}\, cm) = \boxed{2.9851x10^{-3}\, g/cm^2}$$

Finally, comparing all the beams with the aluminum beam,

	Aluminum	Titanium	Steel	Composite
H/Hal	1	0.86	0.71	0.80
WEIGTH/WEIGTHal	1	1.39	1.99	0.44

Part B) Equal bending moment:

$$\text{bending moment} \propto S_u h^2$$

$$h \propto \sqrt{\frac{\text{bending moment}}{S_u}}$$

$$h \propto \left(\sqrt{\text{bending momen}}\right)\left(\sqrt{\frac{1}{S_u}}\right)$$

but as we know the bending moment is equal for all the beams; therefore, $\boxed{h \propto \sqrt{\frac{1}{S_u}}}$

Aluminum Beam:

$$h_{al} \propto \sqrt{\frac{1}{Su_{al}}} = \sqrt{\frac{1}{538x10^6\, Pa}} = \boxed{4.3113x10^{-5}\, m/\sqrt{N}}$$

$$weight_{al} \propto \rho_{al} \cdot h_{al} = (2.78 g/cm^3)(4.3113x10^{-4}\, cm) = \boxed{1.1985x10^{-3}\, g/cm^2}$$

Titanium Beam:

$$h_{Ti} \propto \sqrt{\frac{1}{Su_{Ti}}} = \sqrt{\frac{1}{925x10^6\,Pa}} = \boxed{3.2880x10^{-5}\,m/\sqrt{N}}$$

$$weight_{Ti} \propto \rho_{Ti} \cdot h_{Ti} = \left(4.46g/cm^3\right)\left(3.2880x10^{-4}\,cm\right) = \boxed{1.4664x10^{-3}\,g/cm^2}$$

Steel Beam:

$$h_{st} \propto \sqrt{\frac{1}{Su_{st}}} = \sqrt{\frac{1}{1860x10^6\,Pa}} = \boxed{2.3187x10^{-5}\,m/\sqrt{N}}$$

$$weight_{st} \propto \rho_{st} \cdot h_{st} = \left(7.8g/cm^3\right)\left(2.3187x10^{-4}\,cm\right) = \boxed{1.8086x10^{-3}\,g/cm^2}$$

Composite Beam:

$$h_c \propto \sqrt{\frac{1}{Su_c}} = \sqrt{\frac{1}{2.10x10^9\,Pa}} = \boxed{2.1822x10^{-5}\,m/\sqrt{N}}$$

$$weight_c \propto \rho_c \cdot h_c = \left(1.55g/cm^3\right)\left(2.1822x10^{-4}\,cm\right) = \boxed{3.3824x10^{-4}\,g/cm^2}$$

Now, comparing all the beams with the aluminum beam,

	Aluminum	Titanium	Steel	Composite
H/Hal	1	0.76	0.54	0.51
WEIGTH/WEIGTHal	1	1.22	1.51	0.28

Problem #2:

Steel I-beam (300M):

$$\begin{cases} E = 200x10^9\,Pa \\ \rho = 7.8g/cm^3 \end{cases}$$

$$I = \frac{1}{12}(t)(h)^3 + 2\left[\frac{1}{12}(b)(t)^3 + Area(\bar{y})^2\right]$$

$$I = \frac{1}{12}(t)(h)^3 + 2\left[\frac{1}{12}(b)(t)^3 + (b\cdot t)\left(\frac{h}{2}+\frac{t}{2}\right)^2\right]$$

$$I = \frac{1}{12}(0.005m)(0.2m)^3 + 2\left[\frac{1}{12}(0.05m)(0.005m)^3 + (0.05m)(0.005m)\left(\frac{0.2m}{2}+\frac{0.005m}{2}\right)^2\right]$$

$$\boxed{I_{st} = 8.5875x10^{-6}\,m_4}$$

Bending Stiffness:

$$EI = \left(200x10^9\,Pa\right)\left(8.5875x10^{-6}\,m^4\right)$$

$$\boxed{(EI)_{st} = 1.7175x10^6\,Nm^2}$$

Weight per unit length:

$$\left(\frac{weight}{L}\right) = \left(\frac{\rho\cdot Volume}{L}\right) = \left(\frac{\rho\cdot Area\cdot L}{L}\right)_{st} = \rho\cdot Area$$

$$\left(\frac{weight}{L}\right)_{st} = \rho_{st}A_{st} = \rho_{st}\left[(t\cdot h) + 2(b\cdot t)\right]_{st}$$

$$\left(\frac{weight}{L}\right)_{st} = 7.8g/cm^3\left[(0.5cm)(20cm) + 2(5cm)(0.5cm)\right]$$

$$\boxed{\left(\frac{weight}{L}\right)_{st} = 117g/cm}$$

Aluminum I-beam (7075-T6):

$$\begin{cases} E = 71x10^9\,Pa \\ \rho = 2.78\,g/cm^3 \end{cases}$$

But as we know, the bending stiffness of the aluminum and the steel I-beams should be the same. Therefore,

$$(EI)_{al} = (EI)_{st}$$

$$E_{al} I_{al} = (EI)_{st}$$

$$E_{al} \cdot \left\{ \frac{1}{12}(t)(h)^3 + 2\left[\frac{1}{12}(b)(t)^3 + (b \cdot t)\left(\frac{h}{2}+\frac{t}{2}\right)^2\right]\right\}_{al} = (EI)_{st}$$

$$\frac{1}{12}(t)(h)^3 + 2\left[\frac{1}{12}(b)(t)^3 + (b \cdot t)\left(\frac{h}{2}+\frac{t}{2}\right)^2\right] = \frac{(EI)_{st}}{E_{al}}$$

$$b\left\{2\left[\frac{1}{12}(t)^3 + (t)\left(\frac{h}{2}+\frac{t}{2}\right)^2\right]\right\} = \frac{(EI)_{st}}{E_{al}} - \frac{1}{12}(t)(h)^3$$

$$b_{al} = \frac{\dfrac{(EI)_{st}}{E_{al}} - \dfrac{1}{12}(t)(h)^3}{2\left[\dfrac{1}{12}(t)^3 + (t)\left(\dfrac{h}{2}+\dfrac{t}{2}\right)^2\right]}$$

$$b_{al} = \frac{\dfrac{1.7175x10^6\,Nm^2}{71x10^9\,Pa} - \dfrac{1}{12}(0.005m)(0.2m)^3}{2\left[\dfrac{1}{12}(0.005m)^3 + (0.005m)\left(\dfrac{0.2m}{2}+\dfrac{0.005m}{2}\right)^2\right]}$$

$$\boxed{b_{al} = 0.198479m} \quad \text{or} \quad \boxed{b_{al} = 198.479mm}$$

Now, we need to calculate the weight per unit length of the aluminum I-beam,

$$\left(\frac{weight}{L}\right)_{al} = \rho_{al} A_{al} = \rho_{al}\left[(t \cdot h) + 2(b \cdot t)\right]_{al}$$

$$\left(\frac{weight}{L}\right)_{al} = 2.78g/cm^3\left[(0.5cm)(20cm) + 2(19.8479cm)(0.5cm)\right]$$

$$\boxed{\left(\frac{weight}{L}\right)_{al} = 82.977g/cm}$$

Even though both beams have the same bending stiffness, the aluminum beam weights much less than the steel one. Therefore, the aluminum I-beam is more efficient weight wise.

<u>Composite I-beam (graphite/epoxy AS4/3501-6):</u>

$$\begin{cases} E = 140x10^9\,Pa \\ \rho = 1.55g/cm^3 \end{cases}$$

Once again, the <u>bending stiffness</u> of the composite I-beam needs to be the same that the steel and aluminum ones. As a result,

$$(EI)_c = (EI)_{st}$$

$$E_c I_c = (EI)_{st}$$

$$\frac{1}{12}(t)(h)^3 + 2\left[\frac{1}{12}(b)(t)^3 + (b\cdot t)\left(\frac{h}{2}+\frac{t}{2}\right)^2\right] = \frac{(EI)_{st}}{E_c}$$

$$b_c = \frac{\frac{(EI)_{st}}{E_c} - \frac{1}{12}(t)(h)^3}{2\left[\frac{1}{12}(t)^3 + (t)\left(\frac{h}{2}+\frac{t}{2}\right)^2\right]}$$

$$b_c = \frac{\frac{1.7175x10^6\,Nm^2}{140x10^9\,Pa} - \frac{1}{12}(0.005m)(0.2m)^3}{2\left[\frac{1}{12}(0.005m)^3 + (0.005m)\left(\frac{0.2m}{2}+\frac{0.005m}{2}\right)^2\right]}$$

$$\boxed{b_c = 0.085023m} \quad \text{or} \quad \boxed{b_c = 85.023mm}$$

Just as before, we need to calculate the weight per unit length of the composite I-beam,

$$\left(\frac{weight}{L}\right)_c = \rho_c A_c = \rho_c\left[(t \cdot h) + 2(b \cdot t)\right]_c$$

$$\left(\frac{weight}{L}\right)_c = 1.55 g/cm^3\left[(0.5cm)(20cm) + 2(8.5023cm)(0.5cm)\right]$$

$$\boxed{\left(\frac{weight}{L}\right)_c = 28.679 g/cm}$$

The composite I-beam weights only 1/3 of the aluminum beam or 1/4 of the steel beam even though all three have the same bending stiffness.

Problem #3:

i) Inertia Forces:

Forces balance at c.g. of the airplane

In x-direction:

$\overset{+}{\rightarrow}\sum Fx = 0$ $\qquad D - I_D = 0$ $\qquad \boxed{I_{DX} = D = 100{,}000 lb}$

Inertia force in x-direction

but $I_{DX} = Ma \cdot Ax$. Where,

"Ma" is the mass of the airplane

$$Ax = \frac{I_{DX}}{Ma} = \frac{I_{DX}}{Wa/g} = \frac{100{,}000 lb}{100{,}000 lb/g} = 1g \qquad g = 32.2 ft/\sec^2$$

In y-direction:

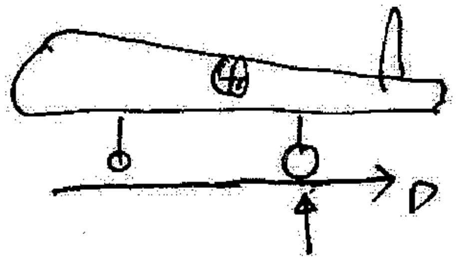
$+\uparrow\sum Fy = 0$ $\qquad 300{,}000 - 100{,}000 = I_{DY}$ $\qquad \boxed{I_{DY} = 200{,}000 lb}$

Inertia force in y-direction

once again, $I_{DY} = Ma \cdot Ay$.

$$Ay = \frac{I_{DY}}{Ma} = \frac{I_{DY}}{Wa/g} = \frac{200{,}000 lb}{100{,}000 lb/g} = 2g$$

$\sum$ of Moment at c.g.



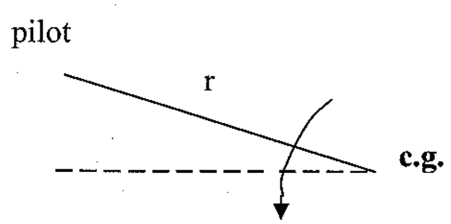
$$\circlearrowright + \sum M = 0$$

$$-100{,}000lb \cdot 120'' - 300{,}000lb \cdot 12'' x7 + I_Y \cdot \alpha = 0$$

$$\text{Inertia moment} = I_Y \cdot \alpha = 37{,}200{,}000lb - in$$

$$\alpha = \frac{37{,}200{,}000lb - in}{40{,}000{,}000lb - \sec^2 - in} = 0.93rad/\sec^2$$

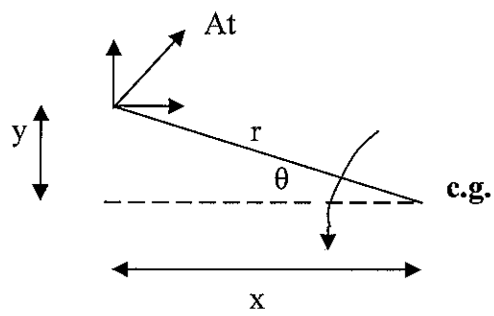
ii) Resultant forces on the pilot:



The forces acting on the pilot consist:

1) Inertia force in x-dir. due to airplane deceleration, Ax
2) Inertia force due to pitching
3) Inertia force in y-dir. due to airplane deceleration, Ay
4) Inertia force in y-dir. due to pilot weight

The Tangential force due to pitching is equal to pilot mass times tangential acceleration.



$$\text{Tangential force} = \text{Mp} \cdot \text{At} = \text{Mp} \cdot \text{r} \cdot \alpha$$

But also,

$$\begin{cases} (Mp \cdot At)Sin\theta = (Mp \cdot r \cdot \alpha)Sin\theta = Mp \cdot \alpha \cdot y \\ (Mp \cdot At)Cos\theta = (Mp \cdot r \cdot \alpha)Cos\theta = Mp \cdot \alpha \cdot x \end{cases}$$

Therefore,

$$I_{PX} = Mp \cdot Ax - Mp \cdot \alpha \cdot y$$

$$I_{PX} = \frac{Wp}{g} \cdot Ax - \frac{Wp \cdot \alpha \cdot y}{g}$$

$$I_{PX} = \frac{180lb}{g} \cdot (1g) - \frac{(180lb)(0.93rad/\sec^2)(40'')}{g} = 180lb - 17lb = \boxed{163lb}$$

And,

$$+\downarrow I_{PY} = Mp \cdot Ay + Wp - Mp \cdot \alpha \cdot x$$

$$I_{PY} = \frac{Wp}{g} \cdot Ay + Wp - \frac{Wp \cdot \alpha \cdot x}{g}$$

$$I_{PY} = \frac{180lb}{g} \cdot (2g) + 180lb - \frac{(180lb)(0.93rad/\sec^2)(372'')}{g} = 360lb + 180lb - 161lb = \boxed{379lb}$$

$$\boxed{\textbf{Resultant force} = \sqrt{(163)^2 + (379)^2} = 410lb}$$

Important Note: Remember to change the units of gravity. Don't use 32.3 ft/sec^2; instead, use 387.6 in/sec^2.