MAE 3315
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SOLUTION #2

SOLUTION HW #2

Problem #1:

Part A) the strain components:

For the normal strains:

$$\varepsilon_x = \frac{\Delta u}{\Delta x} = \frac{x'-x}{\Delta x} = \frac{1.9985m - 2m}{2m} = \frac{-0.0015m}{2m} = \boxed{-750\mu\varepsilon}$$

$$\varepsilon_y = \frac{\Delta v}{\Delta y} = \frac{y'-y}{\Delta y} = \frac{1.4988m - 1.5m}{1.5m} = \frac{-0.0012m}{1.5m} = \boxed{-800\mu\varepsilon}$$

$$\varepsilon_z = \frac{\Delta w}{\Delta z} = \frac{z'-z}{\Delta w} = \frac{1.0009m - 1m}{1m} = \frac{0.0009m}{1m} = \boxed{900\mu\varepsilon}$$

For the shear strains:

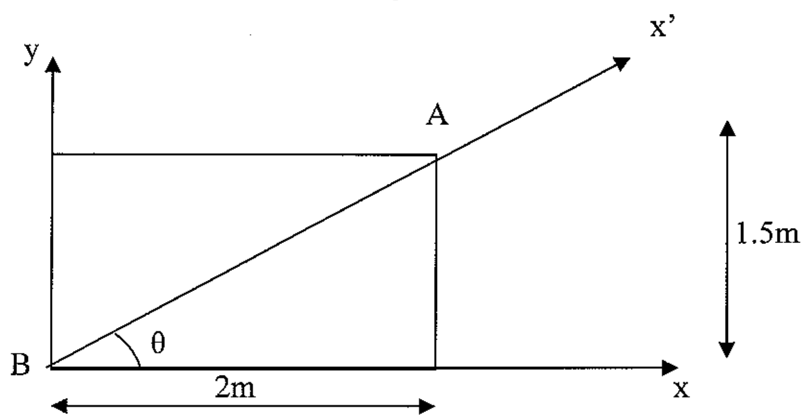
$$\gamma_{xy} = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y} = \frac{y'-y}{\Delta x} + \frac{x'-x}{\Delta y} = \frac{(1.4988m - 1.5m)}{2m} + \frac{(1.9985m - 2m)}{1.5m} = -600\mu\varepsilon - 1000\mu\varepsilon = \boxed{-1600\mu\varepsilon}$$

$$\gamma_{xz} = \frac{\Delta w}{\Delta x} + \frac{\Delta u}{\Delta z} = \frac{z'-z}{\Delta x} + \frac{x'-x}{\Delta z} = \frac{(1.0009m - 1m)}{2m} + \frac{(1.9985m - 2m)}{1m} = 450\mu\varepsilon - 1500\mu\varepsilon = \boxed{-1050\mu\varepsilon}$$

$$\gamma_{yz} = \frac{\Delta w}{\Delta y} + \frac{\Delta v}{\Delta z} = \frac{z'-z}{\Delta y} + \frac{v'-v}{\Delta z} = \frac{(1.0009m - 1m)}{1.5m} + \frac{(1.4988m - 1.5m)}{1m} = 600\mu\varepsilon - 1200\mu\varepsilon = \boxed{-600\mu\varepsilon}$$

Part B) the normal strain in AB direction:

We already know the strain components in the x-y coordinate system. Now, we need to rotate them in order to calculate the normal strain along AB direction.



To calculate the angle,

$$\theta = ArcTan\left(\frac{1.5}{2}\right) = 36.87^\circ$$

Finally,

$$\varepsilon_x' = \varepsilon_x m^2 + \varepsilon_y n^2 + \gamma_{xy} mn$$

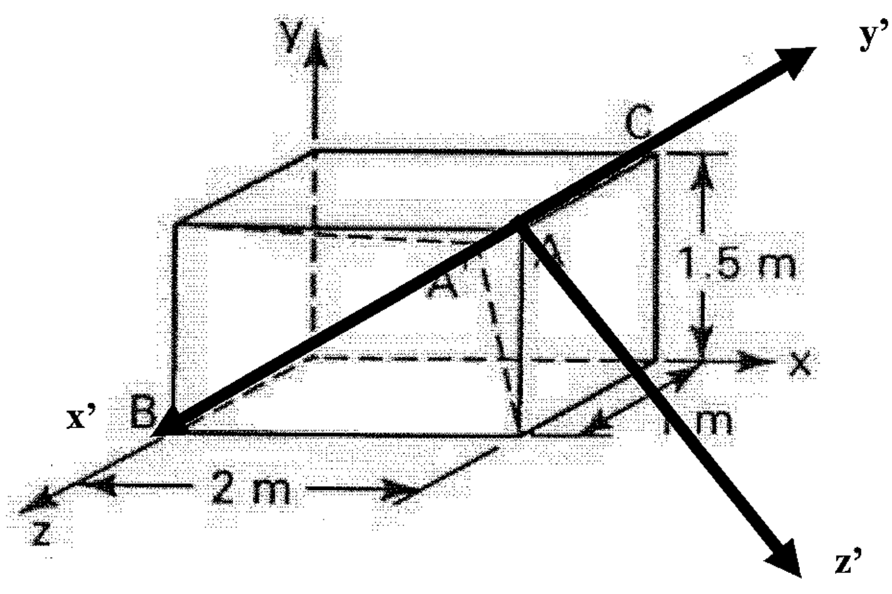
$$\varepsilon_x' = \varepsilon_x Cos^2(\theta) + \varepsilon_y Sin^2(\theta) + \gamma_{xy} Cos(\theta) Sin(\theta)$$

$$\varepsilon_x' = (-750\mu\varepsilon)Cos^2(36.87^\circ) + (-800\mu\varepsilon)Sin^2(36.87^\circ) + (-1600\mu\varepsilon)Cos(36.87^\circ)Sin(36.87^\circ)$$

$$\boxed{\varepsilon_x' = -1536\mu\varepsilon}$$

Part C) the shear strain for perpendicular lines AB and AC:
(Note: This problem is related to 3-D transformation. The 3-D transformation is NOT covered in the class. Hence, you can ignore this problem. The solution is provided for your own interest.)

First, we need to define a new coordinate system,



Note that the direction y' is along the line AC and the direction x' is along the line AB. As you can see, they are 90 degrees from each other. As expected, z' is also perpendicular to x' and y'.

Then, we need to define our direction cosines,

$$\begin{cases} l_1 = Cos(x',x) \\ m_1 = Cos(x',y) \\ n_1 = Cos(x',z) \end{cases}$$

$$\begin{cases} l_2 = Cos(y',x) \\ m_2 = Cos(y',y) \\ n_2 = Cos(y',z) \end{cases}$$

$$\begin{cases} l_3 = Cos(z',x) \\ m_3 = Cos(z',y) \\ n_3 = Cos(z',z) \end{cases}$$

In order to calculate the shear strain in the plane x'-y', we need to rotate the coordinate system as follow,

$$[\varepsilon'] = [T][\varepsilon][T]^T$$

or

$$\begin{bmatrix} \varepsilon_x' & \frac{1}{2}\gamma_{xy}' & \frac{1}{2}\gamma_{xz}' \\ \frac{1}{2}\gamma_{xy}' & \varepsilon_y' & \frac{1}{2}\gamma_{yz}' \\ \frac{1}{2}\gamma_{xz}' & \frac{1}{2}\gamma_{yz}' & \varepsilon_z' \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix} \cdot \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

Please note that the strains matrixes need to be in tensorial form.

From this system, we can get equation (1,2) or (2,1). That is,

$$\gamma_{x'y'} = 2(\varepsilon_x l_1 l_2 + \varepsilon_y m_1 m_2 + \varepsilon_z n_1 n_2) + \gamma_{xy}(l_1 m_2 + m_1 l_2) + \gamma_{yz}(m_1 n_2 + n_1 m_2) + \gamma_{xz}(n_1 l_2 + l_1 n_2)$$

Also, from the figure we get that,

$$\begin{cases} l_1 = Cos(x',x) = Cos(180^\circ + 36.87^\circ) = Cos(216.87^\circ) = -0.80 \\ m_1 = Cos(x',y) = Cos(90^\circ + 36.87^\circ) = Cos(126.87^\circ) = -0.60 \\ n_1 = Cos(x',z) = Cos(90^\circ) = 0 \end{cases}$$

$$\begin{cases} l_2 = Cos(y',x) = Cos(90^\circ) = 0 \\ m_2 = Cos(y',y) = Cos(90^\circ) = 0 \\ n_2 = Cos(y',z) = Cos(180^\circ) = -1 \end{cases}$$

$$\begin{cases} l_3 = Cos(z',x) = Cos(90^\circ - 36.87^\circ) = Cos(53.13^\circ) = 0.60 \\ m_3 = Cos(z',y) = Cos(180^\circ - 36.87^\circ) = Cos(143.13^\circ) = -0.80 \\ n_3 = Cos(z',z) = Cos(90^\circ) = 0 \end{cases}$$

But as we can see, $l_2 = 0$, $m_2 = 0, n_1 = 0$, and $n_3 = 0$. Therefore,

$$\gamma_{x'y'} = \gamma_{yz}(m_1 n_2) + \gamma_{xz}(l_1 n_2)$$

Substituting,

$$\gamma_{x'y'} = (-600\mu\varepsilon)((-0.60)(-1)) + (-1050\mu\varepsilon)((-0.80)(-1))$$

$$\boxed{\gamma_{x'y'} = -1200\mu\varepsilon}$$

Problem #2:

Part A) actual strain and error:

$$\begin{cases} \varepsilon_a = \varepsilon_x m_a^{\ 2} + \varepsilon_y n_a^{\ 2} + \gamma_{xy} m_a n_a \\ \varepsilon_b = \varepsilon_x m_b^{\ 2} + \varepsilon_y n_b^{\ 2} + \gamma_{xy} m_b n_b \\ \varepsilon_c = \varepsilon_x m_c^{\ 2} + \varepsilon_y n_c^{\ 2} + \gamma_{xy} m_c n_c \end{cases}$$

$$\begin{cases} 4500\mu\varepsilon = \varepsilon_x Cos^2(5^\circ) + \varepsilon_y Sin^2(5^\circ) + \gamma_{xy} Cos(5^\circ)Sin(5^\circ) \\ -200\mu\varepsilon = \varepsilon_x Cos^2(50^\circ) + \varepsilon_y Sin^2(50^\circ) + \gamma_{xy} Cos(50^\circ)Sin(50^\circ) \\ 500\mu\varepsilon = \varepsilon_x Cos^2(95^\circ) + \varepsilon_y Sin^2(95^\circ) + \gamma_{xy} Cos(95^\circ)Sin(95^\circ) \end{cases}$$

We have 3 equations and unknowns so can solve them,

$$\boxed{\begin{cases} \varepsilon_x = 4938\mu\varepsilon \\ \varepsilon_y = 62\mu\varepsilon \\ \gamma_{xy} = -4623\mu\varepsilon \end{cases}}$$

To calculate the error, we repeat the same process but with different angles,

$$\begin{cases} 4500\mu\varepsilon = \varepsilon_x Cos^2(0^\circ) + \varepsilon_y Sin^2(0^\circ) + \gamma_{xy} Cos(0^\circ)Sin(0^\circ) \\ -200\mu\varepsilon = \varepsilon_x Cos^2(45^\circ) + \varepsilon_y Sin^2(45^\circ) + \gamma_{xy} Cos(45^\circ)Sin(45^\circ) \\ 500\mu\varepsilon = \varepsilon_x Cos^2(90^\circ) + \varepsilon_y Sin^2(90^\circ) + \gamma_{xy} Cos(90^\circ)Sin(90^\circ) \end{cases}$$

Solving again, we get,

$$\begin{cases} \varepsilon_x = 4500\mu\varepsilon \\ \varepsilon_y = 500\mu\varepsilon \\ \gamma_{xy} = -5400\mu\varepsilon \end{cases}$$

Therefore, the error should be,

$$Error\% - x = \left|\frac{4938 - 4500}{4938}\right| x100\% = \boxed{8.7\%}$$

$$Error\% - y = \left|\frac{62 - 500}{62}\right| x100\% = \boxed{706.5\%}$$

$$Error\% - xy = \left|\frac{-4623 - (-5400)}{-4623}\right| x100\% = \boxed{16.8\%}$$

Part B) reading of the rosette gage at $T = -50^\circ F$:

First, we need to be careful to calculate the drop in temperature from the room temperature (about 70°F) to -50°F. Therefore, $\Delta T = -120^\circ F$.

The important concept in this problem is to realize that thermal strains only affect normal strains. As a result,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{MECHANICAL} + \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{Bmatrix}^{THERMAL}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 4938\mu\varepsilon \\ 62\mu\varepsilon \\ -4623\mu\varepsilon \end{Bmatrix} + \begin{Bmatrix} \alpha \\ \alpha \\ 0 \end{Bmatrix} \Delta T$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 4938\mu\varepsilon \\ 62\mu\varepsilon \\ -4623\mu\varepsilon \end{Bmatrix} + \begin{Bmatrix} 13.0x10^{-6} \\ 13.0x10^{-6} \\ 0 \end{Bmatrix} (-120)$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 4938\mu\varepsilon \\ 62\mu\varepsilon \\ -4623\mu\varepsilon \end{Bmatrix} + \begin{Bmatrix} -1560\mu\varepsilon \\ -1560\mu\varepsilon \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 3378\mu\varepsilon \\ -1498\mu\varepsilon \\ -4623\mu\varepsilon \end{Bmatrix}$$

Now, these are the strain components in x-y coordinate system, to calculate the readings of the rosette gage we need to,

$$\begin{cases} \varepsilon_a = \varepsilon_x Cos^2(5^\circ) + \varepsilon_y Sin^2(5^\circ) + \gamma_{xy} Cos(5^\circ) Sin(5^\circ) \\ \varepsilon_b = \varepsilon_x Cos^2(50^\circ) + \varepsilon_y Sin^2(50^\circ) + \gamma_{xy} Cos(50^\circ) Sin(50^\circ) \\ \varepsilon_c = \varepsilon_x Cos^2(95^\circ) + \varepsilon_y Sin^2(95^\circ) + \gamma_{xy} Cos(95^\circ) Sin(95^\circ) \end{cases}$$

$$\begin{cases} \varepsilon_a = (3378\mu\varepsilon)Cos^2(5^\circ) + (-1498\mu\varepsilon)Sin^2(5^\circ) + (-4623\mu\varepsilon)Cos(5^\circ)Sin(5^\circ) \\ \varepsilon_b = (3378\mu\varepsilon)Cos^2(50^\circ) + (-1498\mu\varepsilon)Sin^2(50^\circ) + (-4623\mu\varepsilon)Cos(50^\circ)Sin(50^\circ) \\ \varepsilon_c = (3378\mu\varepsilon)Cos^2(95^\circ) + (-1498\mu\varepsilon)Sin^2(95^\circ) + (-4623\mu\varepsilon)Cos(95^\circ)Sin(95^\circ) \end{cases}$$

Finally, the readings of the rosette gage will be,

$$\boxed{\begin{cases} \varepsilon_a = 2940\mu\varepsilon \\ \varepsilon_b = -1760\mu\varepsilon \\ \varepsilon_c = -1060\mu\varepsilon \end{cases}}$$

Problem #3:

Part A) normal and shear stress components on the plane ABCD:

As usually, we need the projection of {n} on {x} and {y}.

$$n_x = Cos(\theta) = Cos(60) = \frac{1}{2}$$

$$n_y = Sin(\theta) = Sin(60) = \frac{\sqrt{3}}{2}$$

Therefore, the vector {n} becomes,

$$\{n\} = \begin{Bmatrix} 1/2 \\ \sqrt{3}/2 \\ 0 \end{Bmatrix}$$

As we know, $\{t\} = [\sigma] \cdot \{n\}$,

$$\begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} = \begin{bmatrix} 30 & 20 & -10 \\ 20 & 0 & 10 \\ -10 & 10 & 10 \end{bmatrix} \cdot \begin{Bmatrix} 1/2 \\ \sqrt{3}/2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 32.32 \\ 10 \\ 3.66 \end{Bmatrix}$$

Finally, $\sigma_n = \vec{t} \cdot \vec{n}$,

$$\sigma_n = \vec{t} \cdot \vec{n} = t_x n_x + t_y n_y + t_z n_z = (32.32)\left(\frac{1}{2}\right) + (10)\left(\frac{\sqrt{3}}{2}\right) + (0)(0) = \boxed{24.82 Ksi}$$

Now, to calculate the components of the vector {τ},

$$\tau_x = -Sin(\theta) = -Sin(60) = -\frac{\sqrt{3}}{2} \quad \text{and} \quad \tau_y = Cos(\theta) = Cos(60) = \frac{1}{2}$$

As a result, the vector {τ} becomes,

$$\{\tau\} = \begin{Bmatrix} -\sqrt{3}/2 \\ 1/2 \\ 0 \end{Bmatrix}$$

And then, $S = \vec{t} \cdot \vec{\tau}$,

$$S = \vec{t} \cdot \vec{\tau} = t_x \tau_x + t_y \tau_y + t_z \tau_z = (32.32)\left(-\frac{\sqrt{3}}{2}\right) + (10)\left(\frac{1}{2}\right) + (0)(0) = \boxed{-23 Ksi}$$

Part B) principal stresses and directions:

$$\begin{vmatrix} 30-\sigma & 20 & -10 \\ 20 & 0-\sigma & 10 \\ -10 & 10 & 10-\sigma \end{vmatrix} = 0$$

$$[(30-\sigma)(-\sigma)(10-\sigma) - 2000 - 2000] - [(-100\sigma) + (100(30-\sigma)) + (400(10-\sigma))] = 0$$

$$-\sigma^3 + 40\sigma^2 + 300\sigma - 11000 = 0$$

Therefore, the principal stresses are:

$$\boxed{\begin{cases} \sigma_1 = 40.74 Ksi \\ \sigma_2 = 16.07 Ksi \\ \sigma_3 = -16.80 Ksi \end{cases}}$$

For $\sigma_1 = 40.74 Ksi$:

$$\begin{bmatrix} 30-40.74 & 20 & -10 \\ 20 & 0-40.74 & 10 \\ -10 & 10 & 10-40.74 \end{bmatrix} \cdot \begin{Bmatrix} n_x^{(1)} \\ n_y^{(1)} \\ n_z^{(1)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} -10.74 n_x^{(1)} + 20 n_y^{(1)} - 10 n_z^{(1)} = 0 \\ 20 n_x^{(1)} - 40.74 n_y^{(1)} + 10 n_z^{(1)} = 0 \\ -10 n_x^{(1)} + 10 n_y^{(1)} - 34.74 n_z^{(1)} = 0 \end{cases}$$

From these 3 equations only 2 are independent to each other. Therefore, I pick the 2nd and the 3rd equations from the system and I need 1 more equation, $(n_x^{(1)})^2 + (n_y^{(1)})^2 + (n_z^{(1)})^2 = 1$. From here, I get,

$$\boxed{\begin{Bmatrix} n_x^{(1)} \\ n_y^{(1)} \\ n_z^{(1)} \end{Bmatrix} = \begin{Bmatrix} 0.90 \\ 0.40 \\ -0.16 \end{Bmatrix}}$$

For $\sigma_2 = 16.07 Ksi$:

$$\begin{bmatrix} 30-16.07 & 20 & -10 \\ 20 & 0-16.07 & 10 \\ -10 & 10 & 10-16.07 \end{bmatrix} \cdot \begin{Bmatrix} n_x^{(2)} \\ n_y^{(2)} \\ n_z^{(2)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} -13.93 n_x^{(2)} + 20 n_y^{(2)} - 10 n_z^{(2)} = 0 \\ 20 n_x^{(2)} - 16.07 n_y^{(2)} + 10 n_z^{(2)} = 0 \\ -10 n_x^{(2)} + 10 n_y^{(2)} - 6.07 n_z^{(2)} = 0 \end{cases}$$

Again I need to use only 2 equations from the system. Picking the 2nd and 3rd equations and this additional equation, $\left(n_x^{(2)}\right)^2 + \left(n_y^{(2)}\right)^2 + \left(n_z^{(2)}\right)^2 = 1$, I get,

$$\boxed{\begin{Bmatrix} n_x^{(2)} \\ n_y^{(2)} \\ n_z^{(2)} \end{Bmatrix} = \begin{Bmatrix} -0.05 \\ 0.48 \\ 0.88 \end{Bmatrix}}$$

For $\sigma_3 = -16.80 Ksi$:

$$\begin{bmatrix} 30+16.80 & 20 & -10 \\ 20 & 0+16.80 & 10 \\ -10 & 10 & 10+16.80 \end{bmatrix} \cdot \begin{Bmatrix} n_x^{(3)} \\ n_y^{(3)} \\ n_z^{(3)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} 46.80 n_x^{(3)} + 20 n_y^{(3)} - 10 n_z^{(3)} = 0 \\ 20 n_x^{(3)} + 16.80 n_y^{(3)} + 10 n_z^{(3)} = 0 \\ -10 n_x^{(3)} + 10 n_y^{(3)} + 26.80 n_z^{(3)} = 0 \end{cases}$$

Picking the 2nd and 3rd equations and $\left(n_x^{(3)}\right)^2 + \left(n_y^{(3)}\right)^2 + \left(n_z^{(3)}\right)^2 = 1$, I get,

$$\boxed{\begin{Bmatrix} n_x^{(3)} \\ n_y^{(3)} \\ n_z^{(3)} \end{Bmatrix} = \begin{Bmatrix} 0.43 \\ -0.78 \\ 0.45 \end{Bmatrix}}$$

Part C) corresponding strains:

From Hook's Law,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \cdot \begin{Bmatrix} 30{,}000\,psi \\ 0\,psi \\ 10{,}000\,psi \\ 20{,}000\,psi \\ -10{,}000\,psi \\ 10{,}000\,psi \end{Bmatrix}$$

where, $\nu = 0.3$, $E = 15x10^6\ psi$, and $G = \frac{E}{2(1+\nu)}$.

$$\boxed{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} 1{,}800\,\mu\varepsilon \\ -800\,\mu\varepsilon \\ 67\,\mu\varepsilon \\ 3{,}467\,\mu\varepsilon \\ -1{,}733\,\mu\varepsilon \\ 1{,}733\,\mu\varepsilon \end{Bmatrix}}$$

Part D) Elastic Strain Energy:

$$\Delta U_o = \frac{1}{2}\left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right)$$

$$\Delta U_o = \frac{1}{2}\left((30{,}000)(0.0018) + (10{,}000)(0.000067) + (20{,}000)(0.003467) + (-10{,}000)(-0.001733) + (10{,}000)(0.001733)\right)$$

$$\boxed{\Delta U_o = 79.33\text{in}\cdot\frac{\text{lb}}{\text{in}^3}}$$

Problem #4:

Part A) find σ_n and S:

The vector {n} is given,

$$\{n\} = \begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix}$$

As we know, $\{t\} = [\sigma] \cdot \{n\}$,

$$\begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} = \begin{bmatrix} 10 & 3 & 5 \\ 3 & 1 & 2 \\ 5 & 2 & 4 \end{bmatrix} \cdot \begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} 18/\sqrt{3} \\ 6/\sqrt{3} \\ 11/\sqrt{3} \end{Bmatrix}$$

Finally, $\sigma_n = \vec{t} \cdot \vec{n}$,

$$\sigma_n = \vec{t} \cdot \vec{n} = t_x n_x + t_y n_y + t_z n_z = \left(\frac{18}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{6}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{11}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) = \boxed{11.67 Ksi}$$

To calculate S, we first need to calculate the magnitude of the stress vector {t},

$$\|t\| = \sqrt{t_x^2 + t_y^2 + t_z^2} = \sqrt{\left(\frac{18}{\sqrt{3}}\right)^2 + \left(\frac{6}{\sqrt{3}}\right)^2 + \left(\frac{11}{\sqrt{3}}\right)^2} = 12.66 Ksi$$

Now, to calculate S, we just

$$\|\sigma_n\|^2 + \|S\|^2 = \|t\|^2$$

$$\|S\| = \sqrt{\|t\|^2 - \|\sigma_n\|^2}$$

$$\|S\| = \sqrt{(12.66)^2 - (11.67)^2}$$

$$\boxed{\|S\| = 4.91 Ksi}$$

Part B) direction cosines of {n} and {τ} :

As before, {n} is given,

$$\vec{n} = \begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix}$$

Those are the direction cosines for {n}. Now for {τ},

$$\vec{t} = \sigma_n \vec{n} + S\vec{\tau}$$

$$\vec{\tau} = \frac{\vec{t} - \sigma_n \vec{n}}{S}$$

$$\vec{\tau} = \frac{\begin{Bmatrix} 18/\sqrt{3} \\ 6/\sqrt{3} \\ 11/\sqrt{3} \end{Bmatrix} - (11.67)\begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix}}{4.91}$$

$$\vec{\tau} = \begin{Bmatrix} 0.7443 \\ -0.6667 \\ -0.0788 \end{Bmatrix}$$