MAE3315
Homework No. 3 (SOLUTION)

Problem 3-1

A rectangular rosette gage is mounted at the lower skin along the mid axis of the wing section. The gage readings are 4500με along the elastic axis, 500με in the transverse to elastic axis, and -200με along the other gage direction. The test engineer later found that the gage was mounted a 10 degrees off the reference axial. The Young's modulus and coefficient of thermal expansion of the skin material is given as $E=10.5\times10^6$psi, ν=0.3, and $\alpha=13.0\times10^{-6}$in/in/°F.

a) What is the actual strain? Calculate the error of this measurement?
b) If the airplane is in the flight test where the temperature is at -50°F, answer problem a).

Solution:

a) actual strain and error:

$$\begin{cases} \varepsilon_a = \varepsilon_x m_a^2 + \varepsilon_y n_a^2 + \gamma_{xy} m_a n_a \\ \varepsilon_b = \varepsilon_x m_b^2 + \varepsilon_y n_b^2 + \gamma_{xy} m_b n_b \\ \varepsilon_c = \varepsilon_x m_c^2 + \varepsilon_y n_c^2 + \gamma_{xy} m_c n_c \end{cases}$$

$$\begin{cases} 4500\mu\varepsilon = \varepsilon_x Cos^2(10^\circ) + \varepsilon_y Sin^2(10^\circ) + \gamma_{xy} Cos(10^\circ) Sin(10^\circ) \\ -200\mu\varepsilon = \varepsilon_x Cos^2(55^\circ) + \varepsilon_y Sin^2(55^\circ) + \gamma_{xy} Cos(55^\circ) Sin(55^\circ) \\ 500\mu\varepsilon = \varepsilon_x Cos^2(100^\circ) + \varepsilon_y Sin^2(100^\circ) + \gamma_{xy} Cos(100^\circ) Sin(100^\circ) \end{cases}$$

We have 3 equations and unknowns so can solve them,

$$\boxed{\begin{cases} \varepsilon_x = 5303\mu\varepsilon \\ \varepsilon_y = -303\mu\varepsilon \\ \gamma_{xy} = -3706\mu\varepsilon \end{cases}}$$

To calculate the error, we repeat the same process but with different angles,

$$\begin{cases} 4500\mu\varepsilon = \varepsilon_x Cos^2(0^\circ) + \varepsilon_y Sin^2(0^\circ) + \gamma_{xy} Cos(0^\circ) Sin(0^\circ) \\ -200\mu\varepsilon = \varepsilon_x Cos^2(45^\circ) + \varepsilon_y Sin^2(45^\circ) + \gamma_{xy} Cos(45^\circ) Sin(45^\circ) \\ 500\mu\varepsilon = \varepsilon_x Cos^2(90^\circ) + \varepsilon_y Sin^2(90^\circ) + \gamma_{xy} Cos(90^\circ) Sin(90^\circ) \end{cases}$$

Solving again, we get,

$$\begin{cases} \varepsilon_x = 4500\mu\varepsilon \\ \varepsilon_y = 500\mu\varepsilon \\ \gamma_{xy} = -5400\mu\varepsilon \end{cases}$$

Therefore, the error should be,

$$Error\% - x = \left|\frac{5303 - 4500}{5303}\right| x100\% = \boxed{15.1\%}$$

$$Error\% - y = \left|\frac{-303 - 500}{-303}\right| x100\% = \boxed{265.1\%}$$

$$Error\% - xy = \left|\frac{-3706 - (-5400)}{-3706}\right| x100\% = \boxed{45.7\%}$$

Part B) reading of the rosette gage at $T = -50^\circ F$:

First, we need to be careful to calculate the drop in temperature from the room temperature (about 70°F) to -50°F. Therefore, $\Delta T = -120^\circ F$.

The important concept in this problem is to realize that thermal strains only affect normal strains. As a result,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{MECHANICAL} + \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{Bmatrix}^{THERMAL}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 5303\mu\varepsilon \\ -303\mu\varepsilon \\ -3706\mu\varepsilon \end{Bmatrix} + \begin{Bmatrix} \alpha \\ \alpha \\ 0 \end{Bmatrix} \Delta T$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 5303\mu\varepsilon \\ -303\mu\varepsilon \\ -3706\mu\varepsilon \end{Bmatrix} + \begin{Bmatrix} 13.0x10^{-6} \\ 13.0x10^{-6} \\ 0 \end{Bmatrix} (-120)$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 5303\mu\varepsilon \\ -303\mu\varepsilon \\ -3706\mu\varepsilon \end{Bmatrix} + \begin{Bmatrix} -1560\mu\varepsilon \\ -1560\mu\varepsilon \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{TOTAL} = \begin{Bmatrix} 3743\mu\varepsilon \\ -1863\mu\varepsilon \\ -3706\mu\varepsilon \end{Bmatrix}$$

Now, these are the strain components in x-y coordinate system, to calculate the readings of the rosette gage we need to,

$$\begin{cases} \varepsilon_a = \varepsilon_x Cos^2(10^\circ) + \varepsilon_y Sin^2(10^\circ) + \gamma_{xy} Cos(10^\circ)Sin(10^\circ) \\ \varepsilon_b = \varepsilon_x Cos^2(55^\circ) + \varepsilon_y Sin^2(55^\circ) + \gamma_{xy} Cos(55^\circ)Sin(55^\circ) \\ \varepsilon_c = \varepsilon_x Cos^2(100^\circ) + \varepsilon_y Sin^2(100^\circ) + \gamma_{xy} Cos(100^\circ)Sin(100^\circ) \end{cases}$$

$$\begin{cases} \varepsilon_a = (3743\mu\varepsilon)Cos^2(10^\circ) + (-1863\mu\varepsilon)Sin^2(10^\circ) + (-3706\mu\varepsilon)Cos(10^\circ)Sin(10^\circ) \\ \varepsilon_b = (3743\mu\varepsilon)Cos^2(55^\circ) + (-1863\mu\varepsilon)Sin^2(55^\circ) + (-3706\mu\varepsilon)Cos(55^\circ)Sin(55^\circ) \\ \varepsilon_c = (3743\mu\varepsilon)Cos^2(100^\circ) + (-1863\mu\varepsilon)Sin^2(100^\circ) + (-3706\mu\varepsilon)Cos(100^\circ)Sin(100^\circ) \end{cases}$$

Finally, the readings of the rosette gage will be,

$$\boxed{\begin{cases} \varepsilon_a = 2940\mu\varepsilon \\ \varepsilon_b = -1760\mu\varepsilon \\ \varepsilon_c = -1060\mu\varepsilon \end{cases}}$$

And,

$$\begin{cases} \varepsilon_a = \varepsilon_x Cos^2(0^\circ) + \varepsilon_y Sin^2(0^\circ) + \gamma_{xy} Cos(0^\circ)Sin(0^\circ) \\ \varepsilon_b = \varepsilon_x Cos^2(45^\circ) + \varepsilon_y Sin^2(45^\circ) + \gamma_{xy} Cos(45^\circ)Sin(45^\circ) \\ \varepsilon_c = \varepsilon_x Cos^2(90^\circ) + \varepsilon_y Sin^2(90^\circ) + \gamma_{xy} Cos(90^\circ)Sin(90^\circ) \end{cases}$$

$$\begin{cases} \varepsilon_a = (3743\mu\varepsilon)Cos^2(0^\circ) + (-1863\mu\varepsilon)Sin^2(0^\circ) + (-3706\mu\varepsilon)Cos(0^\circ)Sin(0^\circ) \\ \varepsilon_b = (3743\mu\varepsilon)Cos^2(45^\circ) + (-1863\mu\varepsilon)Sin^2(45^\circ) + (-3706\mu\varepsilon)Cos(45^\circ)Sin(45^\circ) \\ \varepsilon_c = (3743\mu\varepsilon)Cos^2(90^\circ) + (-1863\mu\varepsilon)Sin^2(90^\circ) + (-3706\mu\varepsilon)Cos(90^\circ)Sin(90^\circ) \end{cases}$$

$$\begin{cases} \varepsilon_a = 3743\mu\varepsilon \\ \varepsilon_b = -913\mu\varepsilon \\ \varepsilon_c = -1863\mu\varepsilon \end{cases}$$

$$Error\% - a = \left|\frac{2940 - 3743}{2940}\right| x100\% = \boxed{27.3\%}$$

$$Error\% - b = \left|\frac{-1760 - (-913)}{-1760}\right| x100\% = \boxed{48.1\%}$$

$$Error\% - c = \left|\frac{-1060 - (-1863)}{-1060}\right| x100\% = \boxed{75.8\%}$$

Problem 3-2

A copper tube has 10mm of the wall thickness and 60mm of the radius. The tube is fixed at both ends. If at 20°C no axial force Px exists in the tube, what will Px be when the temperature rises to 120°C? Let E=120GPa, and $\alpha=16.8\text{x}10^{-8}\varepsilon/°C$.

Solution:

The total strain is just the summation of the mechanical and thermal strains together,

$$\varepsilon_x^{TOTAL} = \varepsilon_x^{MECHANICAL} + \varepsilon_x^{THERMAL}$$

The total strain for this problem in the x-direction is zero (because it's constrained in that direction). Therefore,

$$0 = \varepsilon_x^{MECHANICAL} + \varepsilon_x^{THERMAL}$$

$$\varepsilon_x^{MECHANICAL} = -\varepsilon_x^{THERMAL}$$

$$\varepsilon_x^{MECHANICAL} = -\alpha\Delta T = -\left(16.8x10^{-8}\varepsilon°C/\right)\left(100°C\right) = -16.8\mu\varepsilon$$

Finally,

$$\sigma_x = E\varepsilon_x^{MECHANICAL}$$

$$\sigma_x = \left(120x10^9 Pa\right)\left(-16.8\mu\varepsilon\right) = -2.016MPa$$

Now,

$$P_x = \sigma_x A = \left(\pi Dt\right) = \sigma_x\left(\pi 2Rt\right) = \left(-2.016MPa\right)\left(2\pi\left(0.06m\right)\left(0.01m\right)\right) = \boxed{-7600N}$$

Note: There was a typo in the problem statement. The thermal expansion coefficient for copper is $\alpha=16.8\text{x}10^{-6}\varepsilon/°C$.