MAE 3315
Dr. Chan
SOLUTION HW #5

Problem #1:

Part A:

Assume the corner of the thin-walled section is the origin, the distance between centroid and the original y and z are $\bar{y}_c$ and $\bar{z}_c$, respectively.

$$\bar{y}_c = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(h./2)(ht)+(0)(ht)}{ht+ht} = \frac{h}{4}$$

$$\bar{z}_c = \frac{\sum \bar{z}_i A_i}{\sum A_i} = \frac{(0)(ht)+(h/2)(ht)}{ht+ht} = \frac{h}{4}$$

To calculate the inertia of the cross-section, we need to,

we can neglect this term because $t << h$

$$I_y = \frac{(h)(t)^3}{12} + (ht)(h/4)^2 + \frac{(t)(h)^3}{12} + (ht)(h/4)^2 = \frac{h^3t}{16} + \frac{h^3t}{12} + \frac{h^3t}{16} = \boxed{\frac{5}{24}h^3t}$$

≈ 0

$$I_z = \frac{(t)(h)^3}{12} + (ht)(h/4)^2 + \frac{(h)(t)^3}{12} + (ht)(h/4)^2 = \frac{h^3t}{12} + \frac{h^3t}{16} + \frac{h^3t}{16} = \boxed{\frac{5}{24}h^3t}$$

$$I_{yz} = \int yzdA = \int_{A1} yzdA + \int_{A2} yzdA = -\frac{h^3t}{16} + -\frac{h^3t}{16} = \boxed{-\frac{1}{8}h^3t}$$

where,

$$\int_{A1} yzdA = \int_{-z_c}^{h-z_c} y_c ztdz = y_c t\left(\frac{z^2}{2}\right)\Bigg|_{-z_c}^{h-z_c} = y_c t\left(\frac{z^2}{2}\right)\Bigg|_{-h/4}^{3h/4} =$$

$$= -\frac{h}{4}t\left(\frac{(3h/4)^2}{2} - \frac{(-h/4)^2}{2}\right) = -\frac{h}{4}t\left(\frac{9h^2}{32} - \frac{h^2}{32}\right) = -\frac{h^3t}{16}$$

$$\int_{A2} yz\,dA = \int_{-y_c}^{h-y_c} z_c y t\,dy = z_c t\left(\frac{y^2}{2}\right)\Bigg|_{-y_c}^{h-y_c} = z_c t\left(\frac{y^2}{2}\right)\Bigg|_{-h/4}^{3h/4} =$$

$$= -\frac{h}{4}t\left(\frac{(3h/4)^2}{2} - \frac{(-h/4)^2}{2}\right) = -\frac{h}{4}t\left(\frac{9h^2}{32} - \frac{h^2}{32}\right) = -\frac{h^3 t}{16}$$

IMPORTANT NOTE: Please, realize this is not the only way to calculate I_{yz}. This is only to illustrate the way to calculate it through integration. However, the best way to calculate I_{yz} is the following,

$$I_{yz} = \sum_i A_i y_i z_i = (ht)\left(-\frac{h}{4}\right)\left(\frac{h}{4}\right) + (ht)\left(\frac{h}{4}\right)\left(-\frac{h}{4}\right) = \left(-\frac{h^3 t}{16}\right) + \left(-\frac{h^3 t}{16}\right) = \boxed{-\frac{1}{8}h^3 t}$$

Finally, to calculate the neutral axis,

$$\tan\alpha = \frac{I_y M_z - I_{yz} M_y}{I_z M_y - I_{yz} M_z}$$

However, $M_z = 0$.

$$\tan\alpha = \frac{-I_{yz} M_y}{I_z M_y}$$

As a result, the M_y's cancel each other.

$$\tan\alpha = \frac{-I_{yz}}{I_z} = \frac{-\left(-1/8\left(h^3 t\right)\right)}{5/24\left(h^3 t\right)} = \frac{24}{40} = \frac{3}{5}$$

And,

$$\alpha = ArcTan(3/5) = \boxed{31^\circ} \qquad \text{[CLOCKWISE]}$$

NOTE: The way the book define positive α is clockwise; therefore, we should be careful with that.

In addition, the stress distribution can be calculated using the following equation all over the cross-section.

$$\sigma_{xx} = \frac{I_y M_z - I_{yz} M_y}{I_y I_z - I_{yz}^2} y + \frac{I_z M_y - I_{yz} M_z}{I_y I_z - I_{yz}^2} z$$

Again, $M_z = 0$. Therefore,

$$\sigma_{xx} = \frac{-I_{yz} M_y}{I_y I_z - I_{yz}^2} y + \frac{I_z M_y}{I_y I_z - I_{yz}^2} z$$

Part B:

Now, we need to rotate the cross-section 45° counterclockwise.

$$\begin{cases} I_{y'} = I_y Cos^2(\theta) + I_z Sin^2(\theta) + 2I_{yz} Sin(\theta)Cos(\theta) \\ I_{z'} = I_y Sin^2(\theta) + I_z Cos^2(\theta) - 2I_{yz} Sin(\theta)Cos(\theta) \\ I_{y'z'} = I_y Sin(\theta)Cos(\theta) - I_z Sin(\theta)Cos(\theta) + I_{yz}\left[Cos^2(\theta) - Sin^2(\theta)\right] \end{cases}$$

Substituting,

$$I_{y'} = I_y Cos^2(45) + I_z Sin^2(45) + 2I_{yz} Sin(45)Cos(45)$$

$$I_{y'} = \left(\frac{5}{24}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{5}{24}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(-\frac{1}{8}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\boxed{I_{y'} = \frac{1}{12}h^3 t}$$

$$I_{z'} = I_y Sin^2(45) + I_z Cos^2(45) - 2I_{yz} Sin(45)Cos(45)$$

$$I_{z'} = \left(\frac{5}{24}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{5}{24}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2 - 2\left(-\frac{1}{8}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\boxed{I_{z'} = \frac{1}{3}h^3 t}$$

$$I_{y'z'} = I_y Sin(45)Cos(45) - I_z Sin(45)Cos(45) + I_{yz}\left[Cos^2(45) - Sin^2(45)\right]$$

$$I_{y'z'} = \left(\frac{5}{24}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{5}{24}h^3 t\right)\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\boxed{I_{y'z'} = 0}$$

Problem #2:

Part A:

Assume lower-left stringer is the origin; we can obtain the centroid as,

$$\bar{y}_c = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(2)(200)+(1)(200)}{4+2+1+4} = 54.5455cm$$

$$\bar{z}_c = \frac{\sum \bar{z}_i A_i}{\sum A_i} = \frac{(1)(50)+(4)(100)}{4+2+1+4} = 40.9091cm$$

To calculate the moment of inertia,

$$I_y = \sum_i A_i z_i^2 = (4)(40.9091)^2 + (2)(40.9091)^2 + (1)(50-40.9091)^2 + \\ +(4)(100-40.9091)^2 = 24090.909cm^4$$

$$I_z = \sum_i A_i y_i^2 = (4)(54.5455)^2 + (2)(200-54.5455)^2 + (1)(200-54.5455)^2 + \\ +(4)(54.5455)^2 = 87272.727cm^4$$

$$I_{yz} = \sum_i A_i y_i z_i = (4)(-54.5455)(-40.9091) + (2)(200-54.5455)(-40.9091) + \\ +(1)(200-54.5455)(50-40.9091) + (4)(54.5455)(-100+40.9091) = -14545.455cm^4$$

Now to calculate the bending stress in the stringers,

$$\sigma_{xx} = \frac{I_y M_z - I_{yz} M_y}{I_y I_z - I_{yz}^2} y + \frac{I_z M_y - I_{yz} M_z}{I_y I_z - I_{yz}^2} z$$

$$\sigma_{xx} = \frac{(24090.909)(200{,}000) - (-14545.455)(-500{,}000)}{(24090.909)(87272.727) - (-14545.455)^2} y + \\ + \frac{(87272.727)(-500{,}000) - (-14545.455)(200{,}000)}{(24090.909)(87272.727) - (-14545.455)^2} z$$

$$\sigma_{xx} = -1.298077y - 21.538461z$$

Therefore, the bending stress in each stringer is:

y_i (cm)	z_i (cm)	σ_{xx} (N/cm^2)
-54.5455	-40.9091	951.923
145.4545	-40.9091	692.308
145.4545	9.09091	-384.615
-54.5455	59.09091	-1201.923

Part B:

To calculate the neutral axis,

$$\tan\alpha = \frac{I_y M_z - I_{yz} M_y}{I_z M_y - I_{yz} M_z}$$

$$\tan\alpha = \frac{(24090.909)(200{,}000) - (-14545.455)(-500{,}000)}{(87272.727)(-500{,}000) - (-14545.455)(200{,}000)}$$

$$\alpha = ArcTan(0.0602679)$$

$$\alpha = \boxed{3.45^\circ} \quad \text{[CLOCKWISE]}$$

Problem #3:

As usually, first we need to find the centroid. Let's assume lower-left string is the origin.

$$\bar{y}_c = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(80)(4)+(0)(4)}{4+4+4} = 26.6667cm$$

$$\bar{z}_c = \frac{\sum \bar{z}_i A_i}{\sum A_i} = \frac{(0)(4)+(40)(4)}{4+4+4} = 13.3333cm$$

To calculate the moments of bending,

$$M_y = -2PL = -2(200)(500) = -200{,}000Ncm$$

$$M_z = -2PL = -2(200)(500) = -200{,}000Ncm$$

Remember that M_y is positive on positive "y" direction. Therefore, in this problem, M_y should be negative because is pointing in the negative "y" direction. On the other hand, M_z is positive on negative "z" direction. Therefore, for this problem M_z should be negative because is pointing in the positive "z" direction.

Then, we can calculate the moment of inertia,

$$I_y = \sum_i A_i z_i^2 = (4)(13.3333)^2 + (4)(13.3333)^2 + (4)(40-13.3333)^2 = 4266.6667cm^4$$

$$I_z = \sum_i A_i y_i^2 = (4)(26.66667)^2 + (4)(80-26.66667)^2 + (4)(26.66667)^2 = 17066.66667cm^4$$

$$I_{yz} = \sum_i A_i y_i z_i = (4)(-26.66667)(-13.3333) + (4)(-80+26.66667)(13.3333) + $$
$$+ (4)(-26.66667)(40-13.3333) = -4266.6667cm^4$$

The bending stresses in the stringers are,

$$\sigma_{xx} = \frac{I_y M_z - I_{yz} M_y}{I_y I_z - I_{yz}^2} y + \frac{I_z M_y - I_{yz} M_z}{I_y I_z - I_{yz}^2} z$$

$$\sigma_{xx} = \frac{(4266.6667)(-200{,}000)-(-4266.6667)(-200{,}000)}{(4266.6667)(17066.66667)-(-4266.6667)^2}y + $$
$$+\frac{(17066.66667)(-200{,}000)-(-4266.6667)(-200{,}000)}{(4266.6667)(17066.66667)-(-4266.6667)^2}z$$

$$\sigma_{xx} = -31.2500y - 78.1250z$$

Therefore, the bending stress in each stringer is,

y_i (cm)	z_i (cm)	σ_{xx} (N/cm^2)
-26.6667	26.6667	-1250.00
-26.6667	-13.3333	1875.00
53.3333	-13.3333	-625.00

Finally, to calculate the neutral axis,

$$\tan\alpha = \frac{I_y M_z - I_{yz} M_y}{I_z M_y - I_{yz} M_z}$$

$$\tan\alpha = \frac{(4266.6667)(-200{,}000)-(-4266.6667)(-200{,}000)}{(17066.66667)(-200{,}000)-(-4266.6667)(-200{,}000)}$$

$$\alpha = ArcTan(0.4)$$

$$\alpha = \boxed{21.8^\circ} \qquad \text{[CLOCKWISE]}$$

Problem # 4:

$I_{yy} = \sum_i A_i z_i^2$

Stresses on each stringer $\sigma = \frac{M_y z_i}{I_{yy}}$

$\bar{z}_c = \frac{\sum A_i \bar{z}_i}{\sum A_i}$

(Area) x (Number of Stringers)

z-position with respect to the centroid

#	z_{ori}-position (mm)	Number of Stringers	Area (mm^2)	Total Area (mm^2)	$A_i * z_{ori} / A_{total}$	z-position (mm)	I_{yy} (mm^4)	σ (Pa)
1	1200	1	640	640	73.9884	661.5414	280087717	35,669,939
2	1140	2	600	1200	131.7919	601.5414	434222504	32,434,773
3	960	2	600	1200	110.9827	421.5414	213236608	22,729,275
4	768	2	600	1200	88.7861	229.5414	63227119	12,376,744
5	565	2	620	1240	67.4952	26.5414	873515	1,431,099
6	336	2	640	1280	41.4335	-202.4586	52466527	-10,916,452
7	144	2	640	1280	17.7572	-394.4586	199164885	-21,268,983
8	38	2	850	1700	6.2235	-500.4586	425779934	-26,984,443
9	0	1	640	640	0.0000	-538.4586	185560087	-29,033,381

A_{total} = 10380 Z_{bar} = 538.4586

I_{yy} = 1854618897 mm^4

I_{yy} = 0.00185462 m^4

Summation of all the Total Areas

This is $\bar{z}$

Total I_{yy}